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IDENTIFIERS *Comprehensive School Mathematics Program

ABSTRACT

This guide represents the final experimental version of an extended pilot project which was conducted in the United States between 1973 and 1976. The manner of presentation and the pedagogical ideas and tools are based on the works of Georges and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. The CSMP curriculum follows the "spiral approach." The text begins with exercises in the Language of Strings and Arrows. These are intended to teach the skills of classification and provide a language for studying and talking about relationships. The section entitled Geometry and Measurement emphasizes "experience" rather than "mastery." Activities deal with distance and measurement in an unsophisticated sense. Five workbooks are included with problems of varying levels of difficulty all in one booklet. The first ten problems of each booklet are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. The students have the opportunity to work individually with the workbook sections. (Author/SA)

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Final Experimental Version
2-33601

CSMP Mathematics for the Upper Primary Grades
Part II

Teacher's Guide

The Languages of Strings and Arrows
Geometry and Measurement
Workbooks

U.S. DEPARTMENT OF HEALTH,
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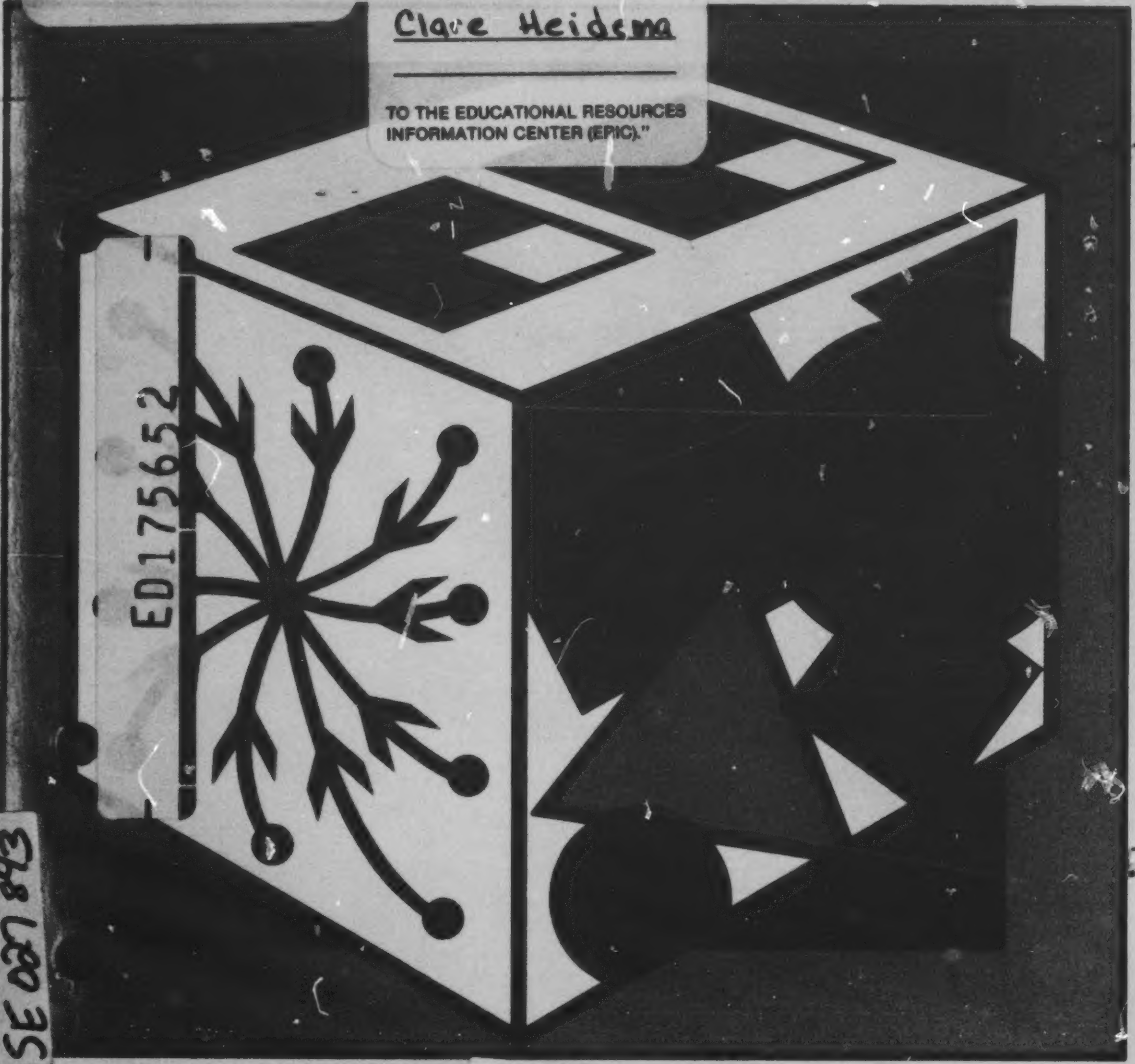

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Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades
Part II

The Languages of Strings and Arrows

CEMREL, Inc.

TABLE OF CONTENTS

INTRODUCTION v

Classification: The Language of Strings v
Relations: The Language of Arrows viii

L-LESSONS

L1 Trees #1 1
Worksheet L1 5
L2 The Empty Set 6
L3 A Valentine Story 13
L4 The String Game #1 23
L5 Trees #2 26
Worksheets L5* and ** 31
L6 Temperature Bar Graphs 32
L7 Composition Games 36
L8 Telling Stories with Arrows 44
L9 Multiples #1 50
L10 Trees #3 57
Worksheet L10 62
L11 The String Game #2 63
L12 Two Dice #1 69
L13 Two Lists 77
L14 Two Dice #2 85
Worksheet L14 92
L15 The String Game #3 93
L16 Multiples #2 95
L17 Telling Stories with Strings 104

APPENDIX: THE STRING GAME 113
Equipment 113
Preparation for the Game; Object and Rules of the Game ... 116
Version A: The String Game with A-Blocks 117
Version B: The String Game with A-Blocks, using "Not" Cards 120

THE LANGUAGES OF STRINGS AND ARROWS

INTRODUCTION

The language of strings is the nonverbal language of sets, that is, collections of objects put together in some way. The language of arrows is the nonverbal language of relations among objects. These languages permeate the CSMP content strands and are used, separately or together, to present an unending variety of numerical and nonnumerical situations.

Classification: The Language of Strings

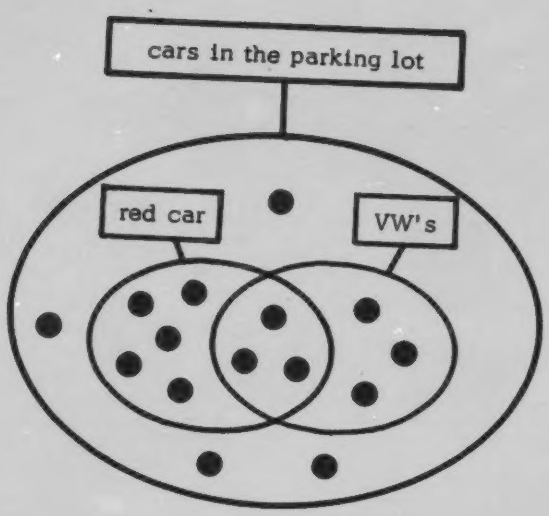
As the word implies, classifying means putting things into classes, or as the mathematician says: sets. If the mathematics of sets has any place in an elementary school mathematics curriculum, it is precisely because it can help students to understand and use the ideas of classification.

The basic idea is simple: given a set S and any object x, either x belongs to S (x is in S) or x does not belong to S (x is not in S). We represent this simple act of sorting -- "in" or "out" -- neatly by using string pictures to illustrate "in" and "out" in a dramatic way. Objects to be sorted are represented by dots, and the sets into which they are sorted are represented by drawing "strings" around dots. A dot inside (the region delineated by) a set's string is for an object in the set and a dot outside a set's string is for an object not in the set. (See for example Lesson L2 The Empty Set.)

This language of strings and dots provides a precise (and nonverbal) way of recording and communicating thoughts about classification. The abilities to classify, to reason about classification, and to extract information from a classification are important skills for everyday life, for intellectual activity, in general, and for the pursuit and understanding of mathematics, in particular. Ordinarily, we are not expected to deal with such problems until our command of our natural language is fully developed and until we face classification problems in adult life. It is not that an ability to classify requires special intelligence, but that it requires a special mode of thinking. The unique quality of the language of strings is that it provides a nonverbal language that is particularly suited to the mode of thinking involved in classification. It frees young minds to think logically and creatively about situations which involve classifying and to report their thinking long before they have extensive verbal skills.

In this strand we present situations and ask carefully phrased questions to continue to advance skills in classification, always remembering that the skills grow out of such experiences. To be able to draw strings and dots is not an objective in itself; to develop the mode of thinking involved is the objective. Thus it is important for us to construct the situations carefully. The sets into which we ask students to classify objects must be determined by well-defined attributes; otherwise, the students have the added problem of deciding whether a certain object does or does not have a certain attribute. For this reason we make extensive use of A-Blocks (Attribute Blocks) and particular sets of numbers in designing classification situations. Students can immediately say whether or not a block is red, whether or not it is a square, and so on. They know whether or not a given number is less than 15, whether or not it is odd, and so on.

One reason for classifying objects is to count the objects that have a certain attribute. Suppose for example, in a parking lot there are eight red cars and six Volkswagons. If four cars in the lot are neither red nor Volkswagons, must there be eighteen cars in the lot? Suppose we count the cars in the parking lot and find there are exactly fifteen. Is this possible? A string picture immediately settles this apparent paradox.



There are three red Volkswagons.

Relations: The Language of Arrows

Relations are interesting and important to us in our everyday lives, in our careers, in school, and in scientific pursuits. We are always trying to establish, explore, and understand relations. In mathematics it is the same; to study mathematics means to study the relations among mathematical objects like numbers or geometric shapes. The tools we use to understand the everyday world are useful to understand the world of mathematics. Conversely, the tools we develop to help us think about mathematical things often serve us in nonmathematical situations.

A serious study of anything requires a language for representing the things under investigation. The language of arrows provides an apt language for studying and talking about relations. Arrow diagrams are a handy graphical representation of a relation, somewhat the same way that a blueprint is a handy graphical representation of a house. By means of arrow diagrams, we can represent important facts about a given relation in a simple, suggestive, pictorial way -- usually more conveniently than the same information could be presented in words. (See for example Lesson L3 A Valentine Story.)

The convenience of arrow diagrams has important pedagogical consequences for introducing children to the study of relations in the early grades. A child can "read" -- and also draw -- an arrow diagram of a relation long before he or she can read or present the same information in words. The difficulty of presenting certain ideas to children lies not in their intellectual inability to grasp the ideas; rather, the limitations are often mechanical, motor problems of writing or reading about the ideas in conventional systems. Arrow diagrams have all the virtues of a good notation: they present information in a clear, natural way; they are attractive, colorful things to look at; they are easy and fun for children to draw. Another educational bonus occurs when arrow diagrams are used imaginatively. A story or problem may be captured by an arrow diagram, and at the same time, the resulting arrow diagram may suggest other, similar situations. This allows students to call upon previous experience or to expand their creativity.

One of the purposes of this strand is to exploit the power of the language of arrows to help children think logically about relations. Again it must be remembered that our goals concern the thinking process and not the mechanism. We resist making the ability to draw prescribed arrows an objective in itself, just as we resist viewing an arrow diagram as just another format for drill problems in arithmetic.

The general aim of The Languages of Strings and Arrows strand is to suggest situations that are inherently interesting and thought provoking, and to give children modes of thinking and appropriate languages with which they can organize, classify, and analyze. In addition to a varied assortment of lessons concerning sets and relations, this strand includes lessons involving systematic methods for solving combinatorial (counting) problems; methods for collecting, recording, and interpreting data in "real life" situations; probabilistic thinking; and networks.

L1 Trees #1

CAPSULE LESSON SUMMARY

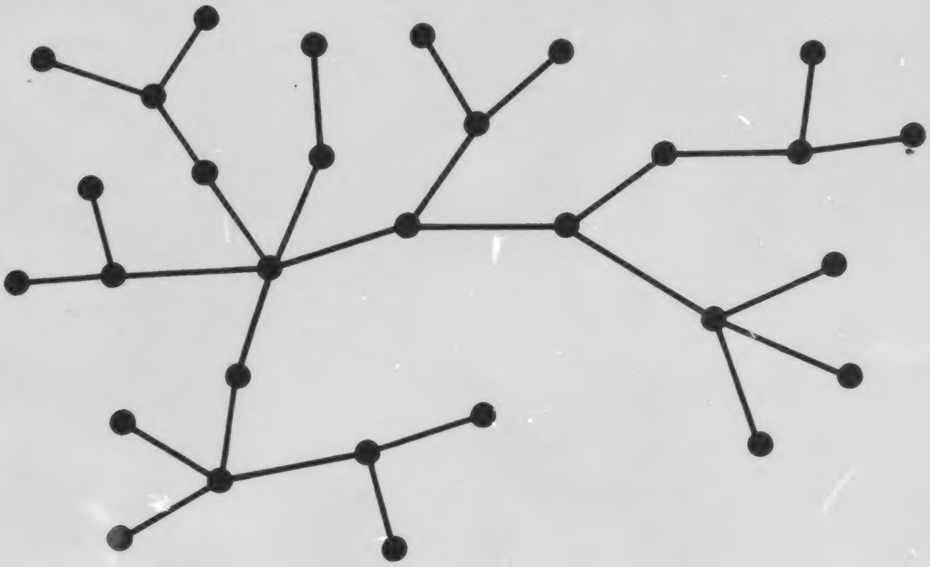
Count the dots and edges in a tree. Observe that in a tree there is just one path between two dots. Choose one of the dots to be a root and direct all the edges away from the root. Choose a second dot to be a root and direct all the edges away from the new root. Observe which edges get the same direction for both roots and which get different directions.

MATERIALS

Teacher: Colored chalk
Student: Worksheet L1 ; colored pencils

DESCRIPTION OF LESSONS

Before the lesson begins, draw this picture on the board.



T: We call this a tree. Why do you think it is called a tree?

Let students freely comment on this question, but do not insist on any specific ideas.

L1

T (pointing out a segment between two dots): This is an edge or branch of our tree. How many edges are there in this tree?

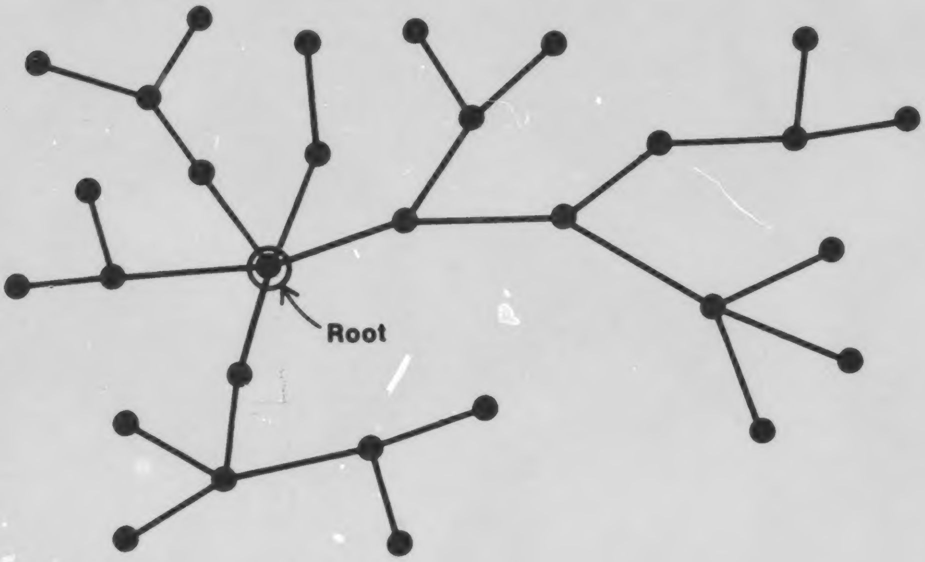
S: Twenty-nine.

T: And how many dots are there?

S: Thirty.

Choose one of the dots and circle it in red.

T: Suppose this is the root of our tree.

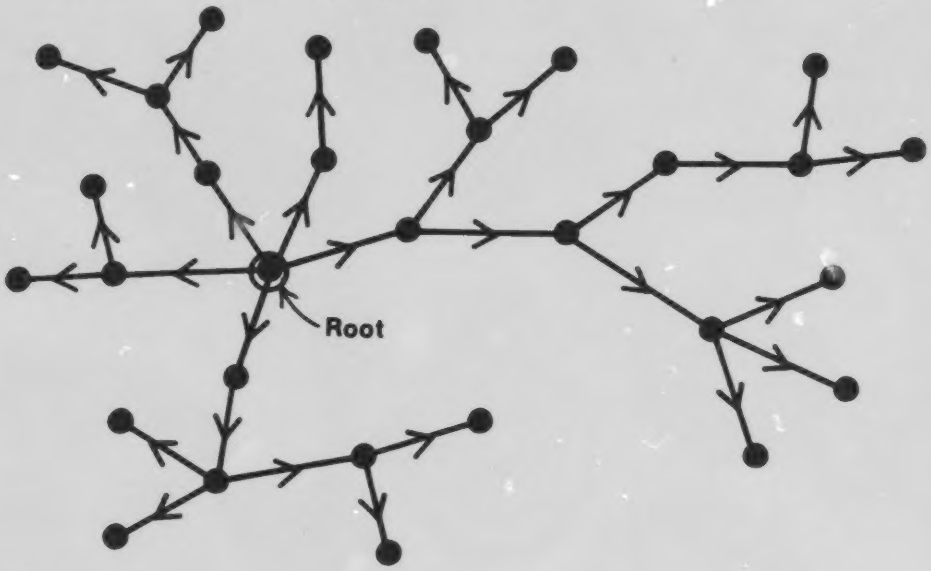


T (pointing to a dot other than the root): Can you find a path in this tree starting at the root and ending at this dot?

Let a student trace such a path. Then, with class discussion, observe that there is only one such path (so long as you do not backtrack). Repeat the question and observation for several other dots.

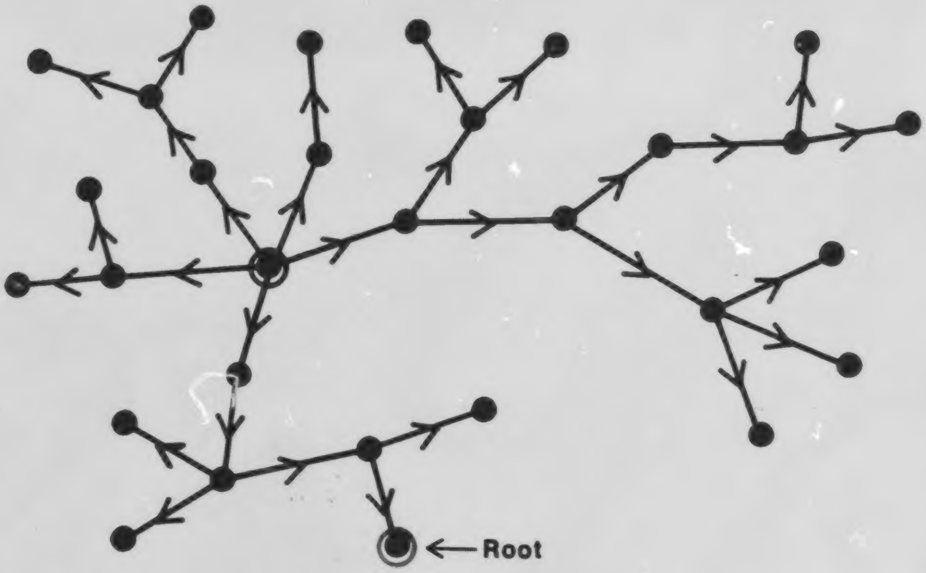
T: Now I would like to draw red arrowheads on all the edges to show the direction of paths starting at the root and going to some other dots. The red arrows will show going away from the root.

Illustrate how to draw the arrowheads on one or two edges yourself, and then call on students to assist you. At first let students add the arrowheads one at a time. After the idea has caught on, allow some students to draw several arrowheads at one time. All the students should participate in this activity. A completed picture is shown below.



Choose a second dot and circle it in blue.

T: Let's change the root in our tree.



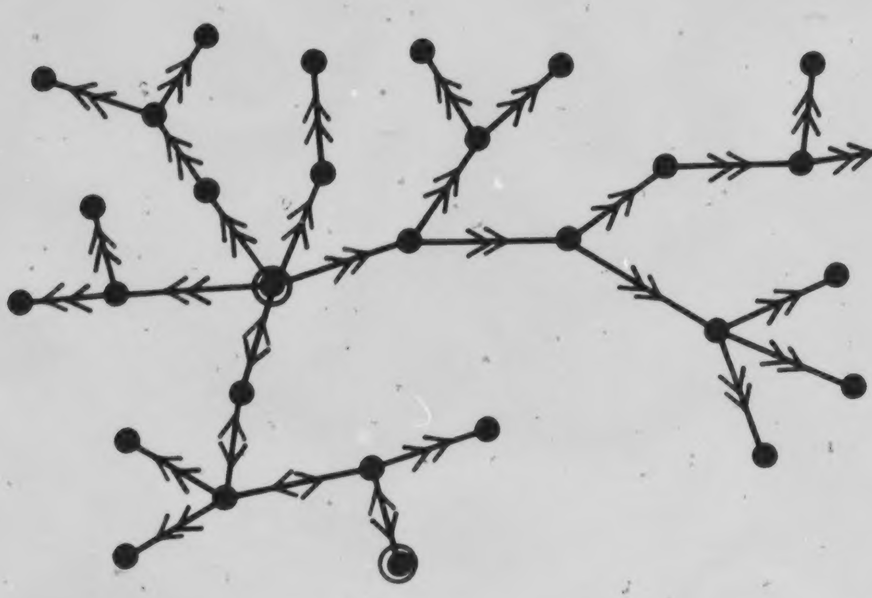
L1

T: Can you find a path starting at the new root (point to the blue circled dot) and ending at this dot, the old root (point to the red circled dot)?

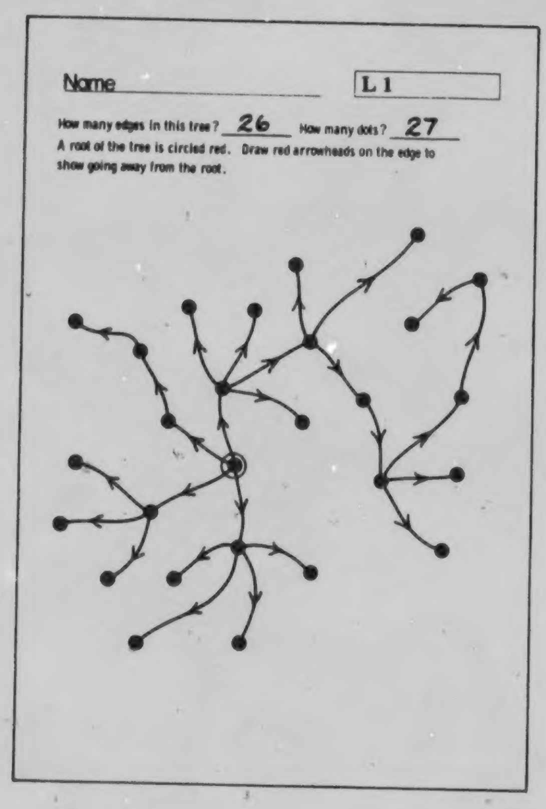
Let a student trace the path and then, with class discussion, observe that the path goes against the red arrowheads. Choose several other dots and invite students to find paths from the new root to those dots. Each time lead a discussion observing that there is only one path and noting when the path goes with the red arrowheads and when it goes against the red arrowheads.

T: Now, let's use this new root and draw blue arrowheads on all the edges to show the direction of paths starting at the root and going to other dots. The blue arrows will show going away from this root.

Illustrate how to draw one or two blue arrowheads yourself, and then call on students to assist. Again, encourage all the students to participate in this activity. A completed picture is shown below.



Worksheet L1 is available for individual work on a similar problem. Instruct students who finish quickly to choose a second dot as a new root, circle it in blue, and then draw corresponding blue arrowheads.



Name ____________________ **L 1**

How many edges in this tree? 26 How many dots? 27

A root of the tree is circled red. Draw red arrowheads on the edge to show going away from the root.

L2 The Empty Set

CAPSULE LESSON SUMMARY

Locate children, pets, and other objects in string pictures. Observe that in some situations there are empty regions and introduce the hatching convention to indicate an empty region. Find different ways of picturing the same situation so that there are no empty regions.

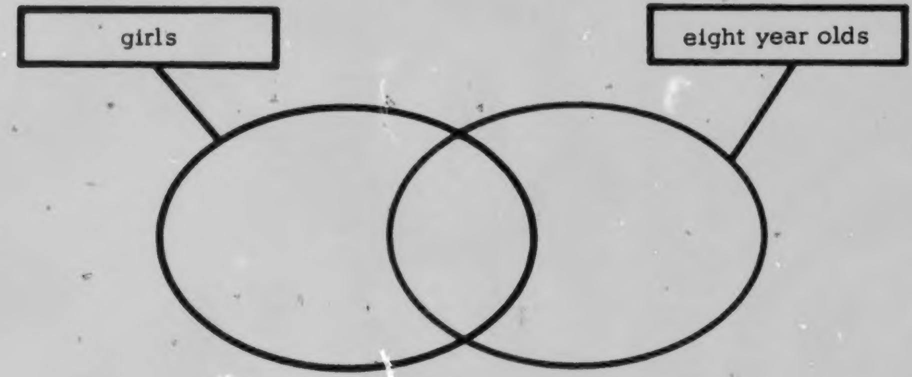
MATERIALS

Teacher: Colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

Draw this picture on the board.

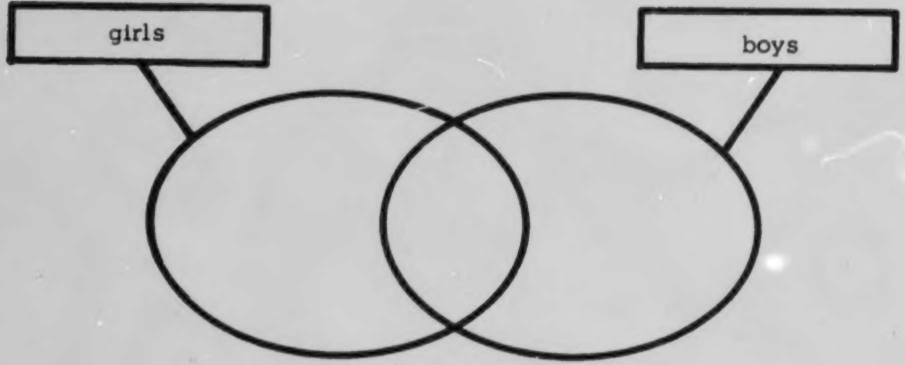


T: The red string is for girls and the blue string is for eight year olds. Think about where you would put a dot for yourself in this picture.

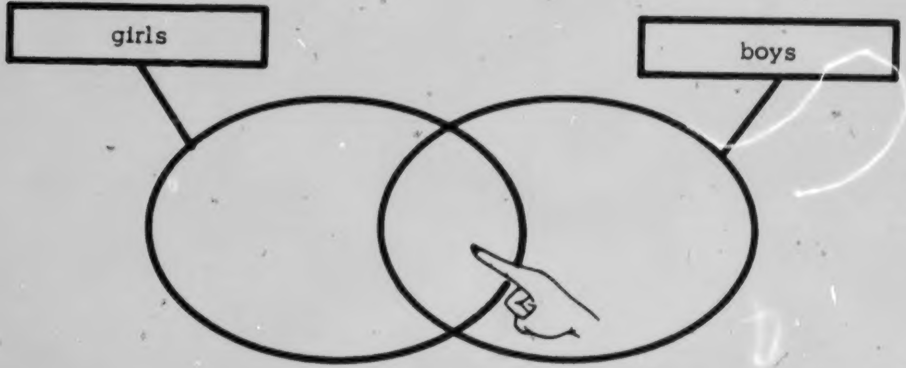
Call on several students to put dots for themselves in the picture. Then draw dots in each of the four regions and ask what (or whom) each dot could be for.

Erase all the dots and change the label of the blue string as you say:

T: Now I'm going to change the label of the blue string. The red string is for girls and the blue string is for boys.



Point to each of the four regions in turn and ask what (who) could be there. Consider the middle region last.

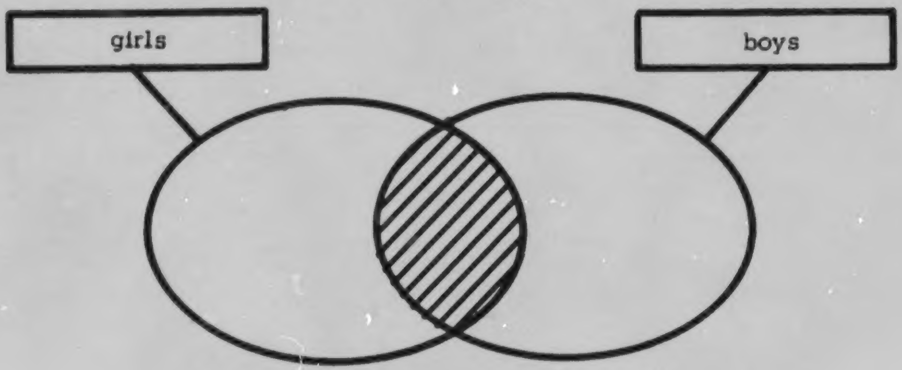


T: Who could be here?

After checking a few suggestions, the class should observe that nothing belongs in that part of the picture.

S: There is no one there because a person cannot be both a girl and a boy.

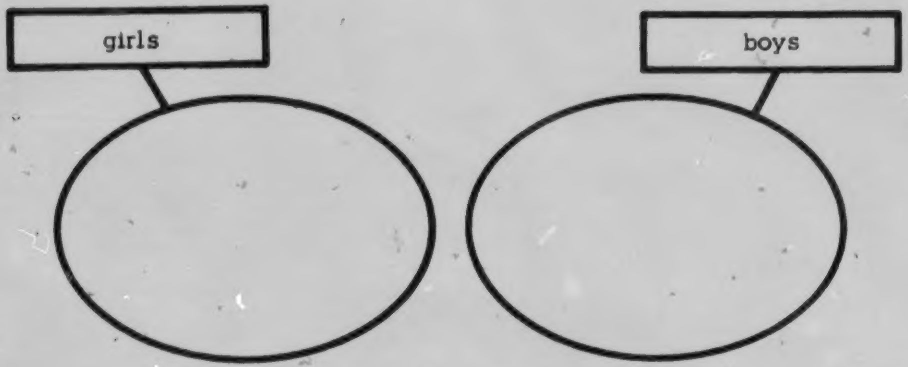
T: Very good. To show this we sometimes hatch the region.



T: This means that it is empty; nothing goes here.

Who can show us another way we could draw the picture?

Perhaps the students will suggest drawing disjoint strings; otherwise, do not force the issue.



Erase the string labels for the picture with overlapping strings and with the middle region hatched.

T: Who can suggest other labels these strings could have?

S: The red string could be for cats and the blue string could be for dogs.

S: The red string could be for even numbers and the blue string could be for odd numbers.

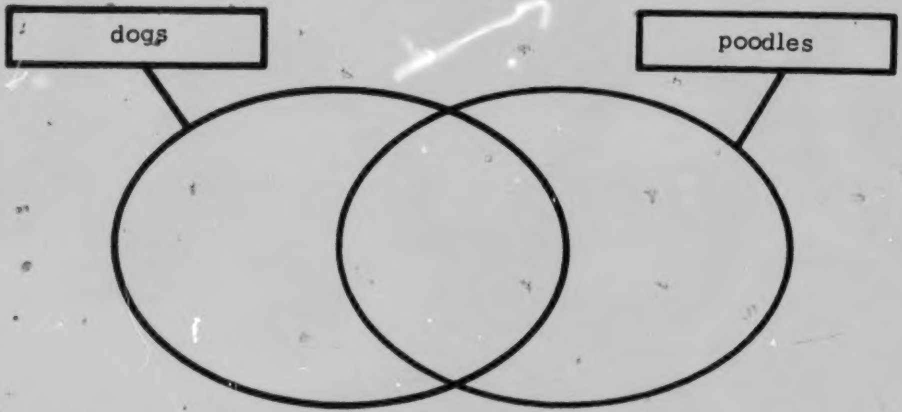
S: The red string could be for numbers less than 10 and the blue string could be for numbers more than 10.

Accept suggestions and let the class check each one. If students do not find some examples of their own, you could start them off by giving a label to one of the strings and asking what the other string could be for.

Erase the board before going on to Exercise 2.

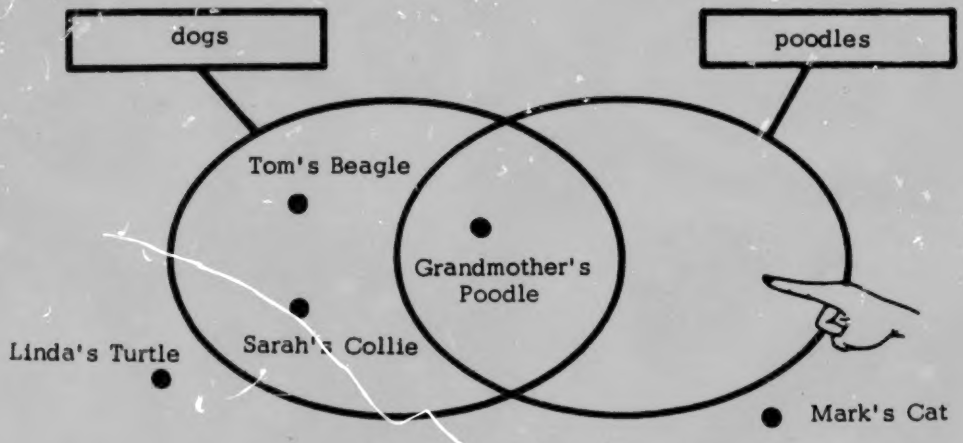
Exercise 2

Draw this picture on the board.



T: The red string is for dogs and the blue string is for poodles. Do you remember the "Where's My Nose?" story? Nick's grandmother had a poodle. Who can draw a dot for her poodle?

Let a student draw a dot and ask for confirmation from the class. Encourage some of your students to put dots for their pets in the picture. Then point to the region inside the blue string but outside the red string (see the next illustration).


dogs
poodles
Tom's Beagle
Grandmother's Poodle
Sarah's Collie
Linda's Turtle
Mark's Cat

T: What could be here?

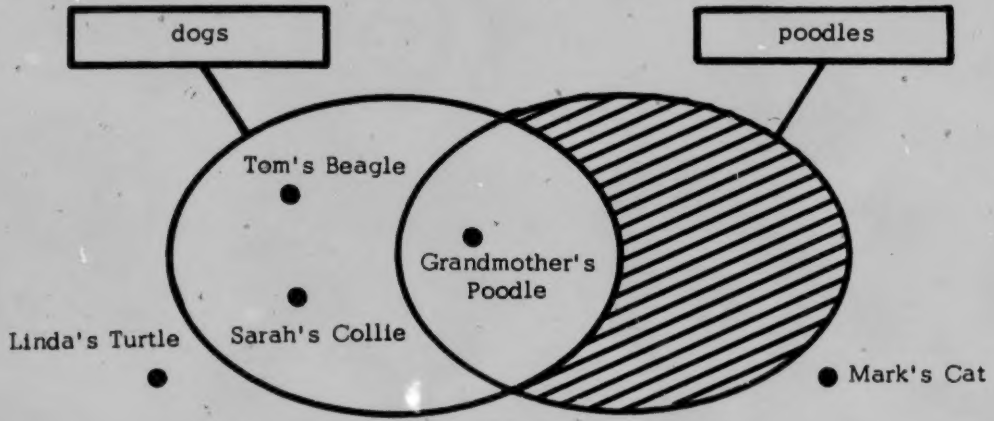
S: Nothing. It would have to be a poodle and poodles are dogs so must be in the middle.

S: That part of the picture is empty.

T: What should we do?

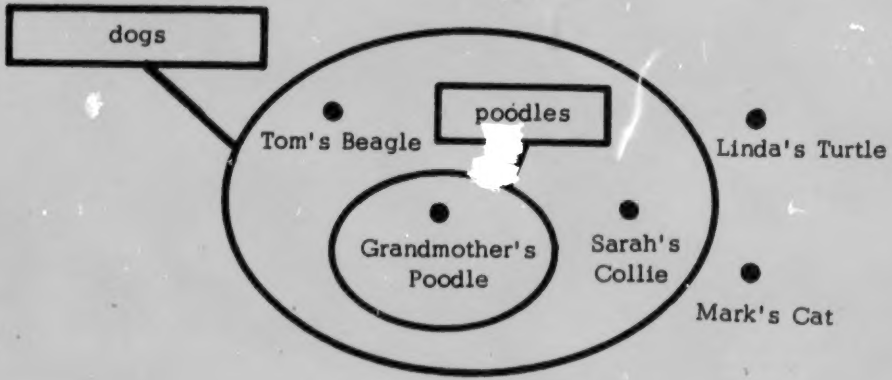
S: Hatch it.

Ask a student to show you where to put the hatching and then add it to your picture.

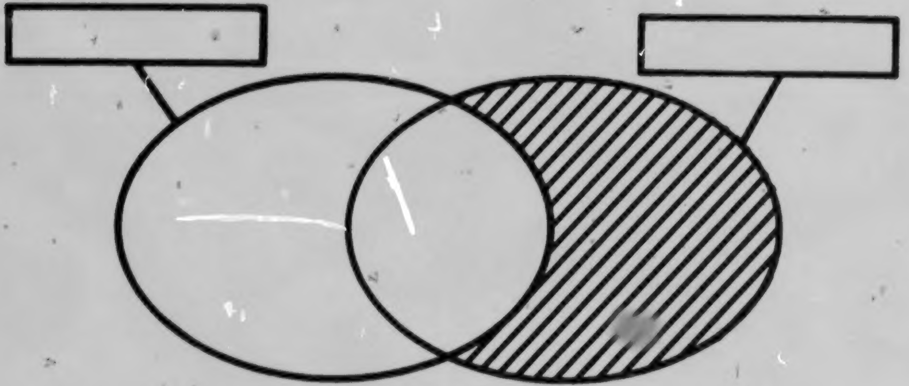

dogs
poodles
Tom's Beagle
Grandmother's Poodle
Sarah's Collie
Linda's Turtle
Mark's Cat

T: Do you know another way we could draw the picture?

If students suggest an alternative drawing, discuss it by putting the same things in the new picture. Again, do not press the class for this drawing.


dogs
poodles
Tom's Beagle
Linda's Turtle
Grandmother's Poodle
Sarah's Collie
Mark's Cat

Erase the board except for this picture.



T: Who can suggest other labels these strings could have?

S: The red string could be for cats and the blue string for Siamese cats.

S: The red string could be for cars and the blue string for Cadillacs.

S: The red string could be for numbers more than 5 and the blue string for numbers more than 10.

L 2

Accept suggestions and let the class check each one. With this situation, it may be difficult for students to find examples. If necessary, lead them by giving one string label and ask them to supply the other.

L3 A Valentine Story

CAPSULE LESSON SUMMARY

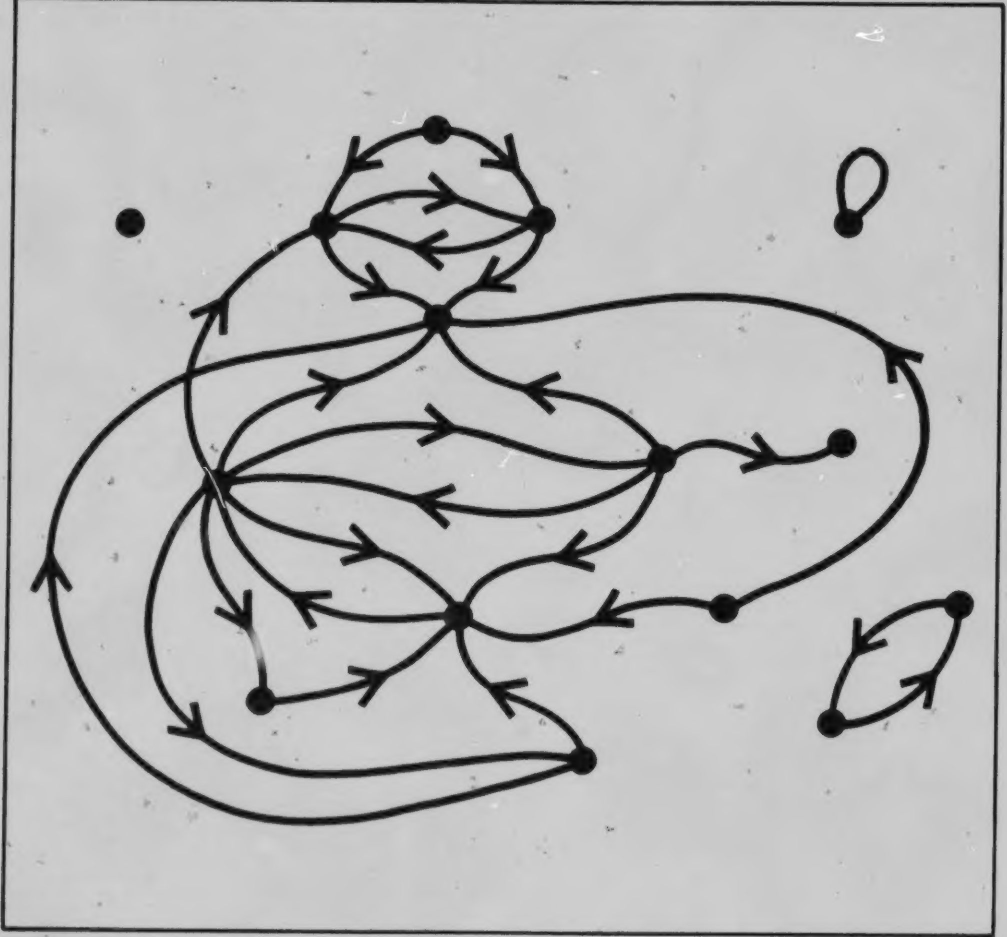
Tell a valentine detective story where information is found in an arrow picture with arrows for "I sent a valentine to you." Give clues about the people in the picture until you (the narrator) are located. Read another valentine story "The Magic Box".

MATERIALS

Teacher : Language Poster #1 for UPG-II
Student : Paper ; colored pencils ; Storybook THE MAGIC BOX

DESCRIPTION OF LESSON

Before the lesson, tape Language Poster #1 for UPG-II to the board. It is important that everyone in the class be able to see the entire poster because the picture is the basis for a type of detective story.



22

L 3

T: What do you think about my picture?

Allow a few minutes for the students to comment freely about the arrow picture. They will probably note such things as the "lonely" dot in the upper left corner, the loop in the upper right corner, the two-cycle in the lower right corner, and the fact that the picture is very complicated.

T: This picture is about a valentine detective story I have for you. What do people do on Valentine's Day?

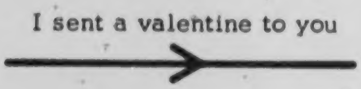
Let the students talk about sending and receiving valentines. If you have arranged your schedule so that this lesson comes on or close to Valentine's Day, the topic of valentines will be very exciting to them.

T: The dots in my picture are for some of the people in my family. How many people from my family are in this picture? (15) What do you think the red arrows are for?

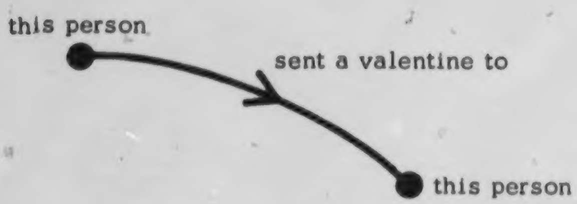
S: Sending valentines.

T: Yes, a red arrow is for "I sent a valentine to you."

Draw and label a key arrow above the picture.



Illustrate at a couple of places in your picture the meaning of the red arrow.



Let the students comment on some of the situations described in your drawing. For example, a student may note that the red loop means a person sent a valentine to himself (or herself). The two-cycle shows two people who sent valentines to each other. Some people sent no valentines; some people received no valentines. There are many interesting situations about which the students will comment; try to limit this discussion to only a few minutes.

T: I am in this picture. Do you know where I am?

The students will probably make several guesses. For each one say:

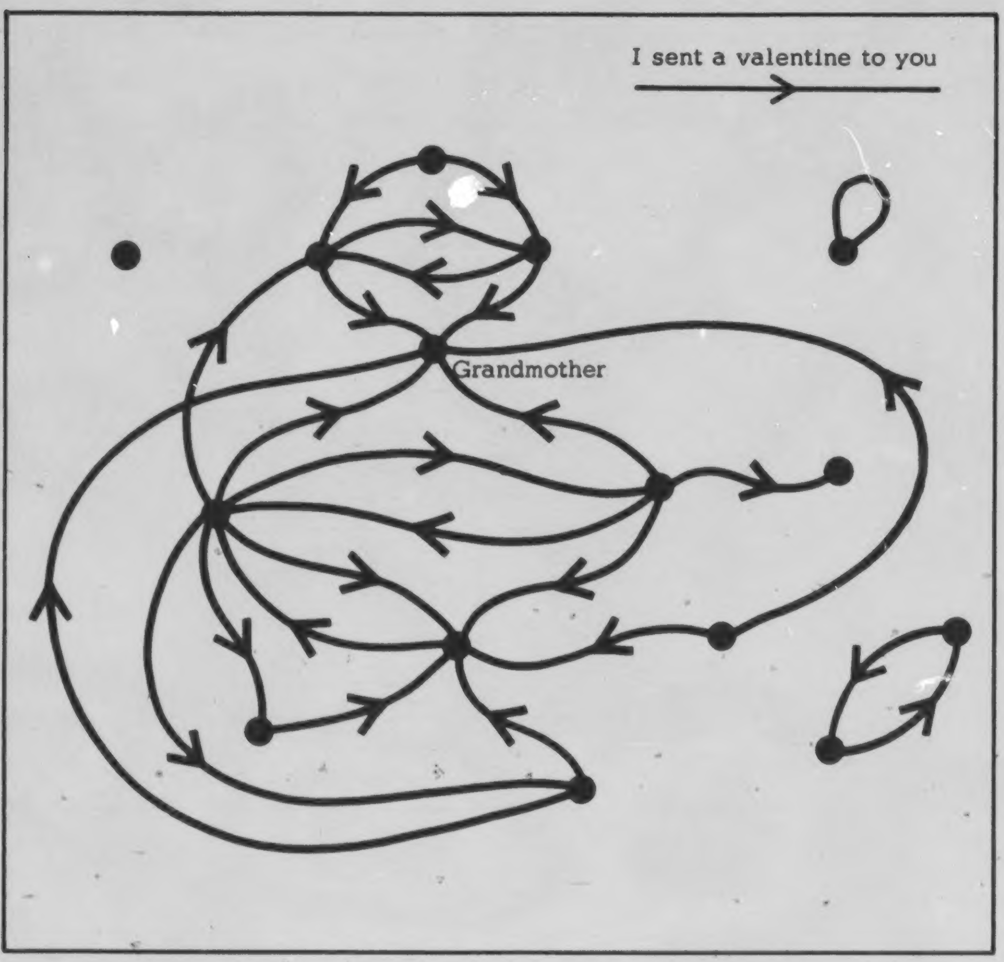
T: That could be me, but you don't know for sure.

Suggest to the class that to find you they need more information about the people in the picture.

T: I will tell you some things about the people in my family. If you are good detectives, you will be able to discover where I am in the picture. My grandmother received the most valentines. Where is my grandmother?

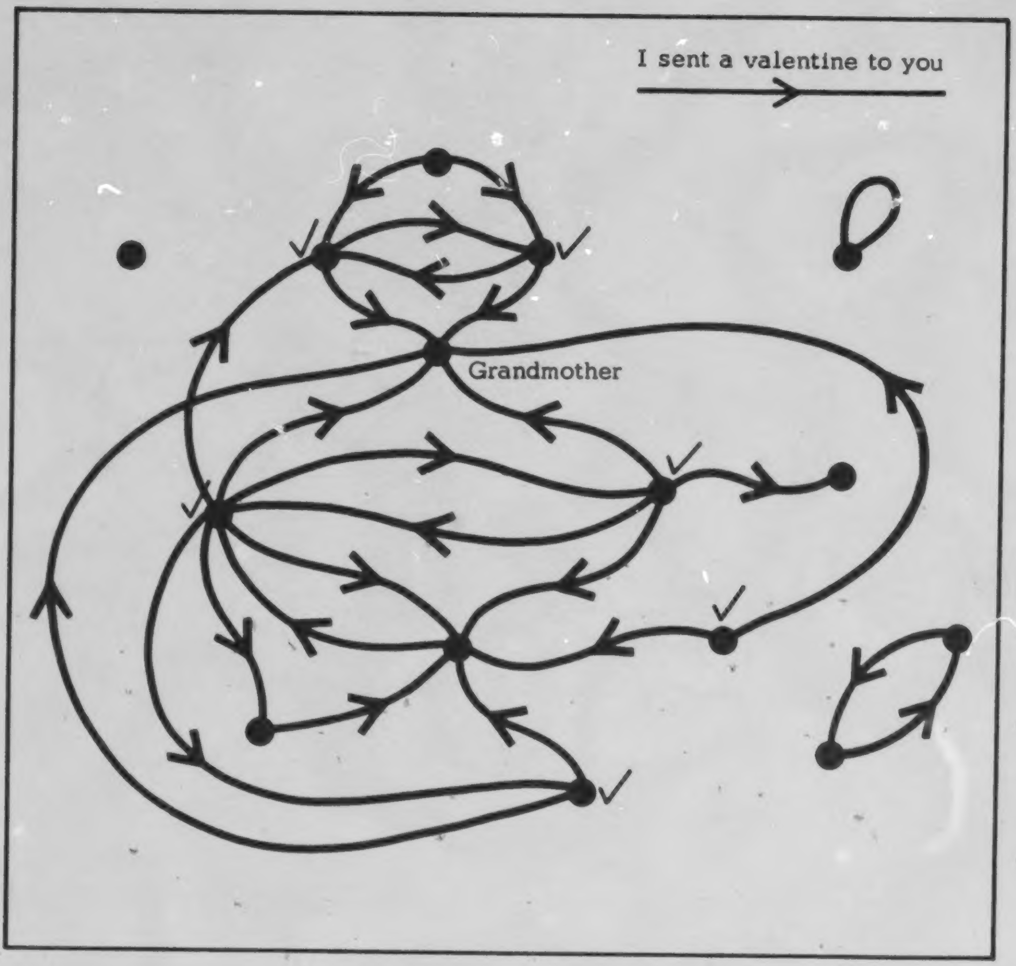
Call on a student to indicate which dot is for your grandmother. You and the class should check that, indeed, no one else in the picture received more valentines. Label the appropriate dot "Grandmother". (See the next illustration.)

L 3



T: How many valentines did grandmother receive? (Six)
I sent a valentine to my grandmother. Where could I be?

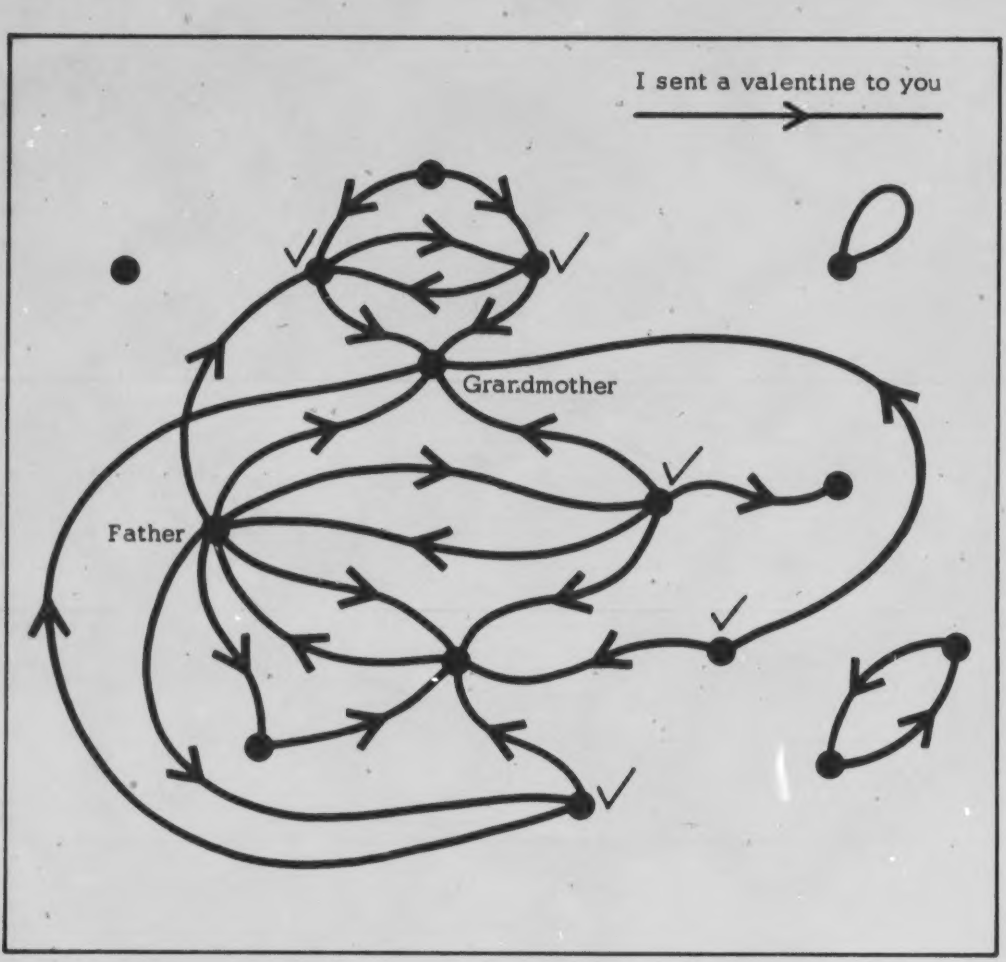
Let the students point to the dots that could be for you. Each time a student points to a dot, ask the class to check (by tracing an arrow) that the person sent a valentine to grandmother and if it is correct, put a check "√" by the dot in pencil. (You will later erase some of the check marks.)



T: The next thing I will tell you about my family is that my father sent the most valentines. Where is my father?

Call on a student to locate the dot for your father and encourage the class to check that it is correct. When there is agreement, erase the check mark by that dot noting that it cannot be for you and your father, and label it "Father". (See the next illustration.)

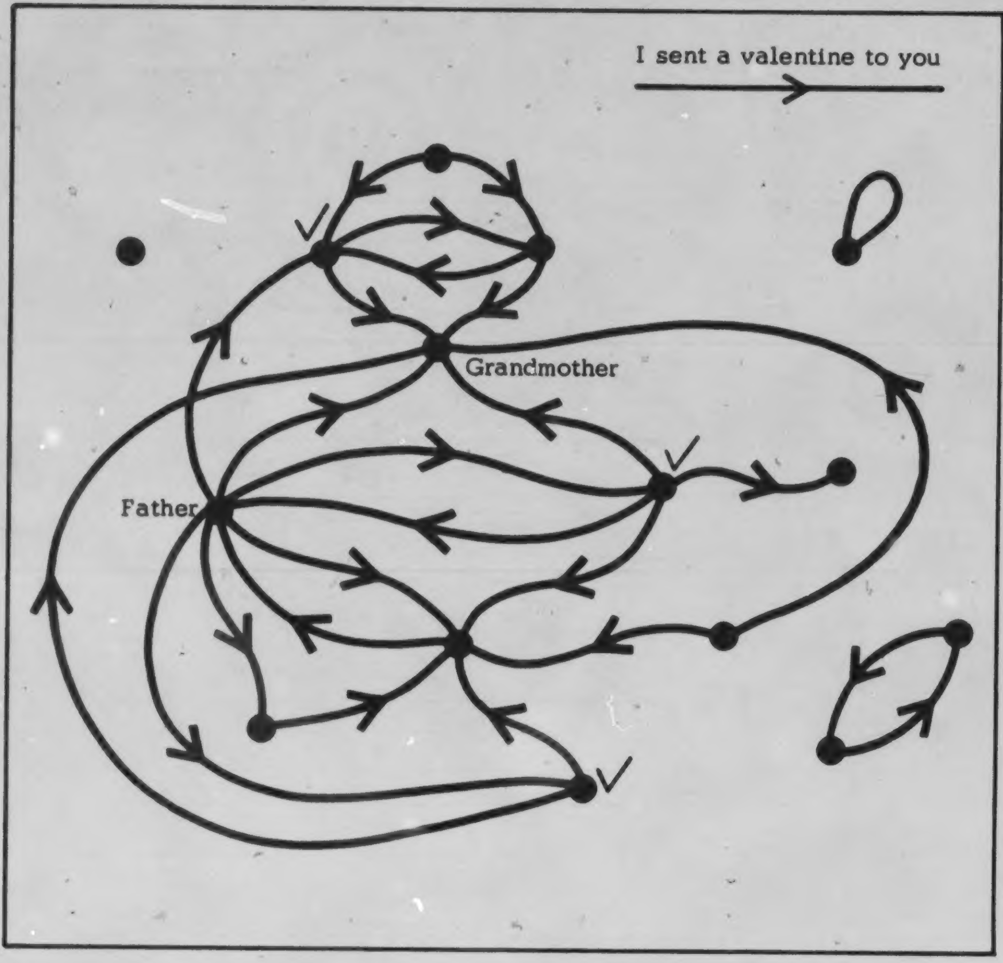
L 3



T: How many valentines did my father send? (Six)

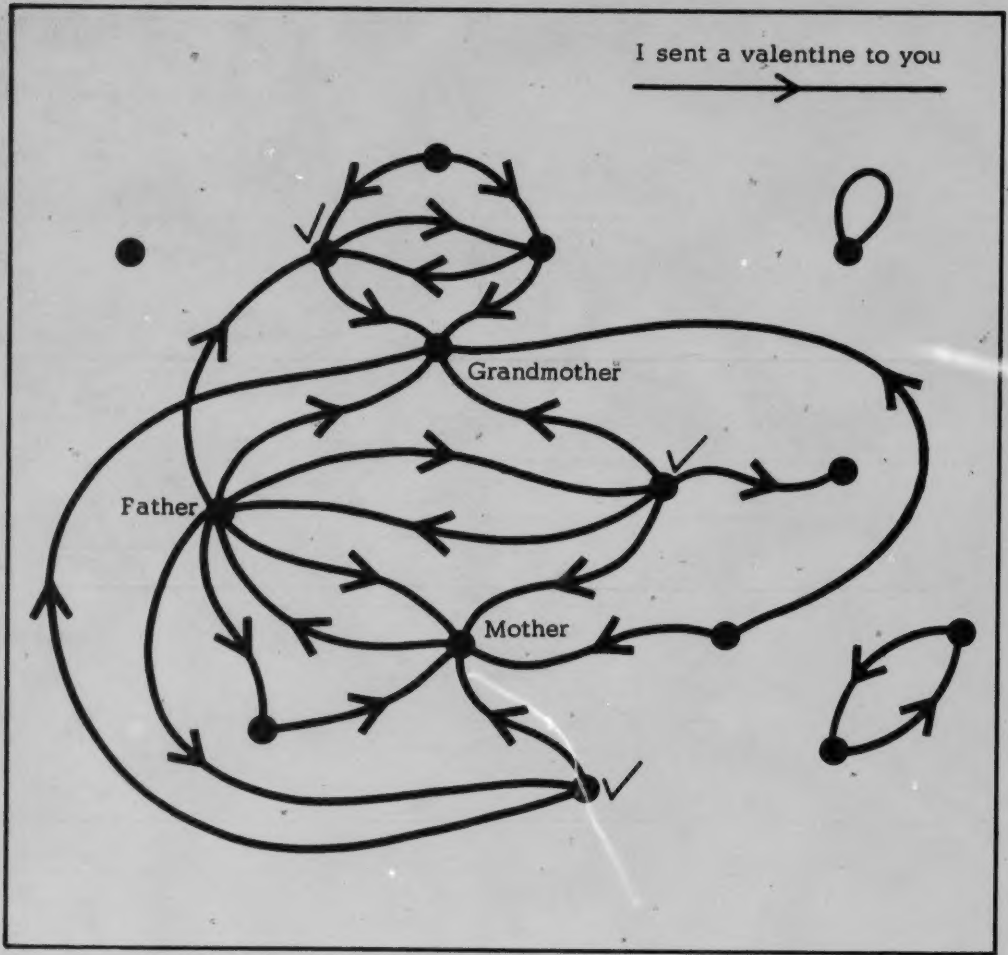
My father sent a valentine to me. Now, where could I be?

Let students point to dots that could be for you. Each time there will be two things to note: that the dot has a check mark by it and that there is an arrow from the dot labeled "Father" to the dot. You should encourage the students to notice that they must look for these two things. Erase the check marks by dots that now cannot be for you. (There will remain three dots still with check marks.)


I sent a valentine to you
Grandmother
Father
✓
✓
✓

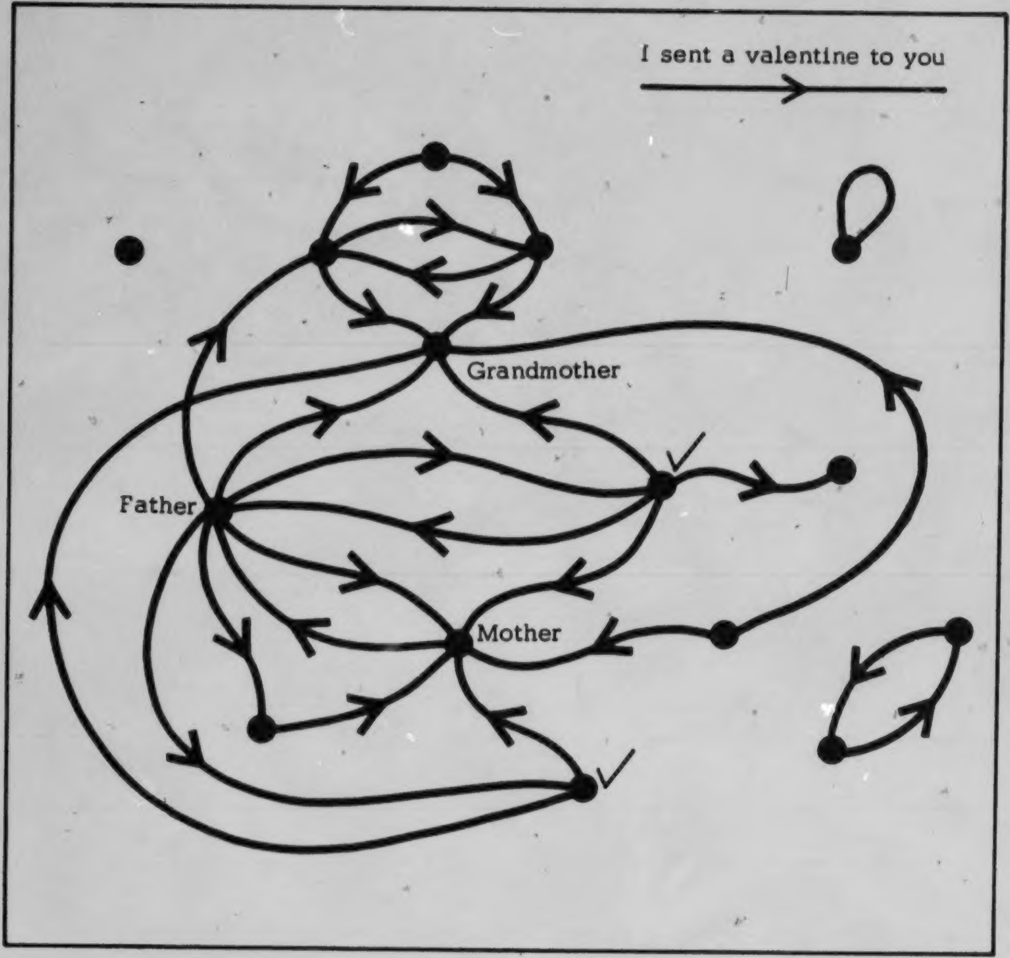
T: Now you know that one of these three dots (point to them) is for me. I will tell you something else. My mother received exactly five valentines. Where is my mother?

Call on a student to locate the dot for your mother, count that five arrows end at that dot, and then label it "Mother". (See the next illustration.)


I sent a valentine to you
Grandmother
Father
Mother
✓
✓
✓

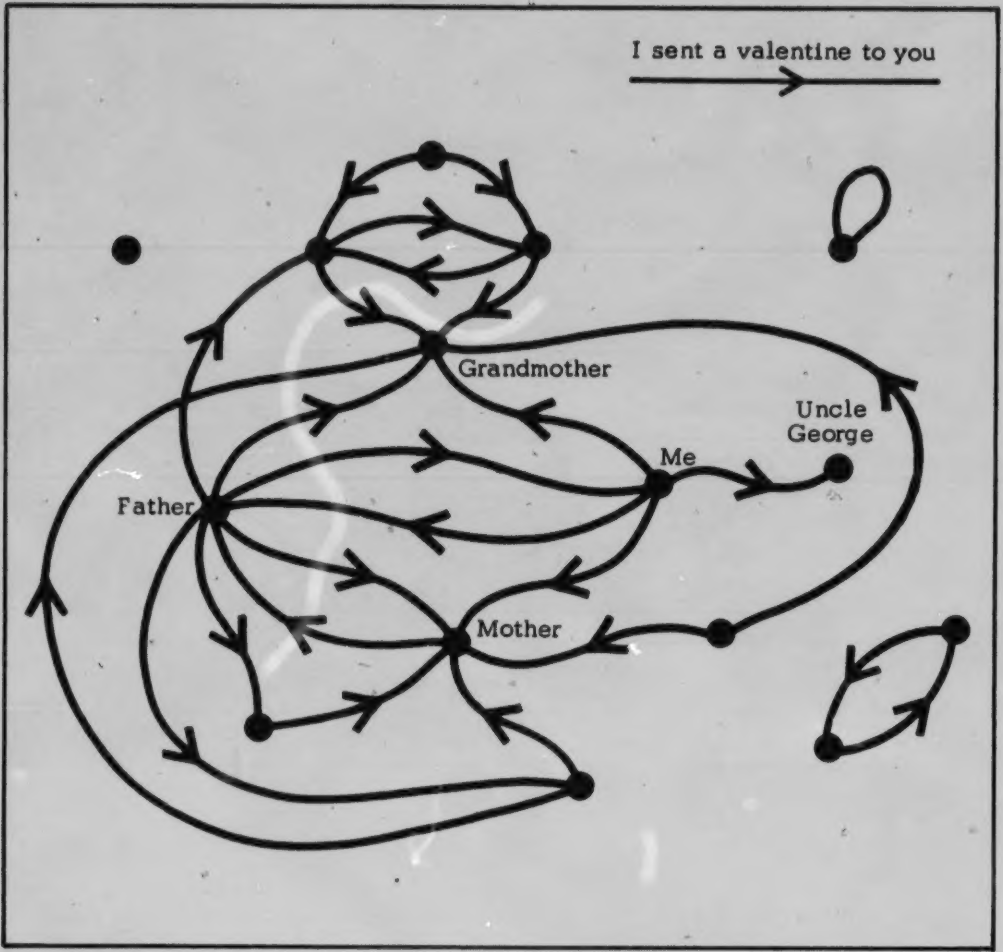
T: I sent a valentine to my mother. Where could I be?

The students should recognize that there are again two things to note: that the dot has a check mark by it and that there is an arrow starting at the dot pointing to the dot labeled "Mother". With this clue, one check mark can be erased; two dots remain that could be for you. (See the next illustration.)


I sent a valentine to you
Grandmother
Father
Mother

T: I have just one more thing to tell you about my family. You should then be able to discover for certain where I am, so listen carefully. I am the only person who sent a valentine to my Uncle George. Where am I and where is my Uncle George?

Call on a student to point to the correct dots and to label them. Be sure to get class agreement.


I sent a valentine to you
Grandmother
Uncle George
Me
Father
Mother

Distribute unlined paper and colored pencils. Ask the students to tell a valentine story about their family or about some of their friends by drawing an arrow picture.

Students who finish quickly may be given the Storybook THE MAGIC BOX to read. This is another valentine story.

L4 The String Game #1

CAPSULE LESSON SUMMARY

Review the A-Block pieces as to size, shape, and color by taking them one at a time out of a box as they are described. Play the A-Blocks String Game.

MATERIALS

Teacher: A-Blocks String Game kit; colored chalk; (optional) magnetic board

Student: None

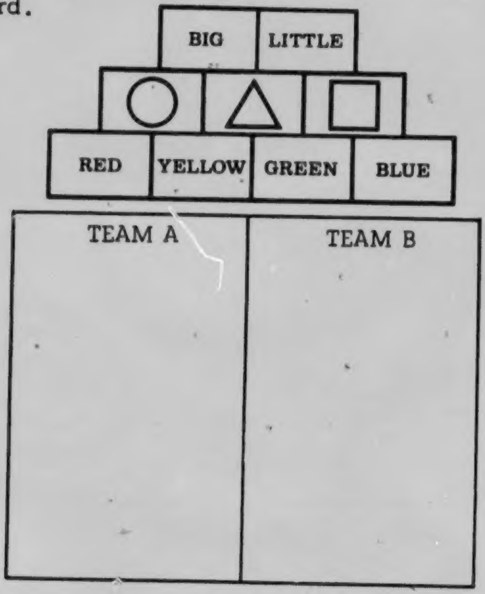
NOTE: Before this lesson begins, be sure you have the materials from the A-Blocks String Game kit prepared and ready for play. See the APPENDIX: The String Game for a description of equipment and preparation for the game.

DESCRIPTION OF LESSON

Put the A-Block pieces (shapes) in a box about the size of a shoe box. Sort them so that each one is easy to locate. Divide the class into two teams and try to arrange that the teams are roughly equal in ability.

Exercise 1

This first exercise serves only as a warm-up and should proceed at a brisk rate. Encourage students to think ahead and to respond quickly when it is their turn. Briefly review the different kinds of A-Block pieces and post a list of attributes above the Team Board.



L 4

T: We are going to play a String Game with these shapes. First you have to help me take the A-Block pieces out of this box. We will take turns ; someone on Team A will tell me which piece to put on Team A's side of the board, and then someone from Team B will tell me which piece to put on Team B's side of the board. Remember, you have to describe a piece that is still in the box and so I know exactly which piece you want me to put on your side of the board.

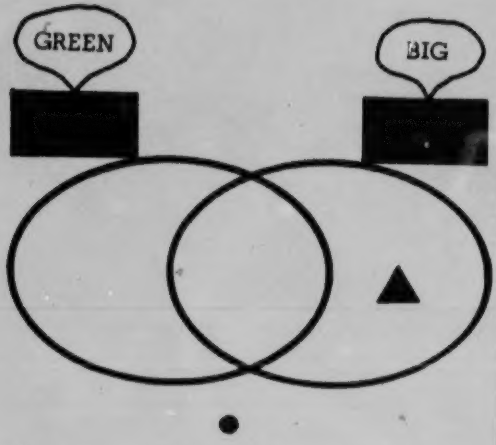
Alternating between the teams, let students describe pieces to put on the Team Board. Insist that the description be complete. If the piece described is already on the Team Board, point it out and ask for another. Try to call on weak students early so that there are many choices open to them. Encourage all the students to be thinking about which pieces they will ask for when you call on them.

Exercise 2

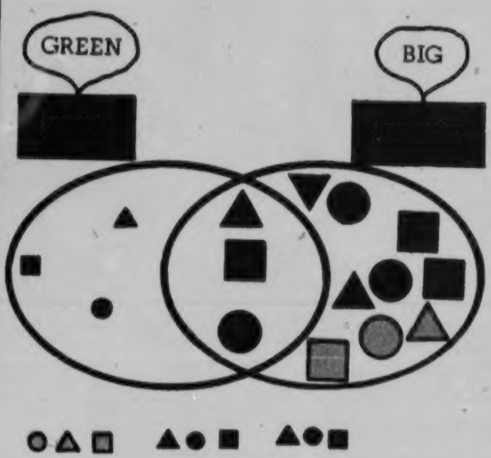
Play one or two rounds of the A-Blocks String Game (two-string version) in the usual way with hidden string labels. The next illustration shows the starting situations and correct placement of all game pieces for two games. In the second game, there is one empty region. It would be appropriate at the end of the game to ask about this region and to review the idea of hatching to indicate an empty region.

Game 1

Starting Situation

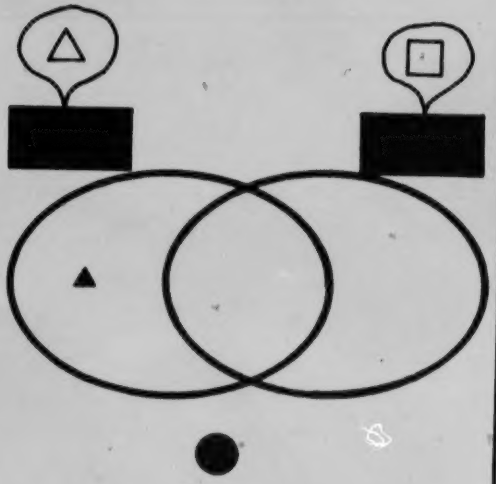


Correct Placement of Pieces

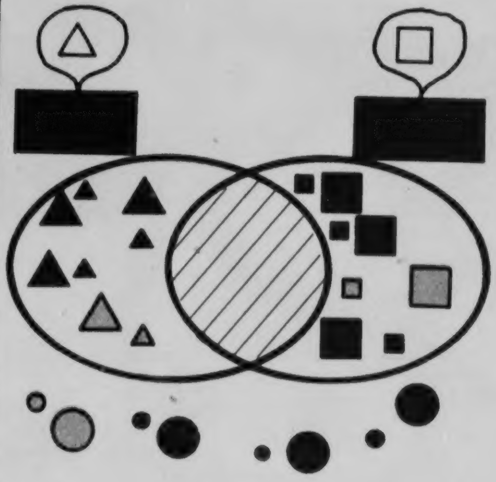


Game 2

Starting Situation



Correct Placement of Pieces



L5

L5 Trees #2

CAPSULE LESSON SUMMARY

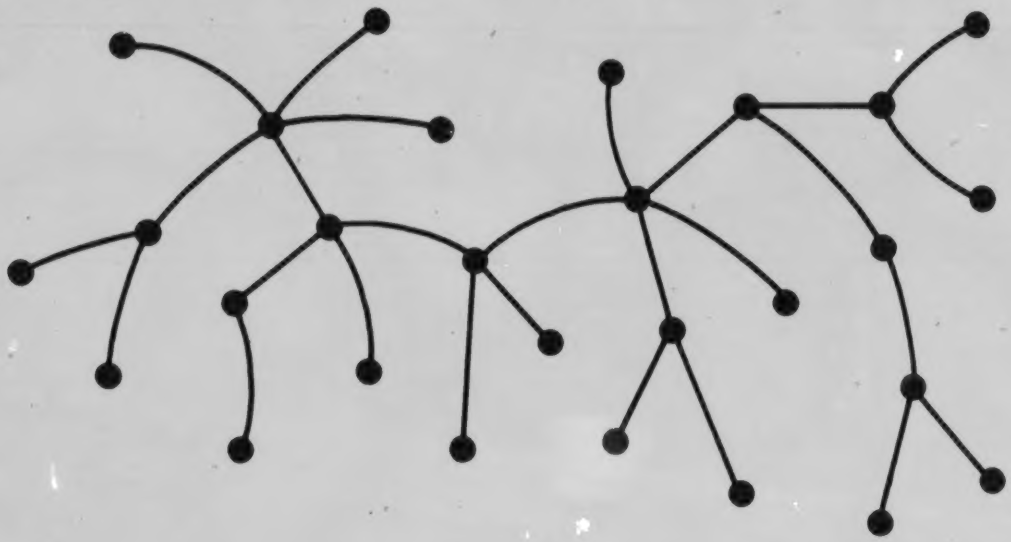
Count the dots and edges in a tree. Choose dots to be roots of the tree and for each root chosen, direct all edges away from the root. Find the distance between two dots in the tree by counting the number of edges in a path between the dots. Decide which is the longest distance between dots in the tree.

MATERIALS

Teacher: Colored chalk
Student: Worksheets L5* and **; colored pencils

DESCRIPTION OF LESSON

Before the lesson begins, draw this picture on the board.



Exercise 1

T: I have drawn a tree on the board. How many edges are there in this tree?

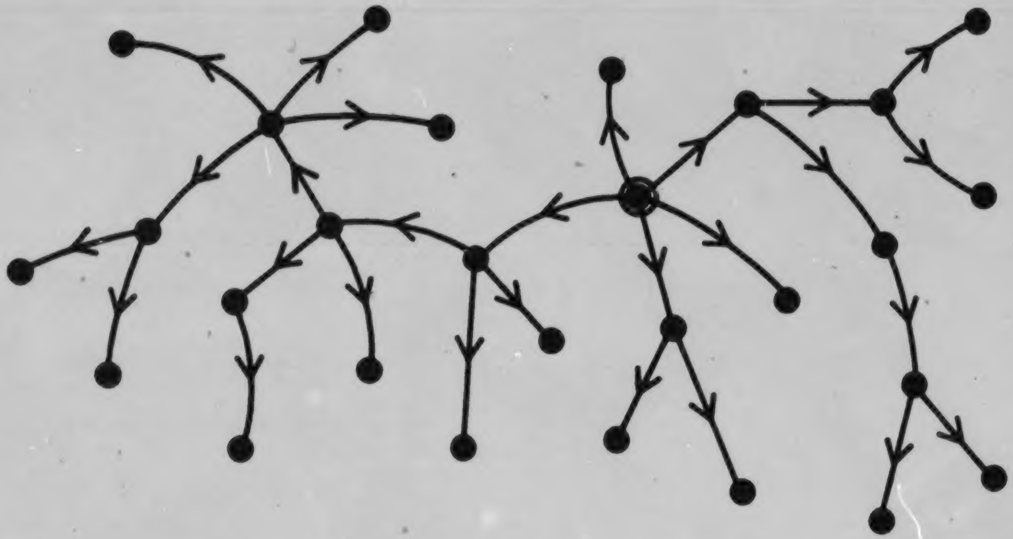
S: Twenty-seven.

T: How many dots?

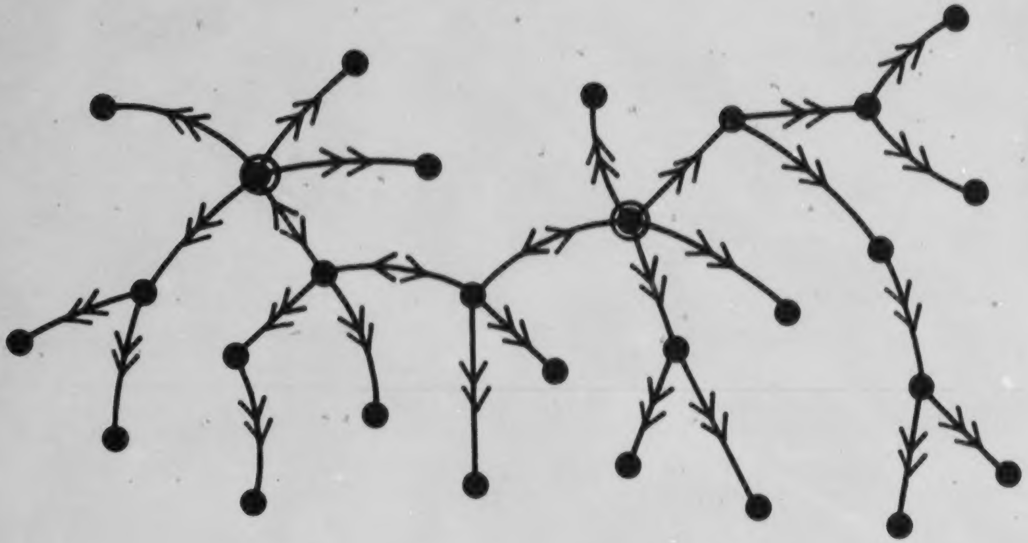
S: Twenty-eight.

T: Who would like to choose a dot at which to root the tree?

Invite a student to choose a dot and circle it in red. Review the idea of finding a path from the root to another dot. Observe that for each dot there is just one path (you are not allowed to backtrack) from the root to the dot. As in Lesson L1, ask students to draw red arrowheads on the edges to show going away from the root. Depending on your students' choice of root, a completed picture might look like the one below.



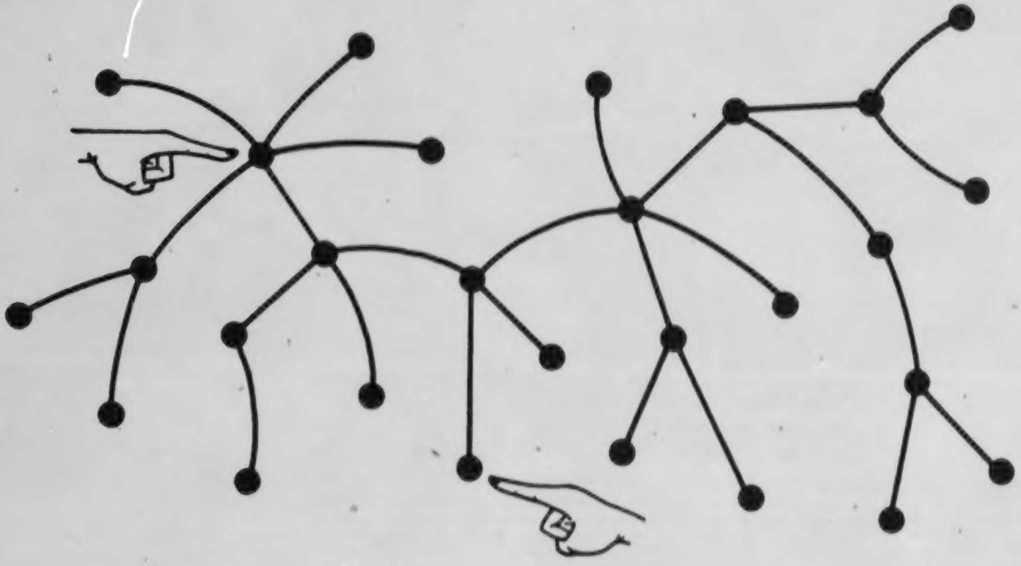
If this exercise proceeds quickly, let another student choose a root. Circle the new root in blue, and then invite the class to draw th[illegible]rresponding blue arrowheads to show going away from the new root. For example, the resulting picture might look like the one in the next illustration.



Erase all the color (arrowheads and circles) from the picture before going on to the next exercise.

Exercise 2

Choose two dots such as indicated below.



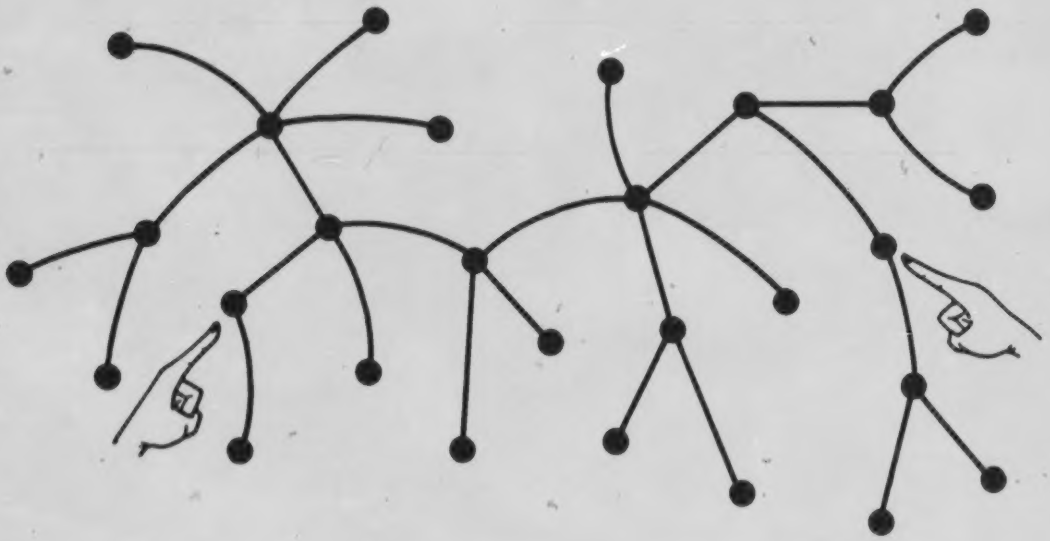
T: How many edges in a path between these two dots?

Call on a student to trace and count the edges in the path.

S: Three.

T: Yes, the distance between these two dots is 3.

Repeat this question for another pair of dots. For example, the dots indicated below are at a distance of 5 from each other.



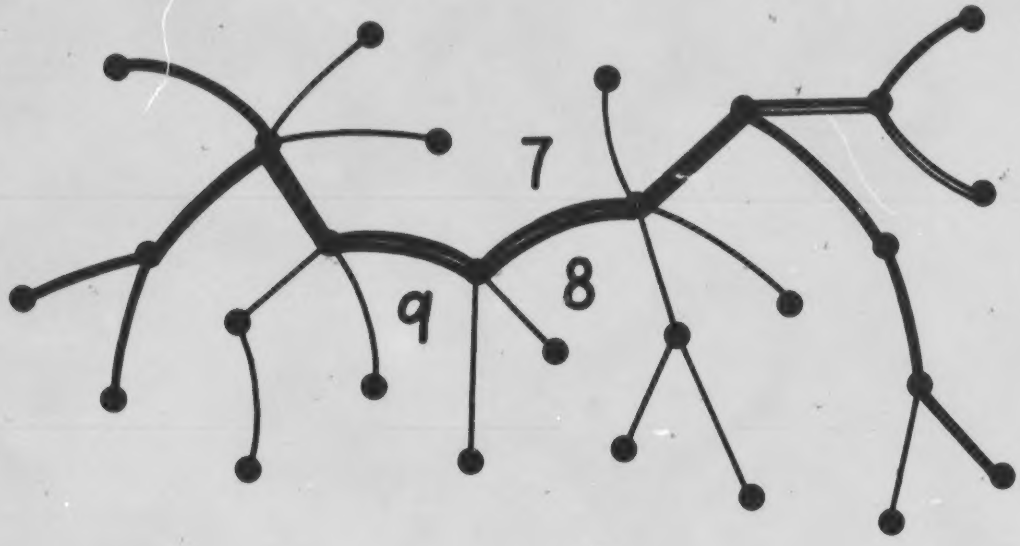
T: Try to find two dots at a distance of 4 from each other.

There are many possible correct responses. Invite several students to locate two such dots. Then repeat the problem for dots at a distance of 2, 3, 1, and 6 apart. NOTE: A dot is a distance of 0 from itself. This idea should be discussed briefly.

T: Now, try to find two dots at a distance apart as large as possible. Find the longest distance between dots in this tree.

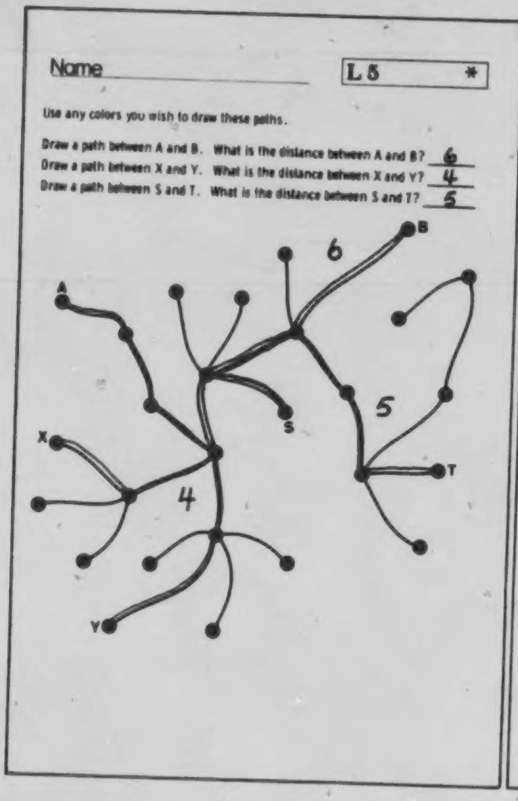
L 5

Let students experiment with this problem. When a student suggests two dots, illustrate the path between them with a heavy colored curve and indicate the length of the path in the same color. After several attempts by students, your picture might look like this.



The class should conclude that the longest distance between dots is 9. Encourage the class to find more than one pair of dots at a distance 9 from each other, but only draw the path for one pair.

Worksheets L5* and ** are available for individual work. Explain the instructions to the class and perhaps do an example collectively.



Name

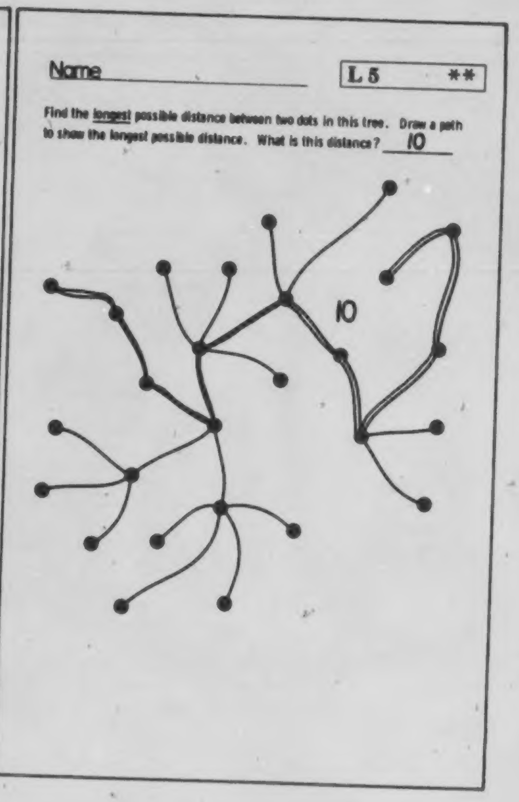
L 5 *

Use any colors you wish to draw these paths.

Draw a path between A and B. What is the distance between A and B? 6

Draw a path between X and Y. What is the distance between X and Y? 4

Draw a path between S and T. What is the distance between S and T? 5



Name

L 5 **

Find the longest possible distance between two dots in this tree. Draw a path to show the longest possible distance. What is this distance? 10

L6

L6 Temperature Bar Graphs

CAPSULE LESSON SUMMARY

Make temperature bar graphs for some given temperature data from two different cities. Compare these graphs to the temperature bar graph your class has been making during the past two months.

MATERIALS

Teacher:	Classroom temperature chart (see Lesson L15 in UPG-I); Language Poster #2 for UPG-II; felt-tip marker
Student:	Temperature data and chart sheet; pencil

DESCRIPTION OF LESSON

Begin the lesson by discussing the classroom temperature chart on which you have been recording daily temperatures during the months of January and February. The chart should be a bar graph for your data similar to those at the end of this lesson. Some questions which you might ask during this discussion are:

- During the past two months, has the temperature usually been above 0°C or below 0°C?
- What is the highest temperature we have recorded?
- What is the lowest temperature we have recorded?
- What is the difference between the highest and lowest temperature?
- Is there any temperature between the high and the low which we did not record?
- What temperature did we record most often?

Display two demonstration temperature charts (blank) from Language Poster #2. Distribute copies of temperature charts and two months of temperature data each for St. Louis and New Orleans. Explain to the class that this is temperature data collected by other second grade classes. They are going to use this data to make bar graphs similar to the one the class has made during January and February (point to the classroom temperature graph).

Together with the class, record for each city the first two or three temperatures in January on the blank charts you have displayed. Ask the students to do this on their papers. Then, let the students work independently recording temperatures and finishing a temperature bar graph for one or both cities. Monitor the work so that students who are having difficulty can get started correctly.

As the students are working on their own graphs, ask them one at a time to go to the demonstration charts and record some temperatures. For example, you could ask one student to record the temperatures in St. Louis for the first week in January. Then ask another student to record the temperatures in St. Louis for the second week in January; and so on.

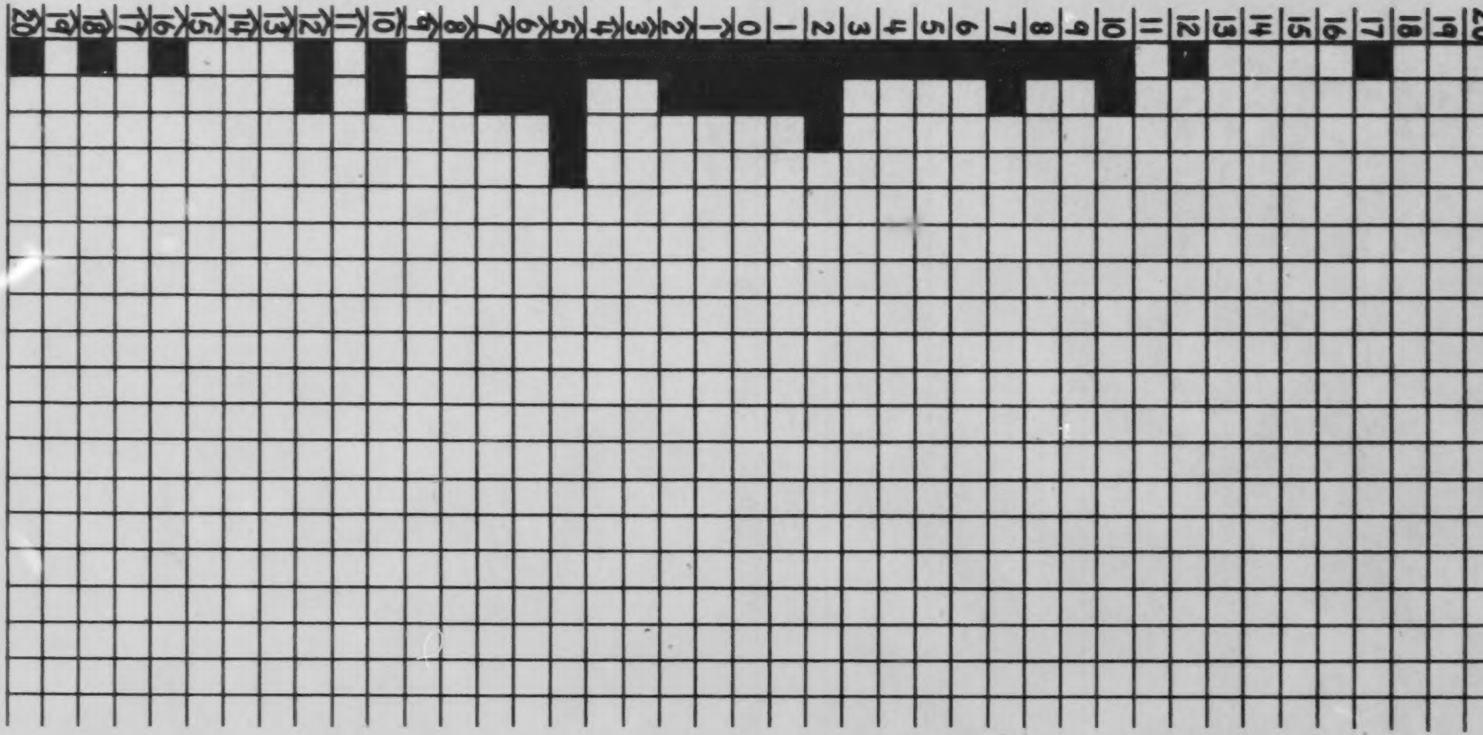
After about 20 or 25 minutes, most students should have completed at least one bar graph, and the class should have completed bar graphs for both cities at the front of the classroom. (Following this lesson description, you will find completed bar graphs for the two sets of data included here.) Compare your classroom temperature graph, the one the class made during the months of January and February, with the ones made during this lesson. You could ask some of the same questions that you asked at the beginning of the lesson. Try to include in the discussion comments on the following questions:

- Which city (your city or St. Louis or New Orleans) had the warmest weather in January and February?
- Which city had the warmest day in January and February?
- Which city had the coldest day in January and February?
- Which city had the greatest variation in temperatures?

If your class has enjoyed this project, you may wish to continue taking and recording the temperature every day during March and April. In this case, you could then compare the January and February temperature graph in your classroom with a March and April temperature graph.

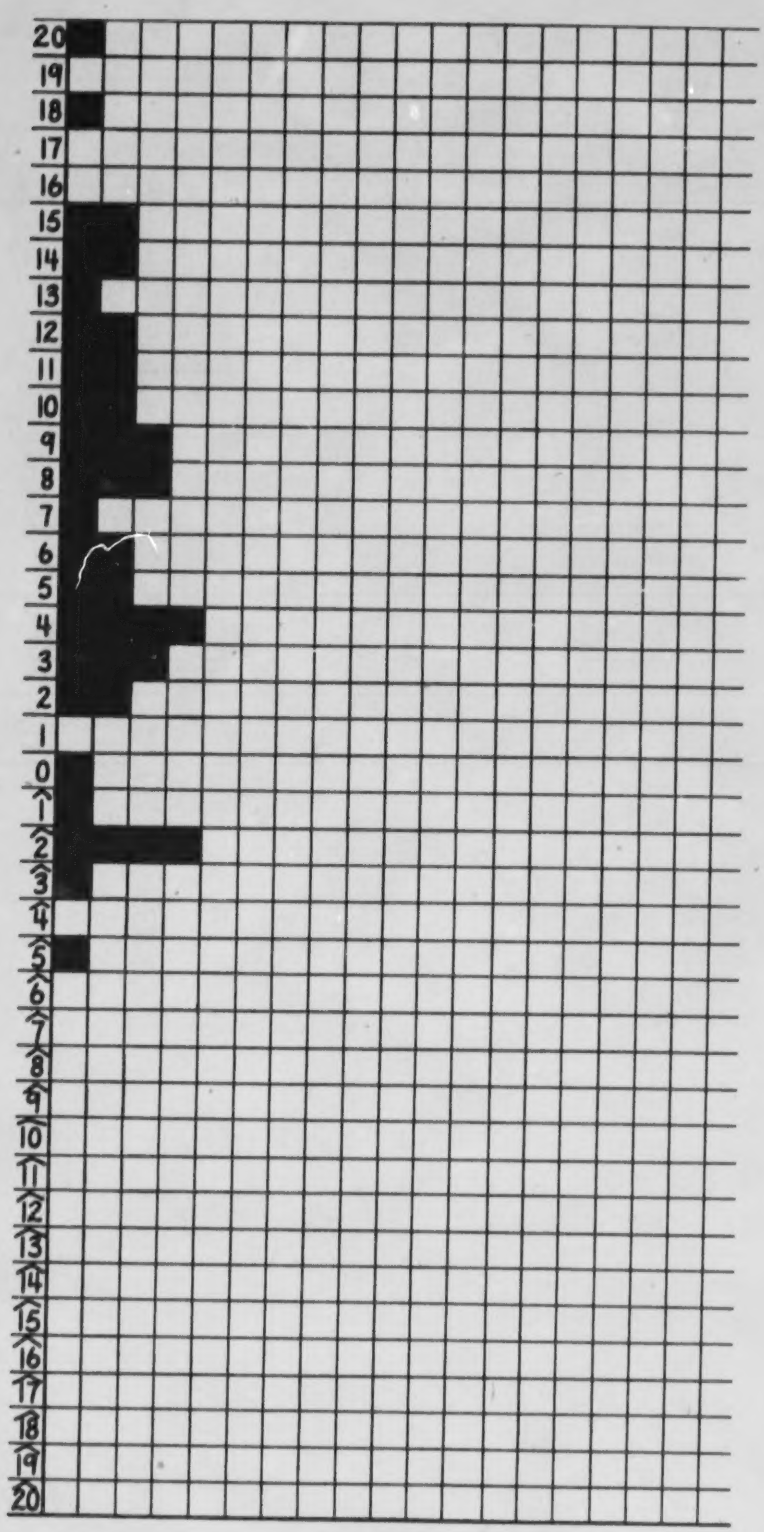
L 6

Temperature Graph for St. Louis, January and February 1977



L 6

Temperature Graph for New Orleans, January and February 1977



L 7

L7 Composition Games

CAPSULE LESSON SUMMARY

Review and play a composition game with the rule: yellow followed by blue is red. Introduce a new rule involving the red arrows; namely, red followed by red is green. Play a game using both rules.

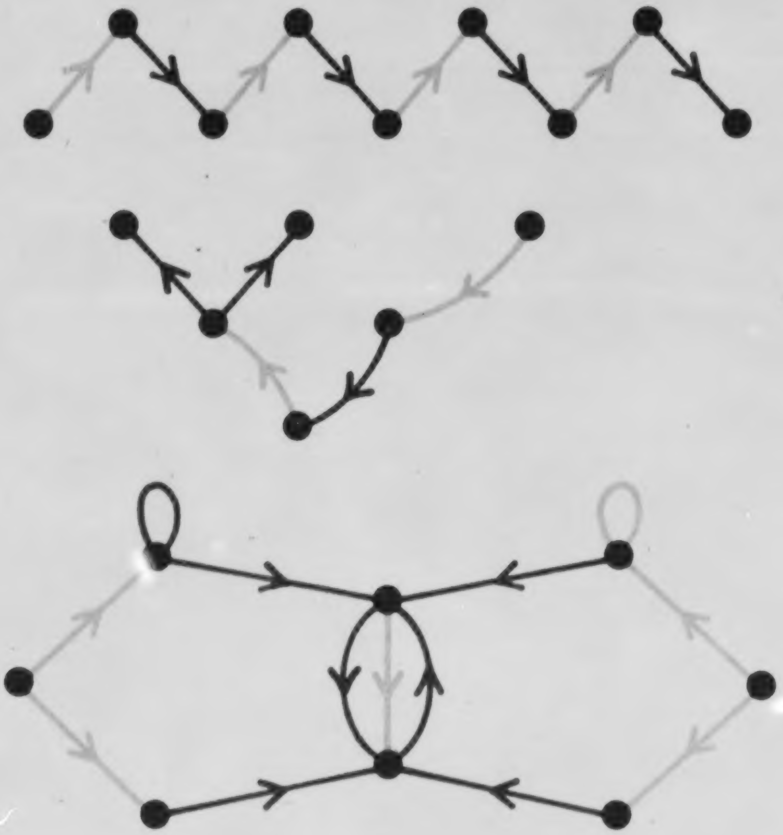
MATERIALS

Teacher: Colored chalk
Student: None

DESCRIPTION OF LESSON

Exercise 1

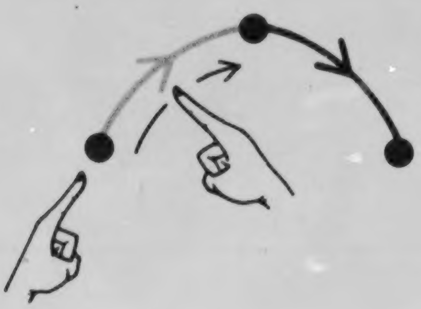
Before the lesson begins, draw this arrow picture on the board. Spread the picture out over the entire board and draw arrows whose colors can be distinguished easily.



T: Do you remember the game we played to draw red arrows? The rule is: when a yellow arrow is followed by a blue arrow, then we can draw a red arrow.

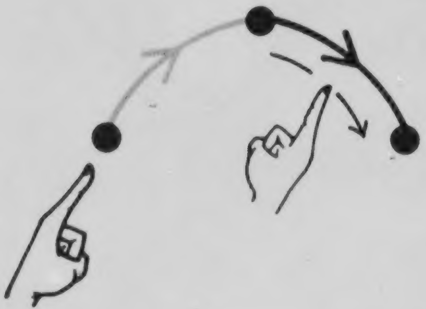
Use the upper left corner of the arrow picture to review the rule of the game. Make the motions described in the right-hand column below as you explain the rule.

T: Each time there is a yellow arrow . . .



Point and hold your left forefinger at a dot where a yellow arrow starts. Follow the yellow arrow in the direction of the arrow with your right forefinger.

T: . . . followed by a blue arrow, . . .



Stop the motion of your right forefinger at the middle dot, tap the dot, and then follow the blue arrow. Hold your right forefinger at the ending dot of the blue arrow.

T: . . . we can draw a red arrow from the dot where the yellow arrow starts . . .

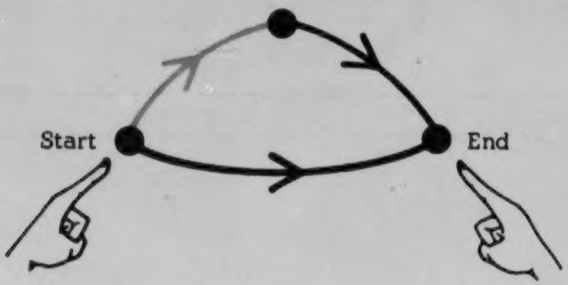
Tap this dot several times with your left forefinger.

. . . to the dot where the arrow ends.

Tap this dot several times with your right forefinger.

L 7

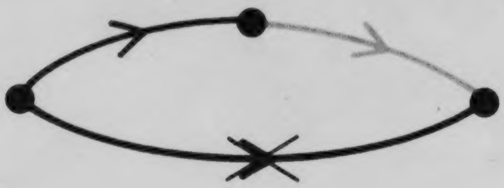
Trace and then draw the red arrow. Remind the class that your left forefinger marks the start of the red arrow and that your right forefinger is at the end of the red arrow.



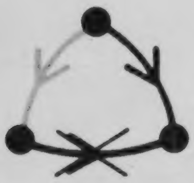
T: Who can show us where we can draw other red arrows?

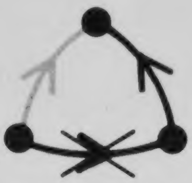
Invite students to show where red arrows can be drawn. Each time ask the student to trace a yellow arrow followed by a blue arrow, and then to trace the anticipated red arrow. Encourage the class to help you catch mistakes such as tracing against the direction of an arrow. If correct, let the student draw the suggested red arrow.

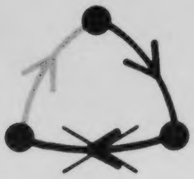
Note on common mistakes and difficulties: The most common mistake at first will occur when students confuse the rule "yellow followed by blue" with "blue followed by yellow."



Another mistake might occur when the directions of the arrows are not correctly followed.

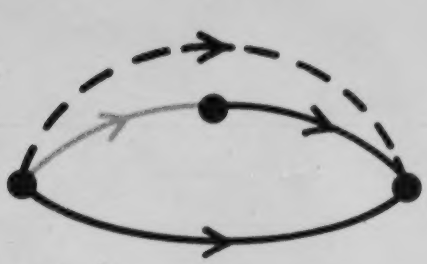






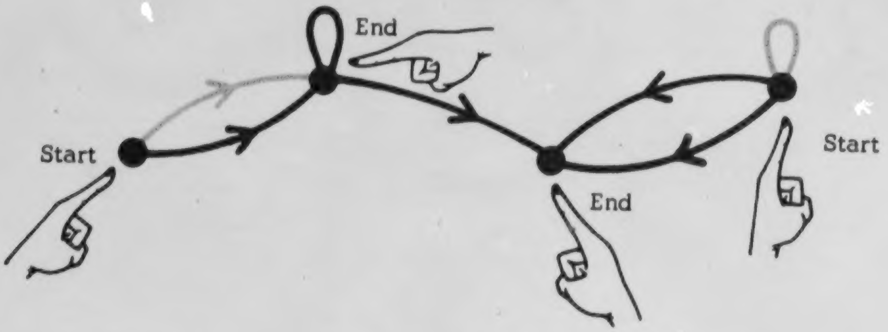
Remind students making such mistakes that they must follow the direction of the arrows. The arrows are like one-way streets and they are not allowed to go the wrong way.

If it happens that a student looks at a portion of the arrow picture and proposes to draw a red arrow as indicated below by the dotted arrow, . . .



another student will probably remark that the red arrow has already been drawn. There is nothing wrong with the dotted arrow; it is merely redundant. That portion of the picture is complete without it.

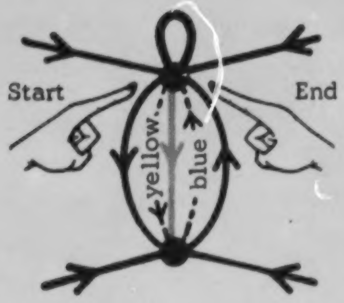
The yellow and blue loops may cause difficulty. If no one discovers one of these red arrows, you could suggest one of them yourself.



It is important here to emphasize where you start and where you end.

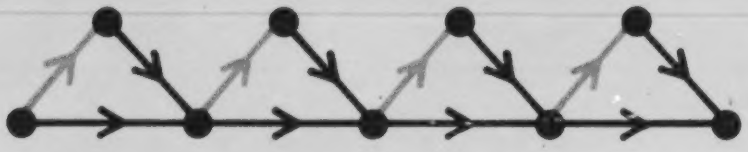
The class might overlook the possibility of a yellow arrow followed by a blue arrow starting and ending at the same dot. In this case, the result is a red loop at that dot. (See the next illustration.)

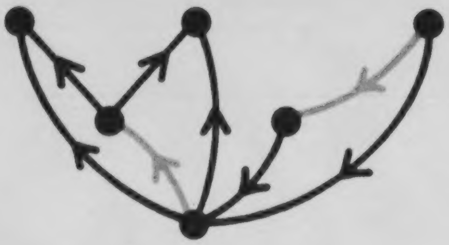
L 7

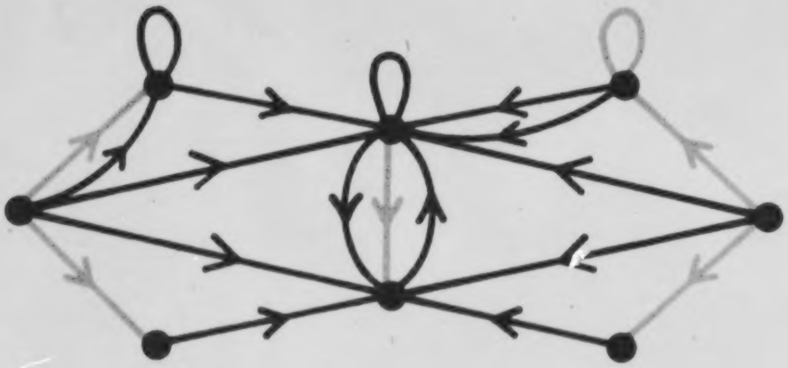


If necessary, point out this possibility yourself and draw the red loop.

When all the red arrows and loops have been drawn, your picture should look like this.



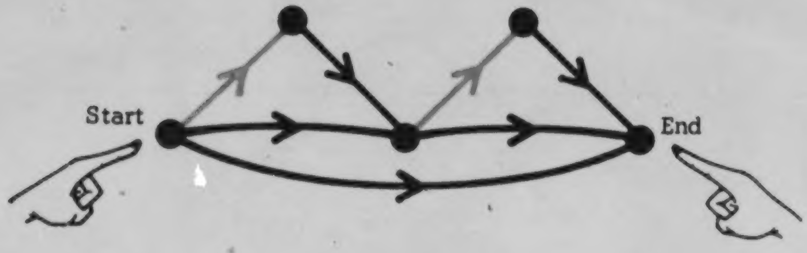




T: Let's introduce a new rule to our game.

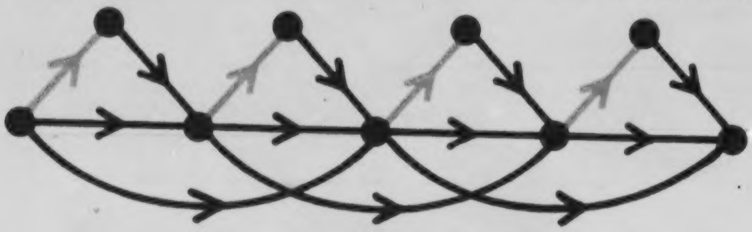
Use the upper left corner of the arrow picture to introduce the new rule. Go through motions similar to those described for the first rule.

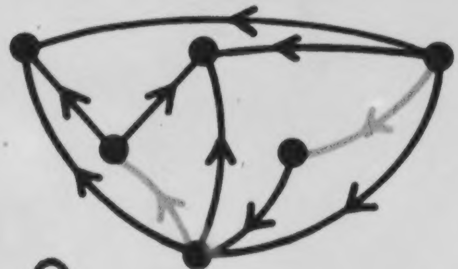
T: A red arrow (trace a red arrow) followed by a red arrow (trace another red arrow) is a green arrow (trace and then draw a green arrow).

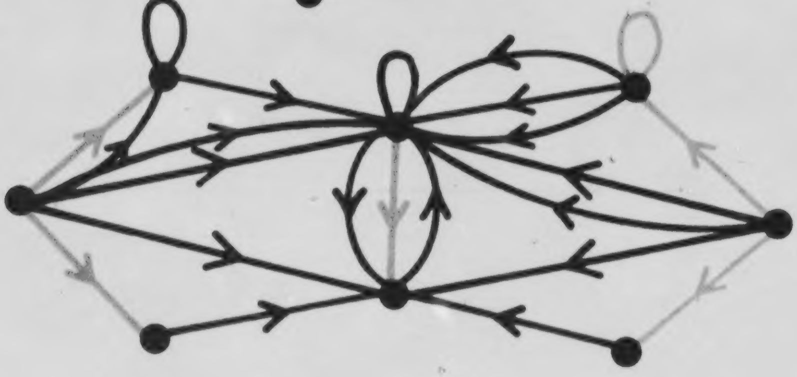


Repeat the rule and then invite students to use the rule to find other places in the picture where green arrows can be drawn. As before, encourage the class to help you catch mistakes. When a student correctly traces a green arrow, let the student draw it.

When all the green arrows have been drawn, your picture should look like this.



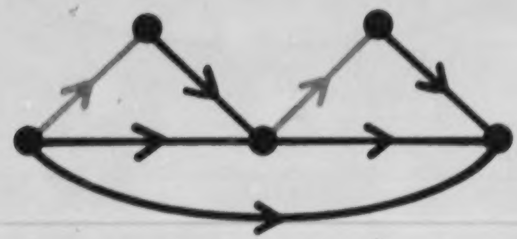




L 7

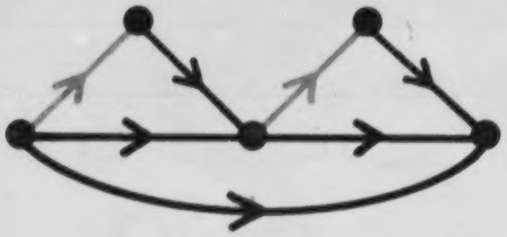
Erase the board except for the upper left corner. This part of the picture serves to remind us of the two rules:

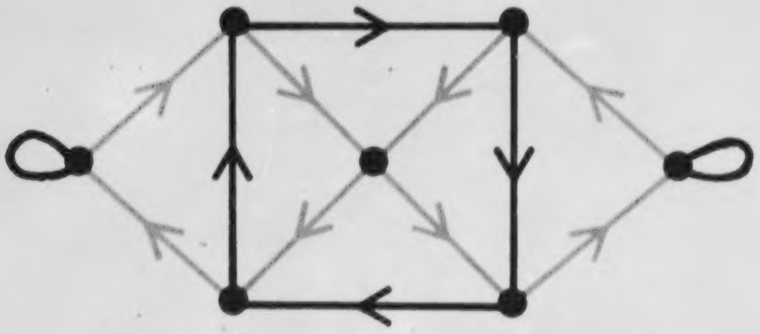
(1) yellow followed by blue is red
(2) red followed by red is green

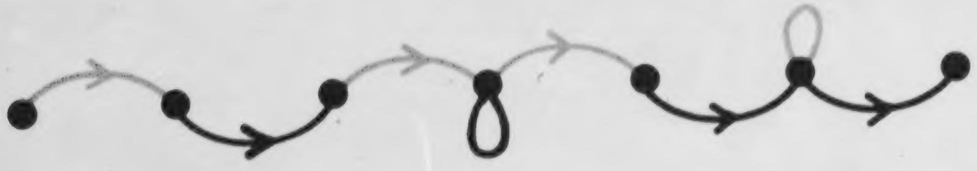


Exercise 2

Draw this picture on the board. As you draw the picture, ask the class to begin looking for places to draw red and green arrows, using the two rules from Exercise 1.

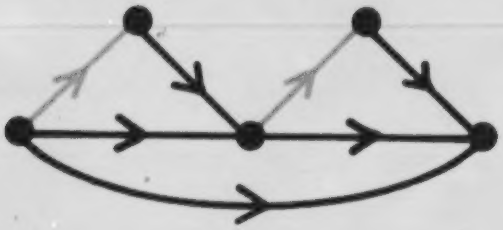


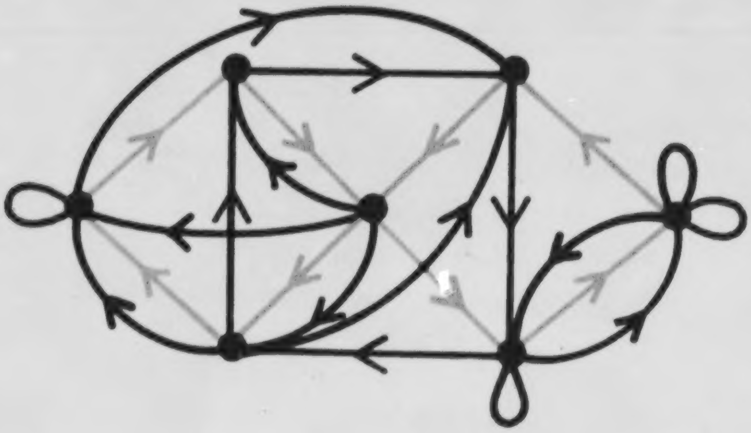


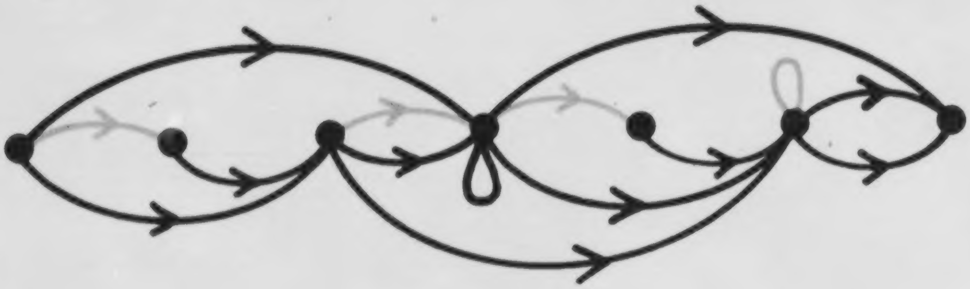


Invite students, one at a time, to trace and then to draw red and green arrows. Encourage all students to participate in this activity. A student might suggest a green arrow before appropriate red arrows have been drawn. In such a case, if the student explains that yellow followed by blue followed by yellow followed by blue is green, accept the green arrow. Otherwise, say that the arrow might be okay, but that the class should try to find the red arrows to check.

A completed picture is shown below.







L 8

L8 Telling Stories with Arrows

CAPSULE LESSON SUMMARY

Present several different arrow pictures and for each one tell stories that the picture could be about. Observe some important details of the story as shown by the picture.

MATERIALS

Teacher: Colored chalk
Student: None

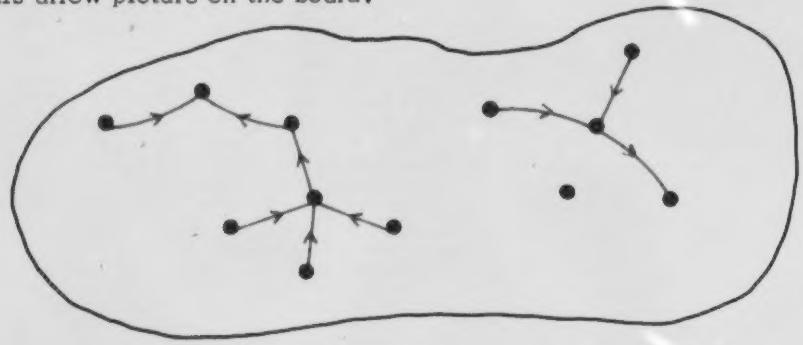
DESCRIPTION OF LESSON

Each exercise presents an arrow picture and students are invited to invent stories. They attempt to describe a story that the arrow picture could be about. After a student tells a story, the other class members give their comments or opinions about whether the picture could be for that situation. You should direct the discussion of a story by asking questions that will help students become more familiar with the arrow language. If students are initially reluctant to tell their own stories or their imaginations "run dry", you may wish to present some stories yourself.

For each exercise, we present dialogue for the discussion of one possible story and we give summaries of some other possible stories. These situations should not limit your discussions. Your students certainly will find different stories and the questions might come from the students. For each picture you present, encourage the class to discuss more than one story in order to reinforce the idea that one arrow picture can represent many different situations.

Exercise 1

Draw this arrow picture on the board.



T: What story could we tell about this picture? What could the dots and the red arrows be for?

Let students comment and ask questions. Lead a discussion of your class' ideas. For example,

S: The dots are for people and they are pointing to their mothers. The red arrows are for "you are my mother."

Allow other students to comment before asking any specific questions.

T: How many people are there in this picture?

S: Twelve.

T: How many can we be sure are mothers?

S: Five.

T (pointing to one of the dots which is both a starting and an ending dot of red arrows): What do we know about this person?

S: She is a mother and one (or more) of her children is in the picture.

T (pointing to the dot at which no red arrow starts or ends): What about this person?

S: That person's mother is not in the picture.

S: That person might be a boy.

S: That person might be a girl but not a mother with children in this picture.

Some other stories which could be told about this picture are summarized below.

- The dots are for numbers and the red arrows are for "I am more than you."

  For such a story, you might discuss possible ways to label dots or additional red arrows that you are sure can be drawn.

- The dots [illegible] for children and the red arrows are for "I sent a message to you."

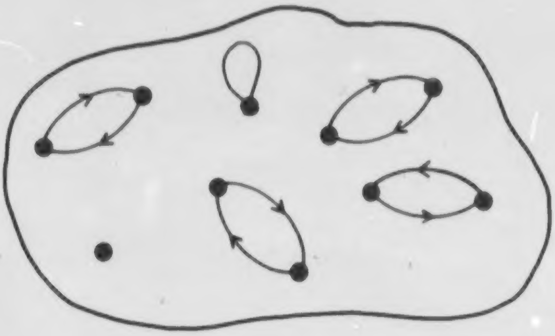
  For such a story, you might discuss which children received just one message or no message or more than one message.

- The dots are for children and the red arrows are for "you are my brother."

  For such a story, you might discuss which dots are surely for boys, where additional red arrows certainly can be drawn, and how many families are represented in the picture.

Exercise 2

Draw this arrow picture on the board.



T: What story could we tell about this picture? What could the dots and the blue arrows be for?

Let students comment and ask questions. Lead a discussion of any suggestions; for example,

S: The dots are for numbers and they are pointing to their 20-freinds. The blue arrows are for "you are my 20-friend."

NOTE: Two numbers are 20-friends if and only if their sum is 20.

Allow other students to react before asking any specific questions.

T (pointing to two dots connected by blue arrows): What numbers could be here?

S: 15 and 5.

S: 0 and 20.

S: 21 and $\hat{1}$.

Let the students suggest several pairs of numbers which are 20-friends.

T (pointing to the dot with a loop): Which number could this be?

S: 10; 10 is its own 20-friend because $10 + 10 = 20$.

T (pointing to the dot with no arrow): Which number could be here?

S: It could be 1 if 19 were not in the picture.

Let the class label the dots in the picture as they wish, but insist that they check the correctness of the labeling.

Some other stories which could be told about this picture are summarized below.

- The dots are for children and the blue arrows are for "you are my best friend."

  For such a story, you might discuss what the child at the loop is saying, what situation is pictured by the four pairs of connected dots, and what knowledge we have about the child whose dot has no arrow starting or ending at it.

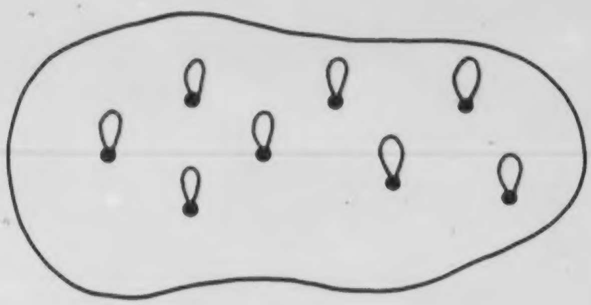
L 8

- The dots are for people and the blue arrows are for "I sent a letter to you."

  For such a story, you might discuss how many letters are sent, what knowledge we have about the person at the loop or the person whose dot has no arrow starting or ending at it, and what situation is pictured by the four pairs of connected dots.

Exercise 3

Draw this picture on the board.



T: What story could we tell about this picture? What could the dots and the green arrows (loops) be for?

S: The dots could be for numbers and the green arrows could be for + 0. Then there would just be loops.

Let the class discuss this possibility before asking specific questions.

T: How many numbers are in the picture?

S: Eight.

T: Which numbers could be in the picture?

S: Any eight numbers. For any number, when you add zero you get that number.

57

Some other stories which could be told about this picture are summarized below.

- The dots are for children and the green arrows are for "I have the same birthday as you."

  For such a story, you might discuss how to select eight children to put in the picture (so that no two have the same birthday).
- The dots are for numbers and the green arrows are for 1 x .
- The dots are for numbers and the green arrows are for "is equal to."

L 9

L9 Multiples #1

CAPSULE LESSON SUMMARY

Play a type of "Guess My Rule" game where the players decide that a given string is for multiples of 3. Locate many integers, positive and negative, in the string picture. Label some dots in a double spiral arrow picture and find that the arrows could be for + 3. Then observe that the numbers on the spiral are the same as those inside the string. Discuss other multiples, for example, multiples of 5 and of 2. Locate numbers in a two-string picture with one string for multiples of 2 and the other for multiples of 5.

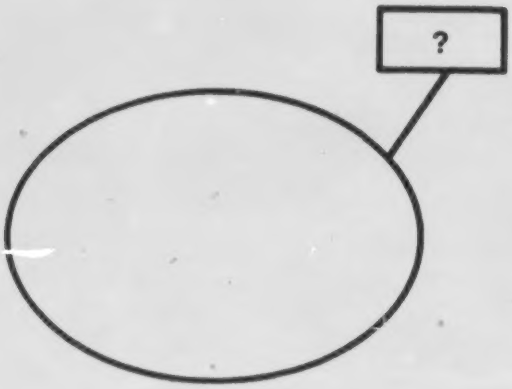
MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Draw a large string on the board.



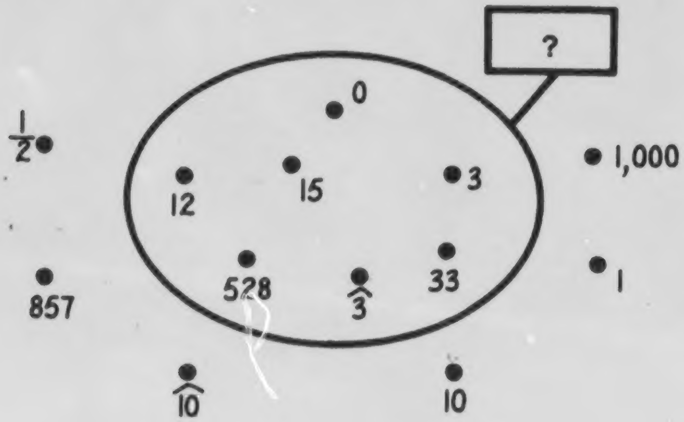
NOTE: The string is for multiples of 3, but do not announce this or use the word "multiples" until later in the lesson.

Tell the class that you are not going to tell them yet what this string is for except that it is for some numbers. They may choose numbers and you will place them correctly in the picture, either inside the string or outside the string. When they think they know what the string is for, they may guess and you will tell them whether or not they are correct.

Give each student a chance to choose a number or to guess what the string is for. That is, go around the room and ask each student, in turn, either to give you a number to put in the picture or, to guess what the string is for.

NOTE: For your information only, an integer is a multiple of 3 if and only if the sum of its digits is a multiple of 3.

After a short while your picture should have many numbers in it and some students may have made some "wild" guesses. For example,



If students guess that the string is for even numbers or odd numbers or more than 10 or less than 1,000 or something else which can easily be seen to be impossible, ask the class to say why the guess is incorrect. For example,

S: The blue string cannot be for even numbers, because 15 is inside and 15 is odd.

S: The blue string cannot be for less than 1,000, because 10 is outside and 10 is less than 1,000.

Although your students may be unfamiliar with the term "multiples of 3", they should soon begin to notice that the numbers inside the string have something to do with counting by threes. When this occurs, invite students to predict where certain numbers will go in the picture.

L9

T: Where should I put 30?

S: Inside the string, because you say "30" when you count by threes.

Invite a student to count by threes starting at 0 and ending at 30.

T: Where should I put 37?

S: Outside the string, because from 30 we go next to 33, 36, 39 and skip over 37.

T: Where should I put 300?

S: Inside the string, because if you count 100 times by threes you'll get to 300.

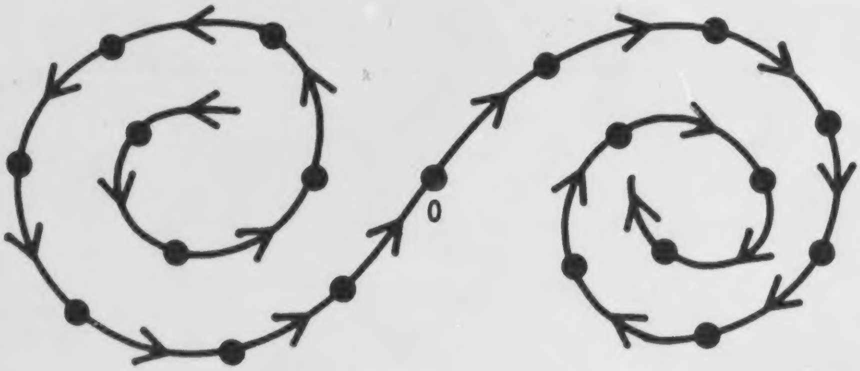
If you already have some negative numbers in the picture, discuss also counting backward by threes and put some more negative numbers in the picture. Otherwise, wait for the next exercise to discuss negative numbers.

T: We have a name for the numbers inside this string. They are called "multiples of 3".

Write this term inside the label for the string. Leave your picture on the board as you will refer back to it in Exercise 2.

Exercise 2

Draw a double spiral arrow picture on the board and label the middle dot "0".

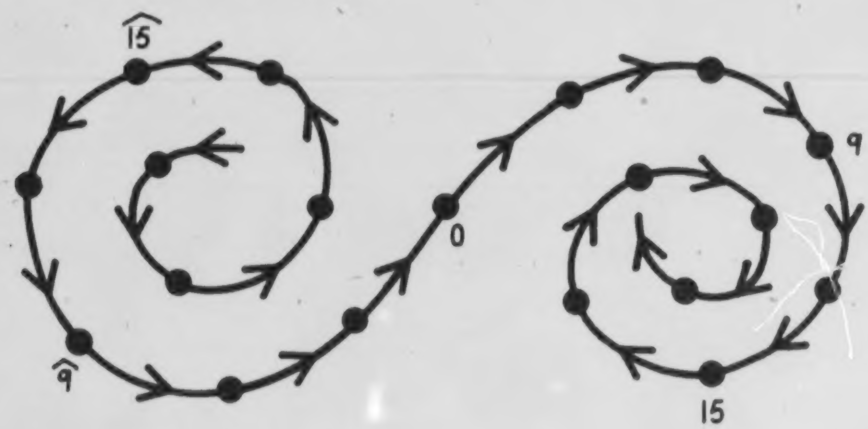


T: What do you think about this picture?

Let the class react. Perhaps they will mention its symmetry, that 0 is in the center, that it could go on and on in both directions, etc. Allow a minute or so of such observations before continuing, but do not insist that they mention these things.

T: Let's put some more numbers in the picture.

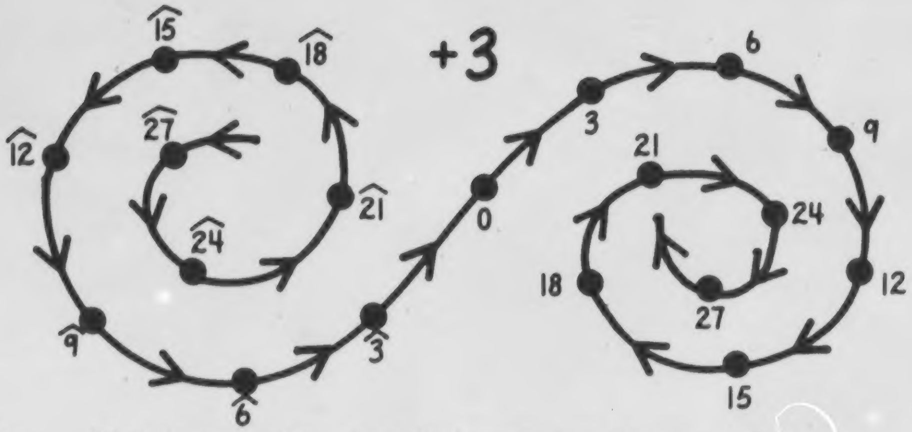
Label dots for 9, $\hat{9}$, 15, and $\widehat{15}$.



T: What could the red arrows be for?

Allow the students to experiment with some possibilities. For example, if a student suggests +9, observe that then 9 would be at the end of the arrow starting at 0, not where it is. Perhaps a student will suggest "is less than" and then you could observe that this is a possibility, but ask for another possibility. When a student says "+3", check that indeed the red arrows could be for +3 and write the key near the picture. Otherwise, put in some more numbers, say 6 and $\hat{6}$, and ask the question again. After it is established that the red arrows are for +3, invite students to label the other dots.

L 9



T: What do you notice about the numbers in this picture?

S: They are alike on both sides; 3 and $\hat{3}$, 6 and $\hat{6}$, 9 and $\hat{9}$, and so on.

S: They are the numbers we say when we count by threes, forward and backward.

S: They are the numbers inside the string, the multiples of 3.

If this last observation is not made, call the class' attention back to the string picture and ask specifically which numbers in the string picture would be in the arrow picture. (The numbers inside the string)

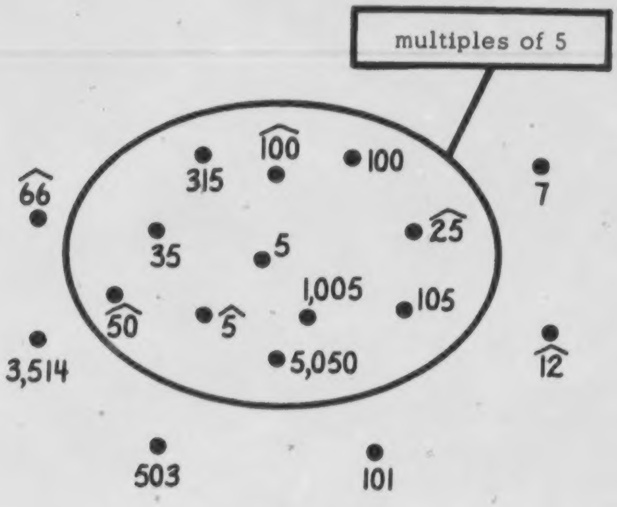
Continue by asking students to predict whether certain numbers are multiples of 3; that is, whether they would be in the double spiral for +3. Include some large numbers such as 300, 333, 900, 906, and 3,000.

At this point, if there are no negative numbers in the string picture, direct the class to observe that all the numbers in the arrow picture are inside the string, including the negative numbers.

Exercise 3

Repeat Exercise 1 for multiples of 5. The students should quite quickly guess that your new string is for multiples of 5 and perhaps will even use the expression "multiples of 5".

After the class discovers that your string this time is for multiples of 5, let them put in some numbers themselves. Encourage students to include both positive and negative numbers and some large numbers.


multiples of 5
$\widehat{100}$
315
100
$\widehat{66}$
7
35
5
$\widehat{25}$
1,005
105
$\widehat{50}$
$\widehat{5}$
3,514
5,050
$\widehat{12}$
503
101

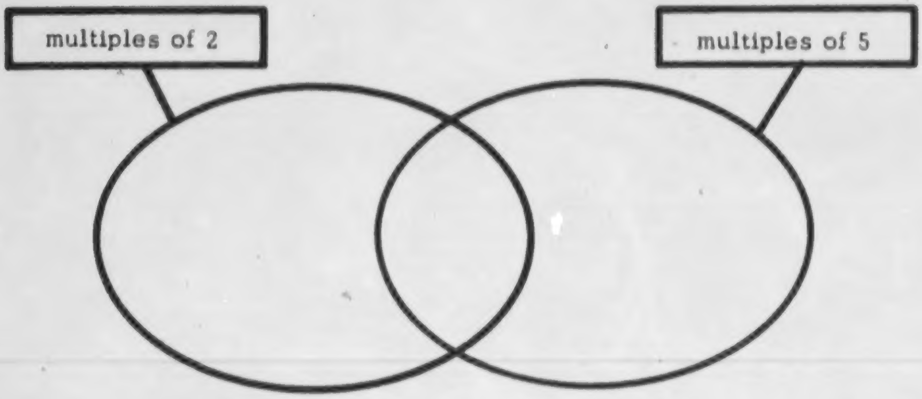
T: If I were to draw an arrow picture as before to include all the multiples of 5, what kind of arrows would I use?

S: +5.

L 9

Exercise 4

Ask the class which numbers are the multiples of 2 (even numbers) and then draw this picture on the board.


multiples of 2
multiples of 5

Instruct the students to draw this picture on their papers and to put in as many numbers as they can.

L10 Trees #3

CAPSULE LESSON SUMMARY

Review the notion of a tree, observing that in a tree there is just one path between two dots. Find the distance between selected pairs of dots in a tree and decide which is the longest distance between dots in the tree. Given three dots in a tree (labeled "A", "B", and "C") find the length of a trip from A to B to C to A.

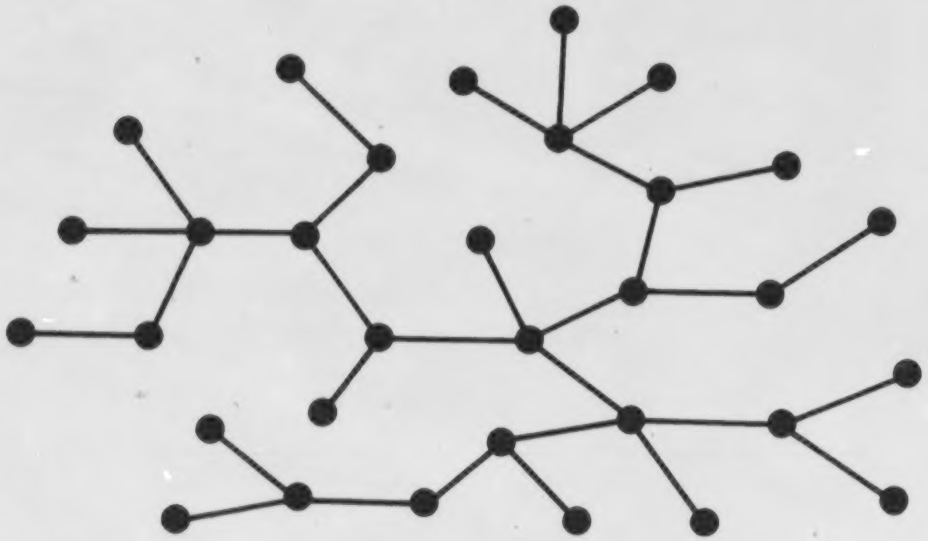
MATERIALS

Teacher: Colored chalk

Student: Worksheet L10; pencil

DESCRIPTION OF LESSON

Before the lesson begins, draw this picture on the board.



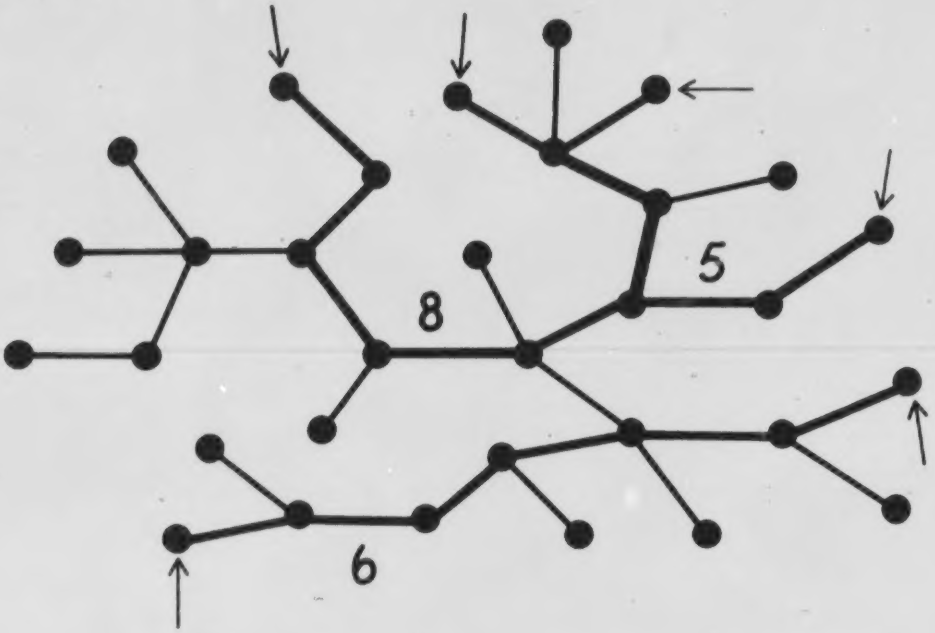
Exercise 1

T: How many edges are there in this tree? How many dots?

S: Thirty-one edges and thirty-two dots.

L 10

Review briefly the idea that in a tree there is exactly one path (if you are not to backtrack) between two dots. For each pair of dots you choose, look also at the number of edges in a path between them and announce this number as the distance between these dots. Some examples are shown in the next illustration.

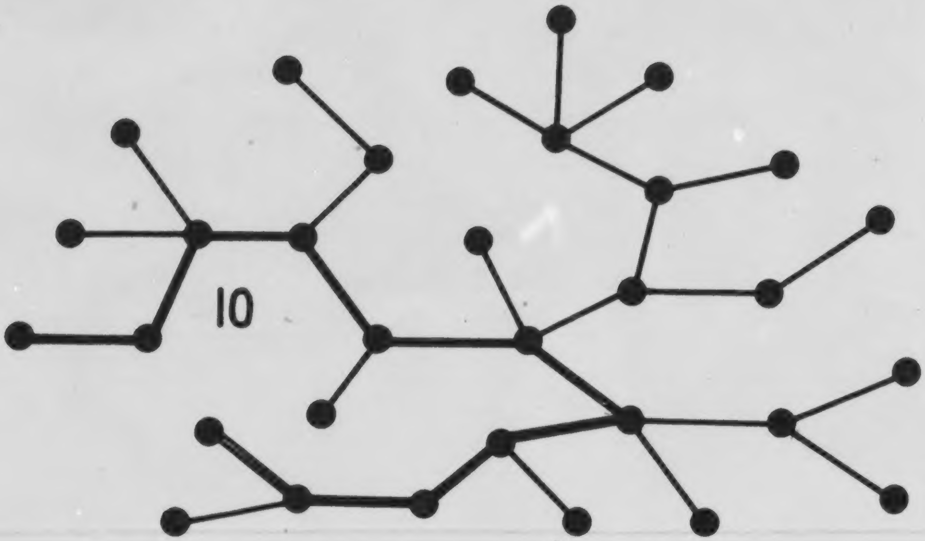


Next, ask the class to find a pair of dots at a specified distance from each other; for example, two dots at a distance of 4 (7, 3, 5, 1) from each other.

If you have drawn any paths in color on the tree, erase them leaving just the original drawing of a tree.

T: Now, try to find two dots at the longest distance apart in this tree.

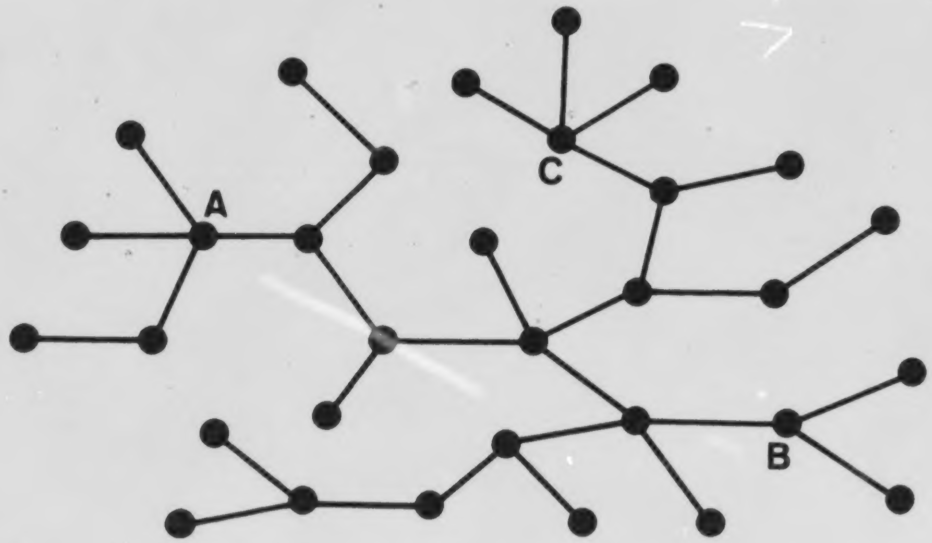
After some trial and error experimenting with this problem, the class should find that the longest distance between two dots is 10 as shown in the next illustration.



If you have drawn any paths in color on the tree, erase them again leaving just the original drawing of the tree.

Exercise 2

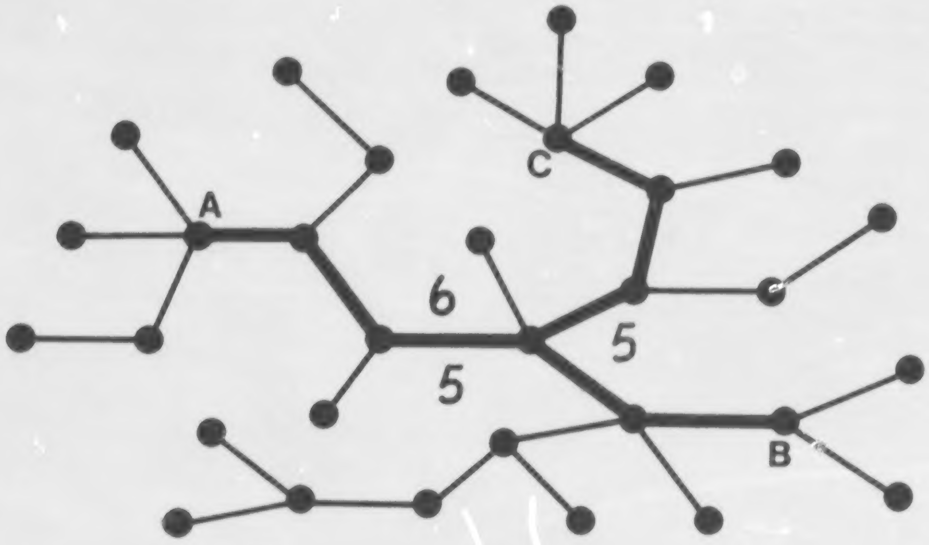
Label three dots "A", "B", "C" as shown below.



L 10

T: Imagine that this tree is the map of a city. The edges are the streets. Anthony lives here (point to "A"). His mother sends him on an errand to the bookstore here (point to "B"). Then he may go to the candy store here (point to "C") before he returns home (point to "A"). How long is the trip Anthony makes? Remember, he goes from A to B, then from B to C, and then from C to A.

First let students work out the solution (16) on their own. Then, invite students to trace the path from A to B, the path from B to C, and the path from C to A. Draw these paths in color on the picture and find the length of each path. Add the three lengths to find the total length of Anthony's trip.



Length of trip	=	Distance from A to B	+	Distance from B to C	+	Distance from C to A
16	=	5	+	5	+	6

Repeat this exercise with another choice of three dots labeled "A", "B", and "C". If the class enjoys such problems, you might wish to repeat the exercise several times and observe that each time the trip has an even number as its length.

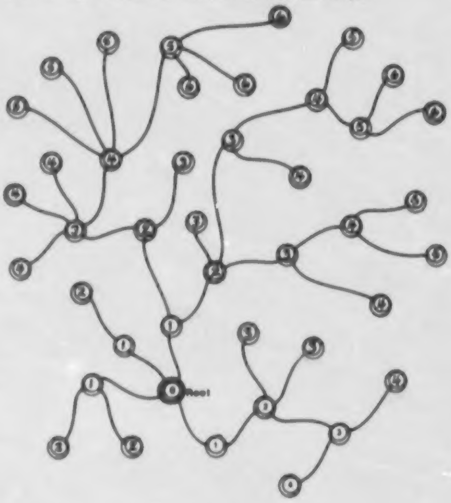
Distribute Worksheet L10. Explain to the class that a root has been marked (circled in red) on this tree. They must find the distance from the root to each of the other dots, and write it inside the dot. Some are done for them; you may wish to check these collectively.

As students complete this work, instruct them to circle in red those dots at an even distance from the root and to circle in blue those dots at an odd distance from the root.

70

Name ______________________ **L 10**

Find the distance from the root (circled red) to each of the other dots in this tree. Write your answers in the circles. Some are done for you.



L11 The String Game #2

CAPSULE LESSON SUMMARY

Introduce the "not" cards as new possibilities for string labels in the String Game. Play a game placing the pieces in a picture with visible labels, both involving "not". Play the String Game in the usual way with hidden labels.

MATERIALS

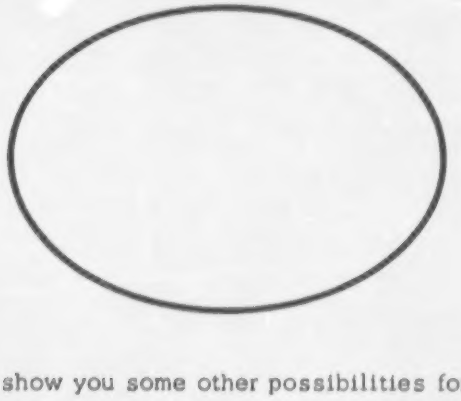
Teacher:	A-Blocks String Game kit; colored chalk; (optional) magnetic board
Student:	None

DESCRIPTION OF LESSON

Prepare the equipment needed to play the String Game with A-Blocks. (See the APPENDIX: The String Game for a complete description of equipment and preparation for the game.) Let the class quickly review the attributes for the A-Blocks; that is, sizes, colors, and shapes.

Exercise 1

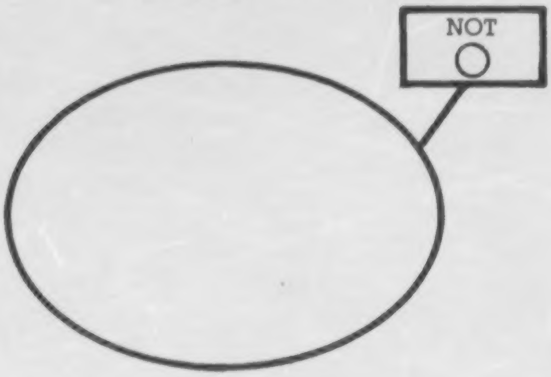
Draw one string on the board.



T: I'm going to show you some other possibilities for string labels.

L 11

Show the class the string card "NOT ○" and label the string accordingly.



T: If this string is for all pieces which are not circles, what would be a piece that goes inside the string?

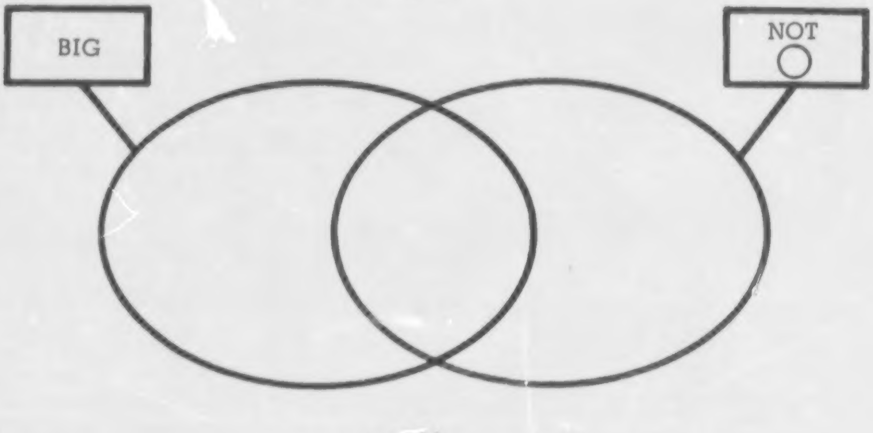
S: The small red square.

T (putting the piece inside the string): Yes, this piece is not a circle. What would be a piece that goes outside the string?

S: A large blue circle.

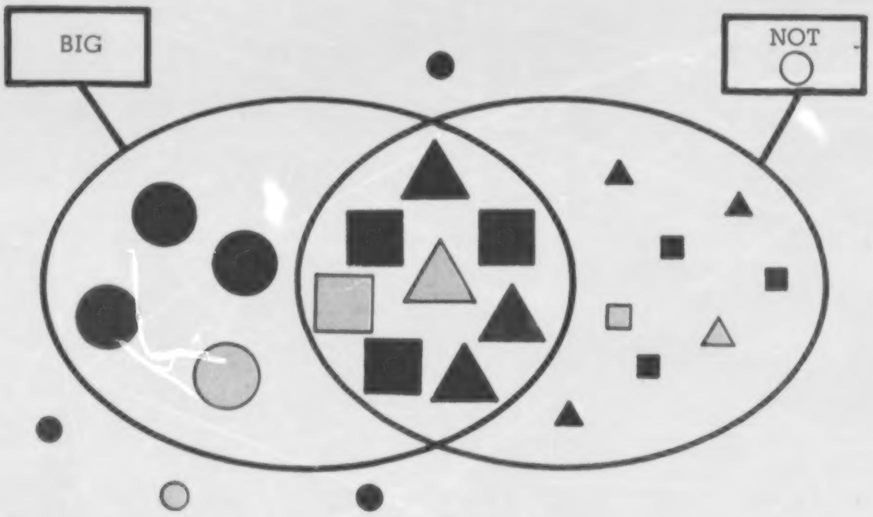
T (putting the piece outside the string): Yes, circles go outside the string.

Let several other students suggest pieces that go inside the string and pieces that go outside the string. Then clear the board of all game pieces and draw a second overlapping string labeled "Big".



L 11

Call on students to place several game pieces of their choice in this picture. Discuss with the class why each placement is correct or incorrect, as the case may be. For your information, the following picture shows correct placement of all the game pieces.



After several pieces are in the picture, at least one in each region, clear the board of the game pieces and erase the string labels.

T: What other new string labels do you think there are?

Show the class each string card as it is mentioned.

S: "Not Red".

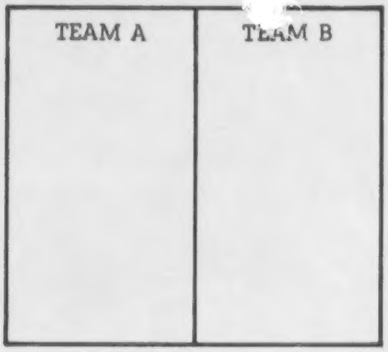
S: "Not □".

Perhaps a student will mention "Not Little" or "Not Big". In any case, discuss why such new string cards are not needed. (Pieces that are not little are the same as those that are big, and pieces that are not big are the same as those that are little.)

L 11

Post the full list of sixteen possible string labels above the Team Board.

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE



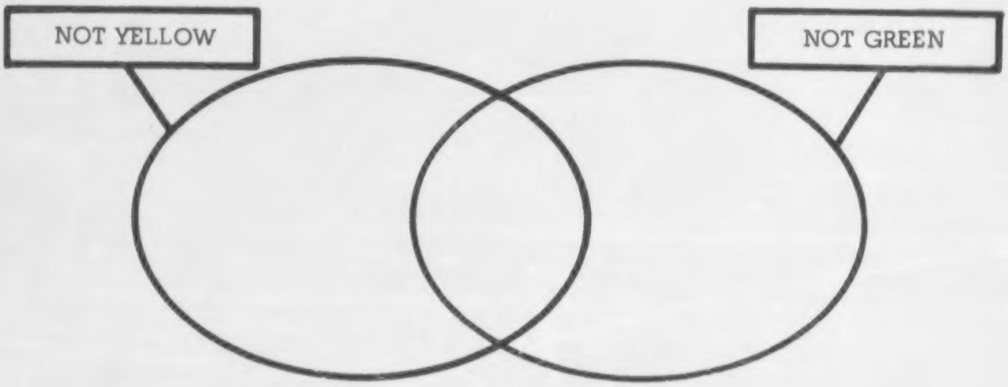
T: Now, when we play the string game, we have all of these possibilities for the string labels.

Exercise 2

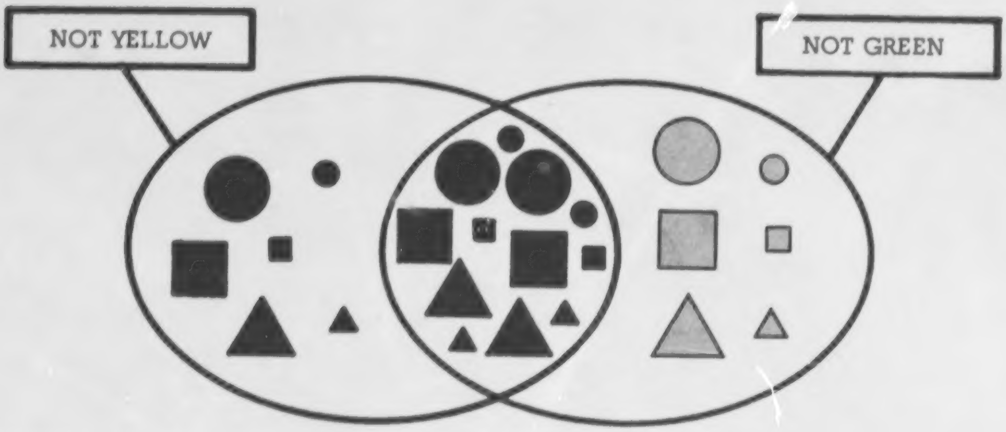
Prepare to play the string game by dividing the class into two teams, Team A and Team B. Distribute the game pieces on the two sides of the Team Board.

T: First we are going to play a warm-up game with your knowing the string labels.

Label the strings as shown below.



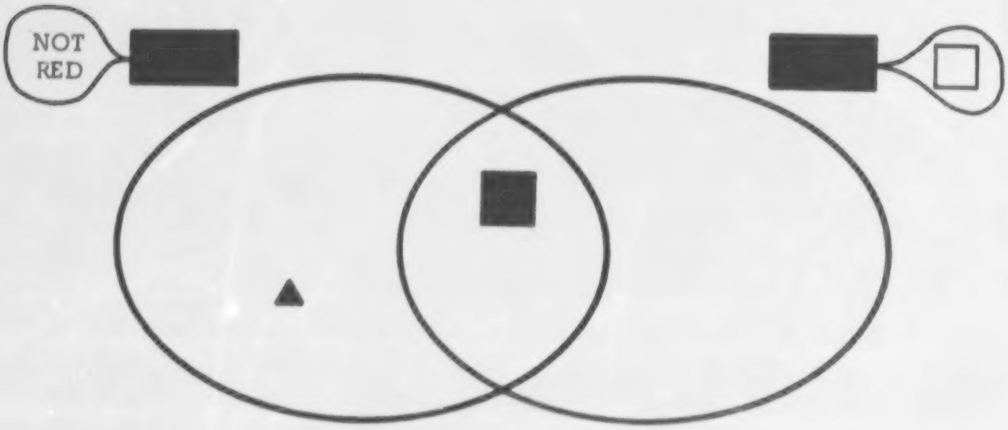
Play the game in the usual way alternating turns between the two teams and among the members of a team. Since the labels are visible, no starting clues are necessary. The first team to place correctly all its game pieces is the winner. For your information, correct placement of all game pieces is shown below.



Exercise 3

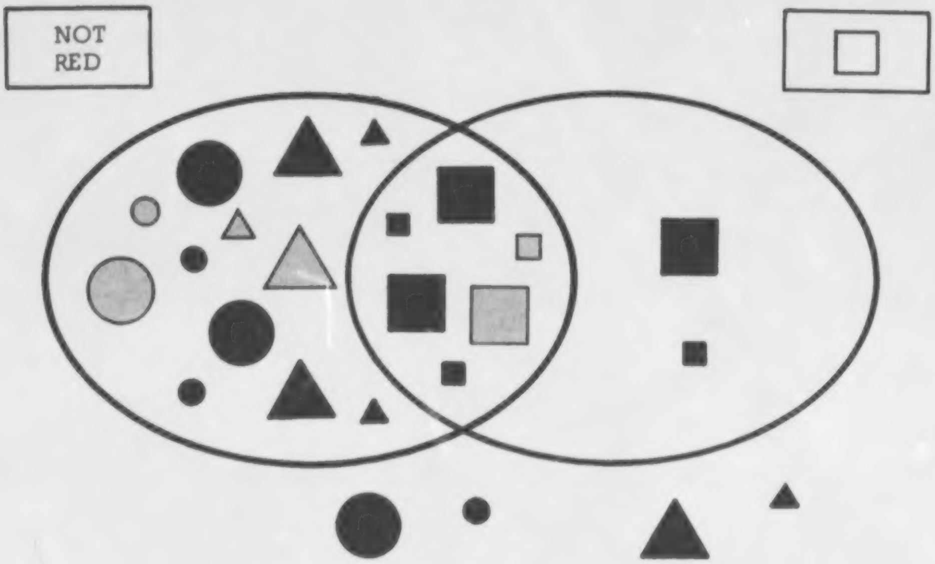
Play a round of the usual two-string version of the String Game with face down string cards. Redistribute game pieces on the two sides of the Team Board. The next illustration shows the starting clues for a game with hidden labels indicated in the bubbles. The second illustration shows correct placement of all game pieces for this game.

Starting Clues



L 11

Correct Placement of Game Pieces



L12 Two Dice #1

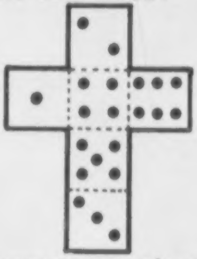
CAPSULE LESSON SUMMARY

Examine a die noting that it is a cube and what is on the six faces of the cube. Discuss the chances of getting various outcomes when tossing one die. Determine all the possible outcomes when tossing two dice and picture them in a coordinate graph. Discuss the possible sums (add the numbers obtained on the two dice) you could get when tossing two dice and find that some sums have a better chance of occurring than others.

MATERIALS

Teacher: Two dice; colored chalk; die cutout
Student: None

NOTE: Before the lesson begins, make a paper cutout of a die as illustrated below. Make the squares between 5 cm and 6 cm on a side. The dotted lines indicate fold lines.



The dice provided in your classroom set of materials are red and white; however, any two different colors could be used.

DESCRIPTION OF LESSON

Hold up the two dice for the class to see.

T: What do we call these things?

S: Dice.

T: Yes; and one of them is a die. What shape is a die?

Perhaps students will say square, in which case you should point out that one side (or face) of a die is a square.

S: A cube.

Pass around the dice and instruct the students to note what they see on a die when their turn comes to examine one of them.

T: Who can describe a die? How many faces (indicate a face on one die) does it have?

S: Six.

T: What are on the faces?

S: Dots — one or two or three or four or five or six.

Let students make other descriptive comments about a die; for example, someone may mention its symmetry, or that each number from 1 to 6 is shown on a die, or that 1 is opposite 6 and 2 is opposite 5 and 3 is opposite 4. Do not insist on these observations, but let students make whatever comments they wish about a die.

Hold up your paper cutout of a cube.

T: Does this look like a die?

S: It's like a die flattened out.

S: You can fold it to make a cube like the die, with from one to six dots on the faces.

Here is a good time to demonstrate how the paper cutout folds to form a cube. Then tape the cutout to the board.

T (hold up one die): Suppose I tossed one die on the desk. What could I get on the top face?

S: 1 or 2 or 3 or 4 or 5 or 6 (dots).

T: Do I have a better chance to get 1 or 6?

S: The same for both.

Lead a discussion in which the class observes that each outcome has the same chance (one out of six) because the six faces would be indistinguishable without dots; the cube is symmetric.

T: Do I have a better chance to get an even number or an odd number?

S: Also the same, because you have three chances out of six to get an even number (2, 4, or 6) and three chances out of six to get an odd number (1, 3, or 5).

T: Do I have a better chance to get a multiple of 2 or a multiple of 3?

S: A multiple of 2; three chances out of six to get a multiple of 2 (2, 4, or 6) and only two chances out of six to get a multiple of 3 (3 or 6).

T (holding up two dice): Suppose I tossed two dice at the same time. What could I get?

S: 1 or 2 or 3 or 4 or 5 or 6 on the red die, and 1 or 2 or 3 or 4 or 5 or 6 on the white die.

Toss the dice three or four times and observe the outcomes; for example, red 3 and white 5, red 6 and white 5, red 2 and white 1, and so on.

T: How many possible outcomes are there?

Let the class grapple with this question for a few minutes. Be sure they understand that red 3 and white 5 is different from red 5 and white 3. After a minute or two, begin a drawing on the board which will picture all the possible outcomes.

T: What could we get on the red die?

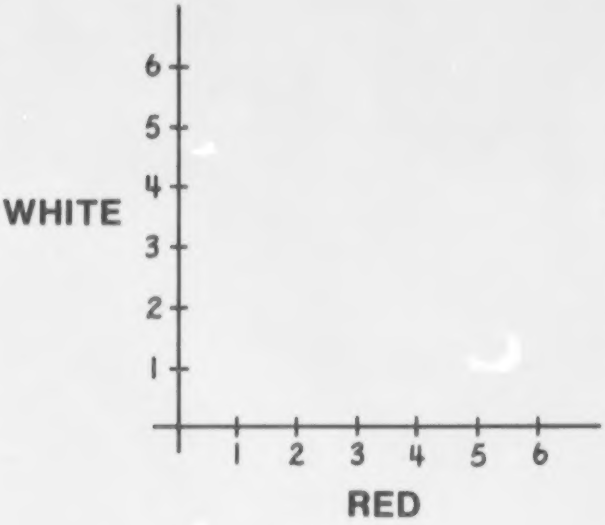
S: 1 or 2 or 3 or 4 or 5 or 6.

Draw a horizontal line with six equally spaced marks and labeled "RED".

T: What could we get on the white die?

S: 1 or 2 or 3 or 4 or 5 or 6.

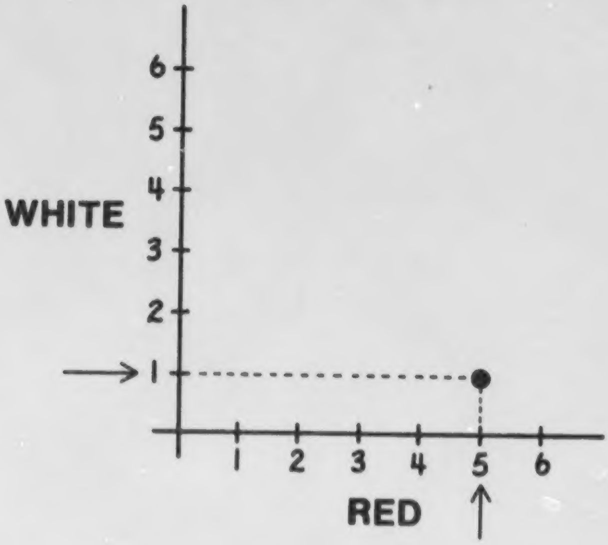
Draw a vertical line as in the next illustration, again with six equally spaced marks and labeled "WHITE".



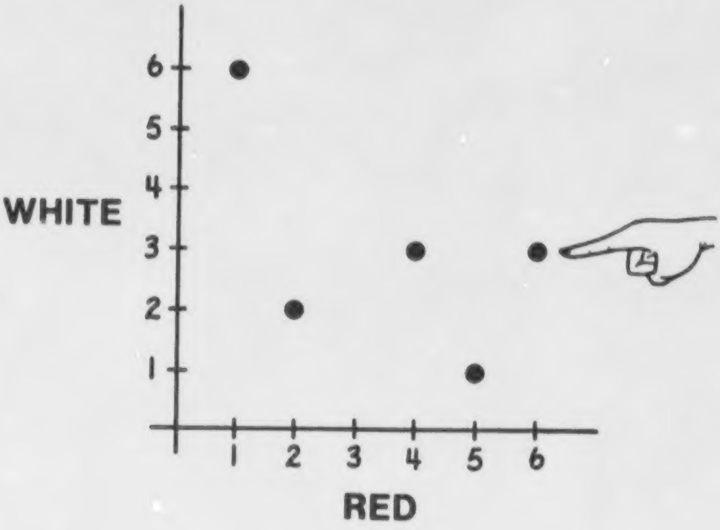
T: Give me one possible outcome.

S: 5 on the red die and 1 on the white die.

Illustrate how one locates a dot for this outcome in the picture.


WHITE
1
2
3
4
5
6
RED
1
2
3
4
5
6

Ask for several more possible outcomes and let students locate dots for these outcomes. Then draw a dot and ask which outcome that dot is for. For example,

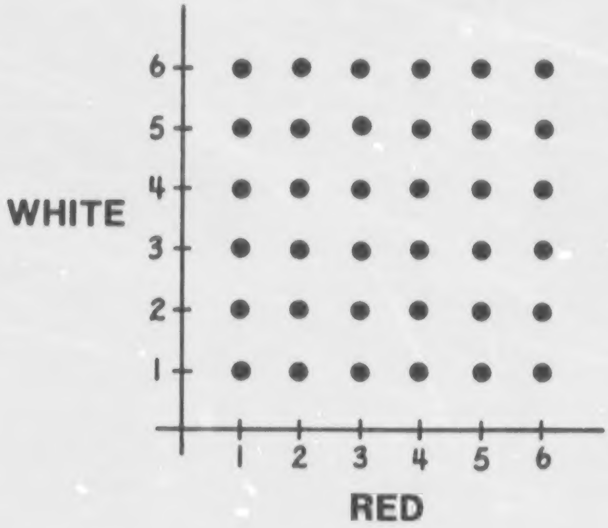

WHITE
1
2
3
4
5
6
RED
1
2
3
4
5
6

T: Which outcome is this dot for?

S: Red 6 and white 3.

L 12

Repeat this kind of question several times. When you have eight to ten dots for outcomes in your picture, ask again how many possible outcomes there are. Eventually the class should recognize that there are thirty-six outcomes, and then you should let them quickly draw in the remaining dots.



T: Now we have a picture of all the possible outcomes. How many are there?

S: Thirty-six.

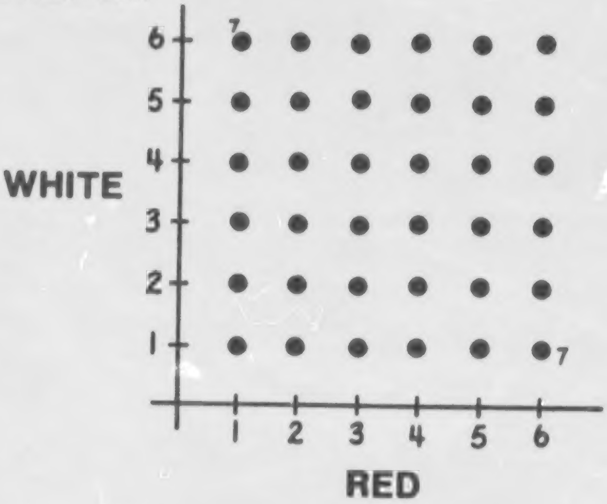
T: Suppose we toss the two dice and add the numbers we get. What sums could we get?

S: We could get 7, 1 + 6 = 7.

T: Are you thinking of 1 on the red die and 6 on the white die or 1 on the white die and 6 on the red die?

S: It doesn't matter; 1 on red and 6 on white or 6 on red and 1 on white, both give a sum of 7.

As different outcomes are mentioned, point them out in the picture and write the sum next to the dot.

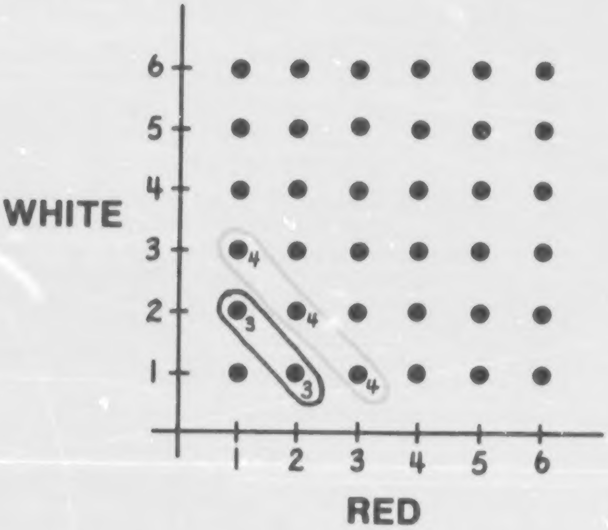


T: What is the largest sum we could get? (12) The smallest? (2)

Lead the class to find that any whole number from 2 to 12 is a possible sum and locate at least one point for each possible sum.

T: Would we have a better chance to get a sum of 3 or a sum of 4?

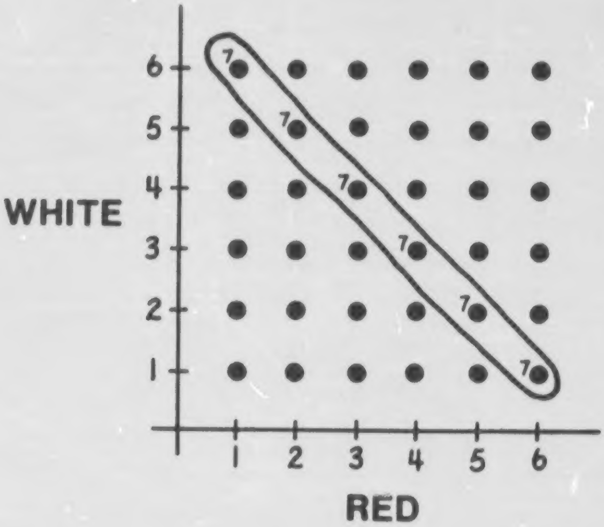
The students should locate all the dots for a sum of 3 and all the dots for a sum of 4 to determine that 4 has a better chance (three out of thirty-six versus two out of thirty-six).



L 12

T: Which sum would we have the best chance of getting?

After examination of all the possible sums, the class should discover that the sum of 7 has the best chance (six out of thirty-six).


WHITE
6
5
4
3
2
1
7
7
7
7
7
7
1
2
3
4
5
6
RED

L13 Two Lists

CAPSULE LESSON SUMMARY

Determine how many names are on two lists when the number on each list is known and where there is some duplication of names on the lists. Make up and solve some similar problems.

MATERIALS

Teacher: Colored chalk
Student: Paper; colored pencils

DESCRIPTION OF LESSON

Begin the lesson by asking each student to write the names of five to ten classmates on a piece of paper. Insist that this be done secretly. As the students complete their lists (with at least five but not more than ten names), collect the lists and look for two which have two or three names in common. In our development class, the following two lists were chosen; therefore, this lesson description uses these lists. You will, of course, need to adjust the lesson to fit your choice of students' lists.

Shenna's list	Daniel's list
Angela	Shenna
Juana	Jason
Kristy	Angela
Sonja	Colin
Lynn	Gary
	Eric
	Kristy

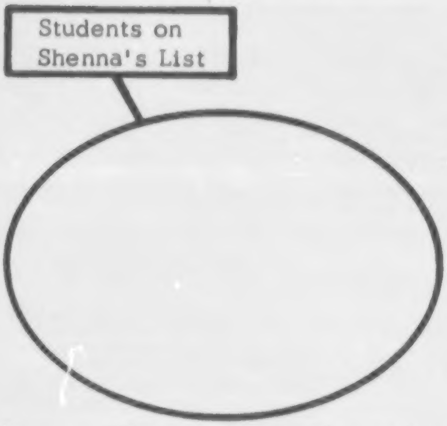
T: I have chosen two of your lists; Shenna's list has five names and Daniel's list has seven names. For the moment, I won't tell you whose names are on these lists, and Shenna and Daniel must keep my secret too. Can you predict how many of you have your names on these lists?

86

L 13

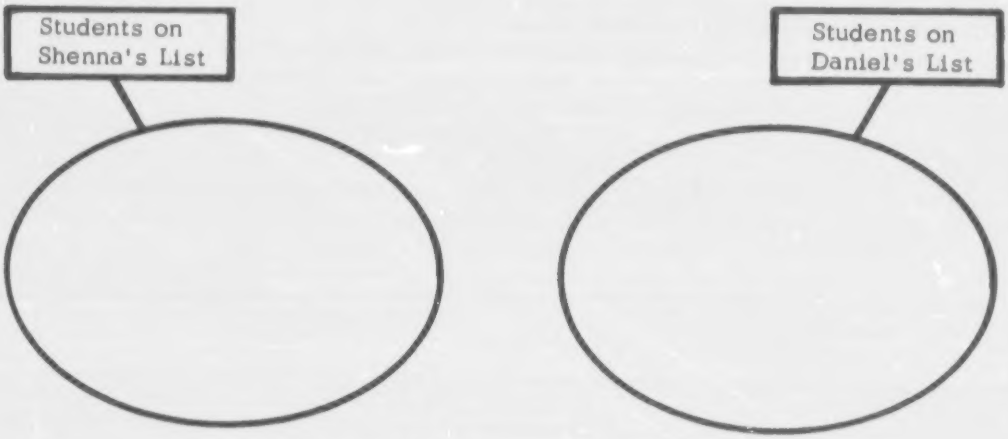
Every prediction is our development class was twelve. It is likely that your students will respond similarly by adding the number of names on the two lists. Perhaps some will observe that they are not sure because a name might be on both lists. Let the students make their predictions without comment from you.

T: Let's draw a picture to see if you are correct. I shall draw a red string for the students whose names are on Shenna's list.



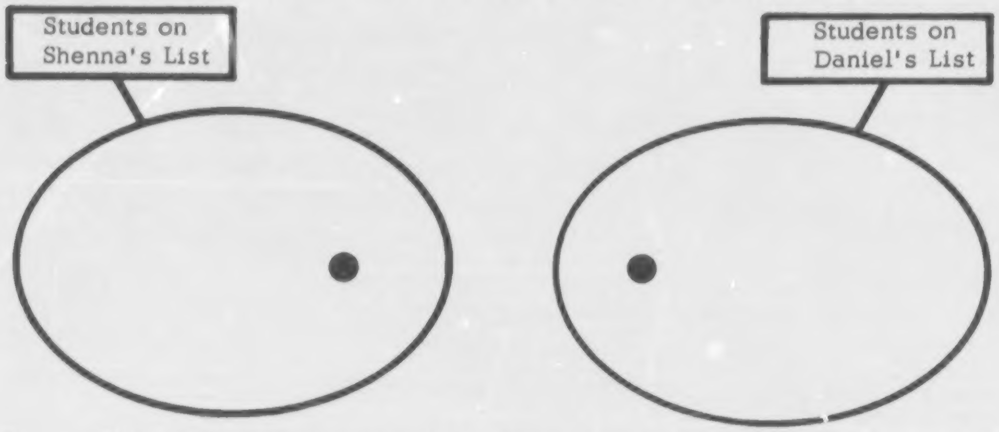
T: Who can draw a blue string for the students on Daniel's list?

Let a student draw a blue string on the board. There are several possible pictures that could result and you will have to adjust your next questions to the situation. As in our development class, you might get this picture and then initiate a discussion similar to that which follows.



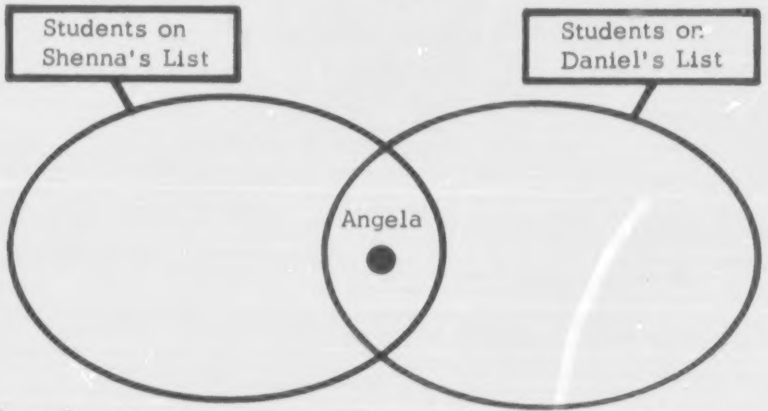
T: I have a problem. Angela's name is on both Shenna's list and Daniel's list. Where should we put a dot for Angela?

A student might suggest that two dots be drawn, one in each string.


Students on Shenna's List
Students on Daniel's List

T: But Angela is just one person. There should be only one dot for Angela. How can we solve the problem?

Perhaps after a couple of other attempts, a student will suggest overlapping the strings and putting a dot in the middle. Let the student (Angela in this case) whose dot is being discussed draw and label the dot.


Students on Shenna's List
Students on Daniel's List
Angela

NOTE: If overlapping strings are drawn immediately in your class, go on with the lesson without mention of the previous problem.

T: Now I shall read the names on Shenna's list. The first one is "Angela" and we have a dot for Angela in our picture. The next one is "Juana". Juana, can you draw a dot for yourself in our picture?

L 13

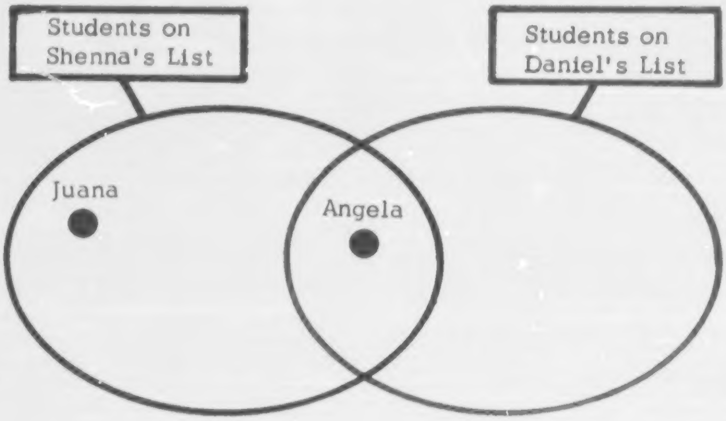
The student could respond in different ways. Perhaps she will guess where to put the dot, or she might hesitate. If a guess is made, ask the student to explain how she knows where to put the dot. Encourage the student to recognize that she cannot be sure where to put the dot.

T: You are right to think about the situation. Perhaps you can ask me a question so you will be sure where to put your dot.

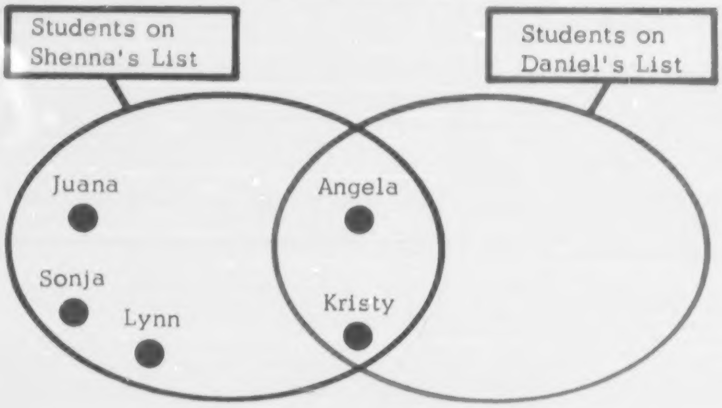
Juana: Am I also on Daniel's list?

T: That's a very good question. No, you are only on Shenna's list.

Let the student draw and label the dot and ask the class to check that it is correct.


Students on Shenna's List
Students on Daniel's List
Juana
Angela

Continue in this way for all of the names on one list.


Students on Shenna's List
Students on Daniel's List
Juana
Angela
Sonja
Lynn
Kristy

At this point you could ask the students whether they still think their predictions are correct. If some students wish to change their predictions, ask them to explain why. Again do not comment on the correctness of their responses.

T: Now I shall look again at Daniel's list. How many names are on his list?

S: Seven.

T: How many students from Daniel's list are already in our picture?

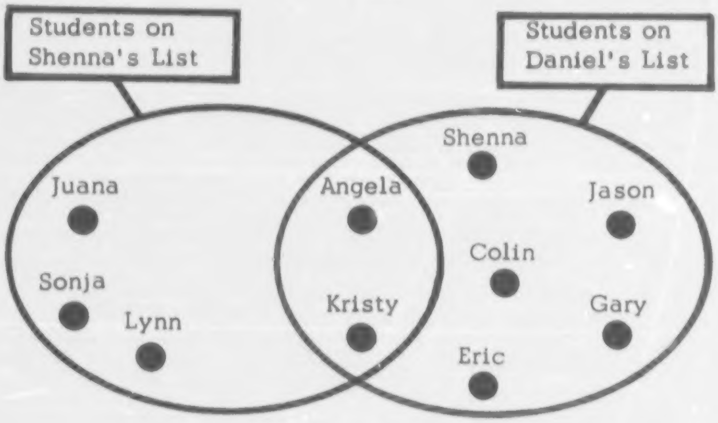
S: Two, Angela and Kristy.

T: That's right. So how many other students did Daniel list?

S: Five.

T: Without knowing their names who can put dots for these five students in our picture?

Let a student do it. Then read the other names on Daniel's list one at a time and let the respective students label their dots.


Students on Shenna's List
Students on Daniel's List
Juana
Sonja
Lynn
Angela
Kristy
Shenna
Jason
Colin
Gary
Eric

90

L 13

T: Look at our picture. How many students are there altogether on the two lists?

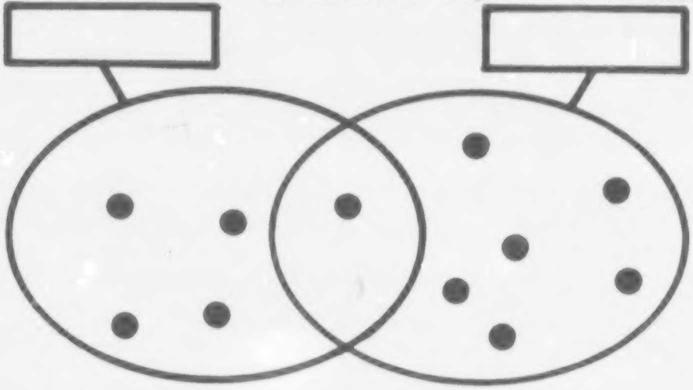
S: Ten.

T: Many of you predicted twelve. Do you understand why it is ten and not twelve?

S: Angela and Kristy are on both lists; two people are in the middle.

Erase the students' names from the picture; erase one of the dots in the middle and replace it with an additional dot in each string (as in the next illustration) as you say:

T: Suppose there had been just one name on both lists, but there were still five names on one list and seven names on the other list. Then how many students would there be altogether on the lists?



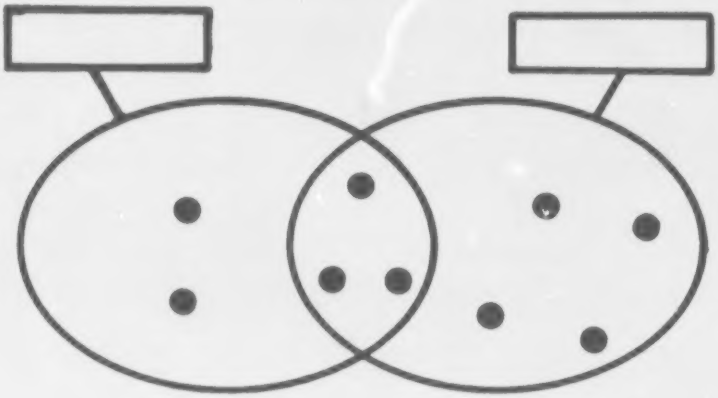
S: Eleven.

Erase the dots from your picture.

T: Think about this problem and try to draw a picture. Suppose there were three names on both lists, but still five names on one list (point to the red string) and seven names on the other list (point to the blue string). How many students would there be altogether on the lists?

91

Let the students work independently for a few minutes. After many students have solved the problem, ask one of them to put the dots in the picture on the board.



T: How many dots? How many students altogether on the lists?

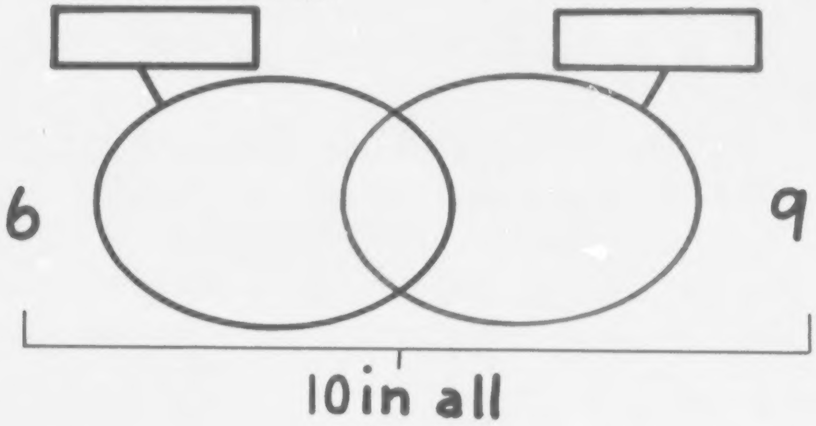
S: Nine.

Erase the dots in the picture.

T: I am going to change the problem. This time I will tell you how many students there are on each list and how many students altogether. You have to try to draw a picture.

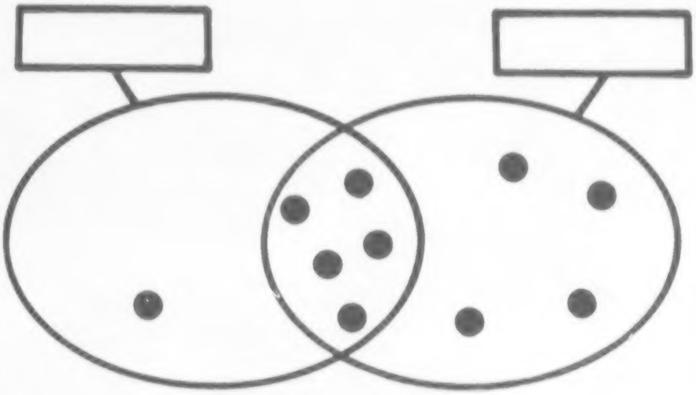
Write the numbers next to the respective strings as you mention them.

T: There are six people on this list (point to the red string) and nine people on this list (point to the blue string). Altogether there are ten people on the lists.



L 13

Let the students work independently on the problem. This is a harder problem Do not expect everyone to solve it. Encourage the students to use trial and error methods. As you observe the individual attempts, ask students to check their own work by counting the number of dots they have drawn. After many students have found a solution, ask one of them to put the dots in the picture on the board.



Students who finish quickly could be challenged with other similar problems.

93

L14 Two Dice #2

CAPSULE LESSON SUMMARY

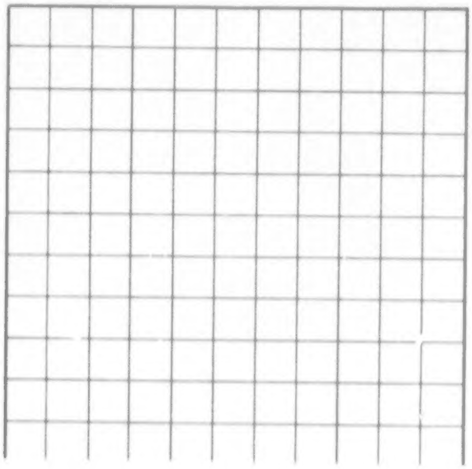
Review the structure of a die and the possible sums you could get when tossing two dice. Make a bar graph of the results (sums) obtained in about thirty trials of the experiment of tossing two dice, and make some observations from reading the graph. Color a six by six array of squares, each square associated with a possible outcome from the tossing two dice experiment, according to some scheme where a color indicates a sum. Use the completed picture to discuss the chances of getting various sums.

MATERIALS

Teacher: Two dice; colored chalk
Student: Worksheet L14; colored pencils

DESCRIPTION OF LESSON

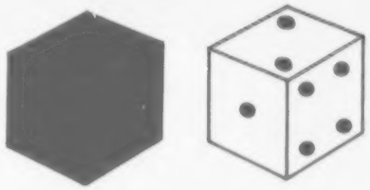
Before the lesson begins, sketch an 11 by 10 square grid on the board as shown below. It is not necessary that there be perfect squares in this picture.



Exercise 1

Hold up the two dice for the class to see. Review the fact that a die is a cube and on each of the six faces there is a number of dots, either 1 or 2 or 3 or 4 or

5 or 6. Indicate that you are going to toss the two dice and then add the numbers of dots on the top faces. Do this one or two times to be sure the students understand. For example,



T: 3 on the red die and 2 on the white die; 3 + 2 = 5 so the result is 5.
What other sums could we get?

S: 6.

T: How could we get 6?

S: 3 on both dice.

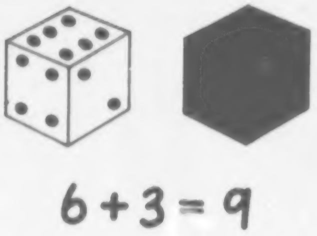
S: 4 on the red die and 2 on the white, or 4 on the white die and 2 on the red die.

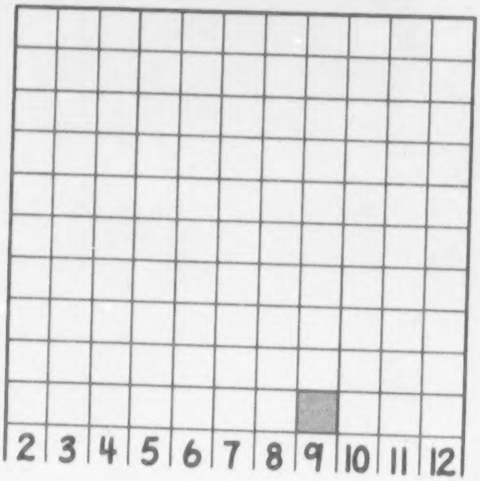
Continue accepting possible outcomes until every whole number from 2 to 12 is mentioned. Ask that students mention one way to get each sum, but do not insist that they give several ways in cases where there are more than one. This idea occurs in a later part of the lesson.

Distribute Worksheet L14 and ask that students look at the front side. Observe that all the possible sums are listed along the bottom edge of the grid, on the worksheet, and put these numbers below your grid on the board.

T: Each of you is going to have a turn tossing the two dice and noting the sum. We will all record the results on this grid by filling in a square above a number each time it is the sum.

Demonstrate by starting the activity yourself. Toss the two cubes and note the sum. Then fill in a square above the number. For example,

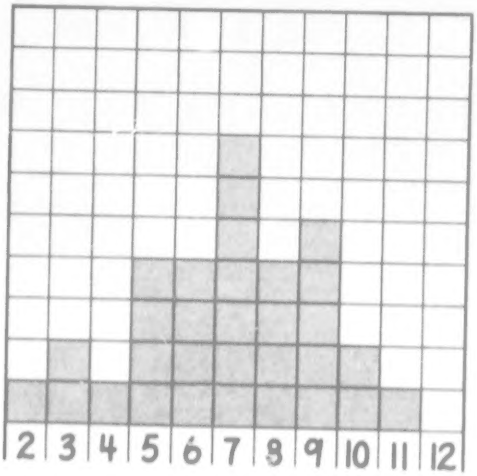




T: Before we go on, predict which number will have the highest tower of filled in squares after we have all had our turns. Write your prediction at the bottom of the page.

Pass the two dice around the room letting each student take a turn. Encourage students to shake the dice in their hands before tossing and then to give them a good roll on the desk. Keep a record at the board of all the results and ask the students to do likewise on their papers.

For a class of thirty students, a possible graph is shown below.



L 14

At the end of this activity, ask the class which sum occurred most often (the one with the highest tower of filled in squares). Let students compare the result to their predictions. You might also ask some of the following questions about the graph.

- Which sum occurred least often?
- How many times did we get a sum of 10? . . . 5? . . . 8?
- How many more times did we get 9 than 3?
- How many times did we get sums larger than 7? . . . smaller than 7?
- How many times did we get an even number as the sum? . . . an odd number?

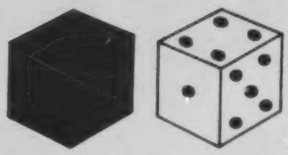
Exercise 2

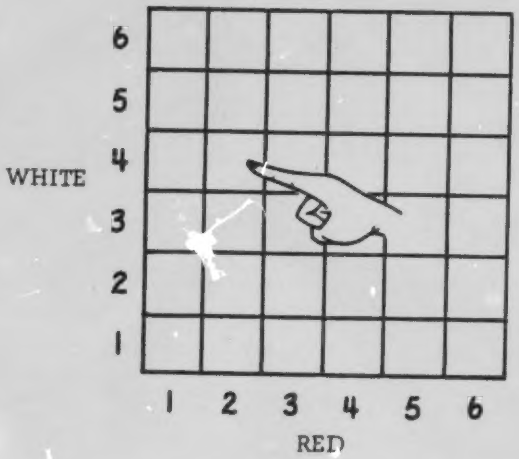
Instruct the students to turn over their worksheets. Sketch a six by six grid on the board like the one on the worksheet. Assign a color code to each of the eleven possible sums when you toss two dice. You could let the students help with this assignment as it will be limited by their collections of colored pencils. For example,

2	■	black	**7**	▨	yellow strip
3	■	green	**8**	▨	blue strip
4	■	red	**9**	▨	red strip
5	■	blue	**10**	▨	green strip
6	▒	yellow	**11**	▨	black strip
			12	□	white

Explain how each square in the grid is for an outcome when tossing two dice; it indicates what is on the red die and what is on the white die.

Toss the two dice and then point to the appropriate square; for example,





T: This square is for 2 on the red die (point to "2" at the bottom edge) and 4 on the white die (follow the squares above "2" up to the fourth square opposite "4" along the side edge).

Using our color code, how should this square be colored?

S: Yellow, because 4 + 2 = 6.

Color the square at the board and instruct students to do the same on their worksheets. Call on students to suggest other possible outcomes (by tossing; if they wish) and then to locate the appropriate square. Color the squares according to your color code. After three or four squares have been colored, ask for an outcome giving a specific sum.

T: What could be on the red die and on the white die if the sum is 10.

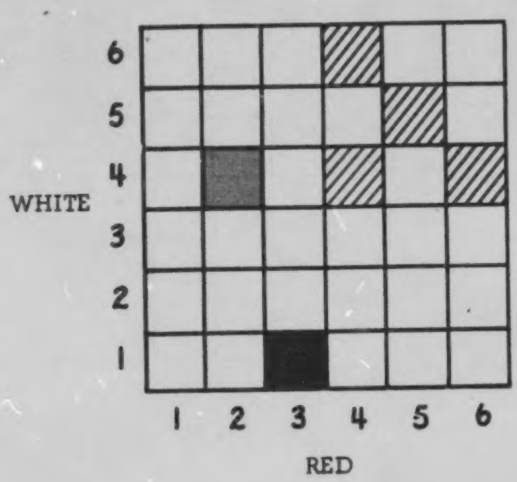
S: 5 on both dice.

S: 6 on the red die and 4 on the white die.

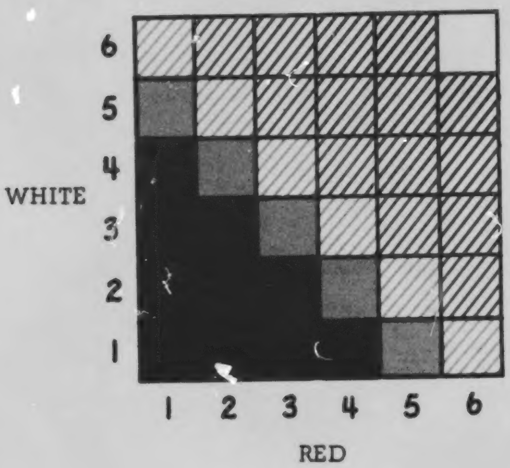
S: 4 on the red die and 6 on the white die.

L 14

As the outcomes are mentioned, locate and color the appropriate squares according to your color code.



Continue in this way until all thirty-six squares have been colored.

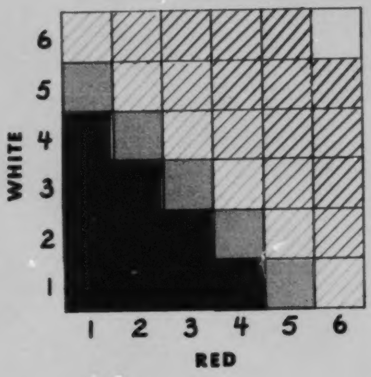


Encourage students to make some observations. If time permits, lead a discussion of the colored square including some of these questions.

- Does this picture suggest why we often got a sum of 7 when we tossed the dice?
- How many small squares are there? How many are colored solid blue? What are the chances of getting a sum of 5? (Four out of thirty-six.)

How many are blue striped? What are the chances of getting a sum of 8? (Five out of thirty-six)

- Do we have a better chance to get a sum of 5 or to get a sum of 8?
- Which sum do we have the best chance of getting?
- What are the chances of getting a sum larger than 7? . . . smaller than 7?


WHITE
6
5
4
3
2
1
1
2
3
4
5
6
RED

The coloring will depend on your class' choice of a color code, assignment of a color to each possible sum.

101

L15 The String Game #3

CAPSULE LESSON SUMMARY

Play the String Game with A-Blocks using sixteen possible string labels.

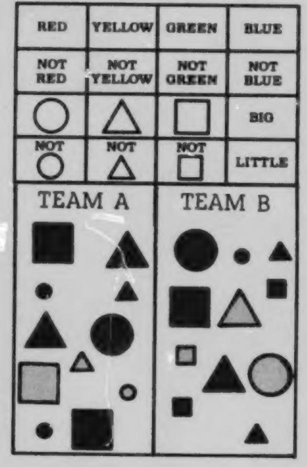
MATERIALS

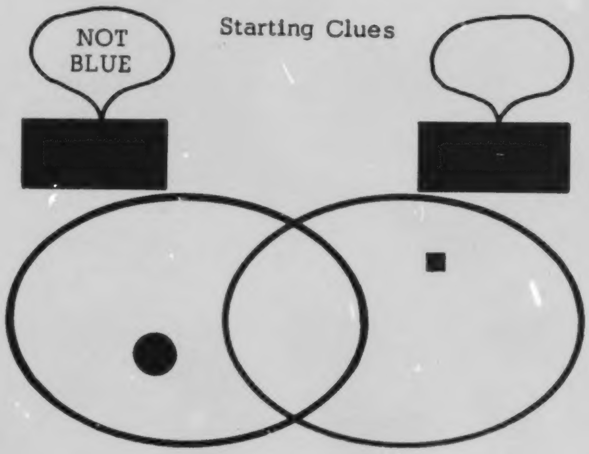
Teacher: A-Blocks String Game kit; colored chalk

Student: None

DESCRIPTION OF LESSON

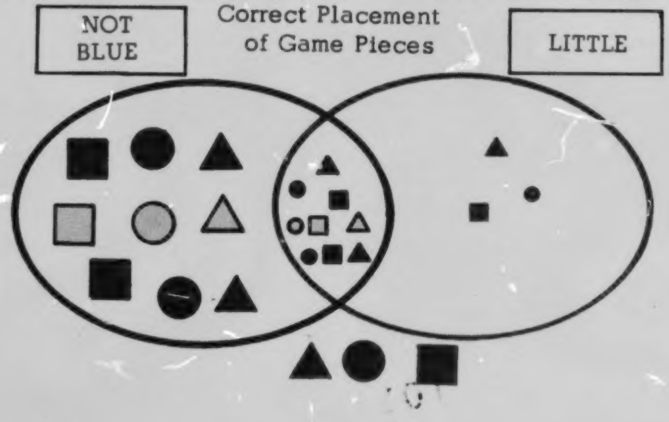
Prepare to play the String Game with A-Blocks in the usual way. (See the APPENDIX: The String Game for a complete description of equipment and preparation for the game.) Play several games as time allows. Two possible games, both using the Version B poster list of string labels, are suggested here; there are many other examples in the Appendix. Try to choose games which are challenging for most of your students but are not overly difficult.


RED
YELLOW
GREEN
BLUE
NOT RED
NOT YELLOW
NOT GREEN
NOT BLUE
BIG
NOT
NOT
NOT
LITTLE
TEAM A
TEAM B

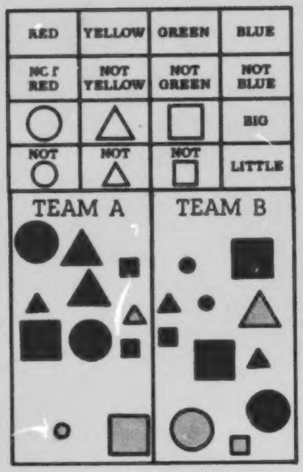


Starting Clues

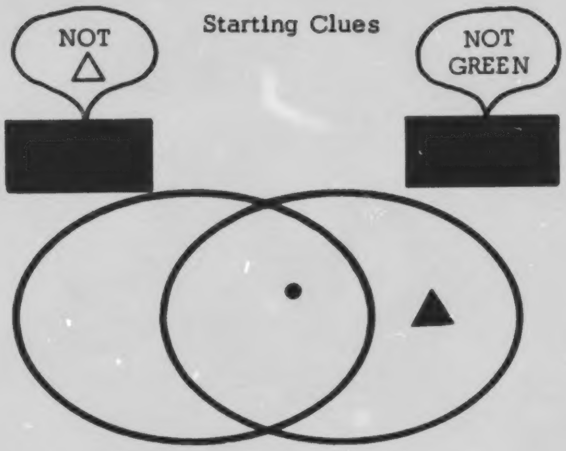
Correct Placement of Game Pieces


NOT BLUE
LITTLE

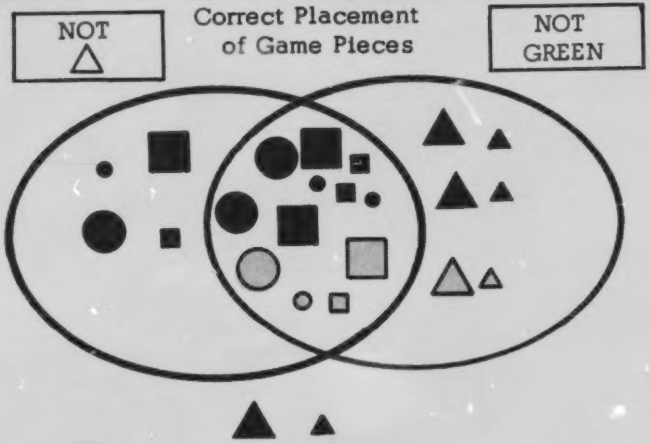
L 15


RED
YELLOW
GREEN
BLUE
NOT RED
NOT YELLOW
NOT GREEN
NOT BLUE
BIG
NOT
NOT
NOT
LITTLE
TEAM A
TEAM B

Starting Clues


NOT △
NOT GREEN

Correct Placement of Game Pieces


NOT △
NOT GREEN

L16 Multiples #2

CAPSULE LESSON SUMMARY

Put numbers in a string picture until it is known what the strings are for. Draw a string picture in which one string is for multiples of 10 and the other is for multiples of 5. Discover that the picture could be drawn with one string inside of the other. Solve a detective story which involves three coins, a string picture, and an arrow picture.

MATERIALS

Teacher: Coins (one quarter, one dime, and one penny); colored chalk

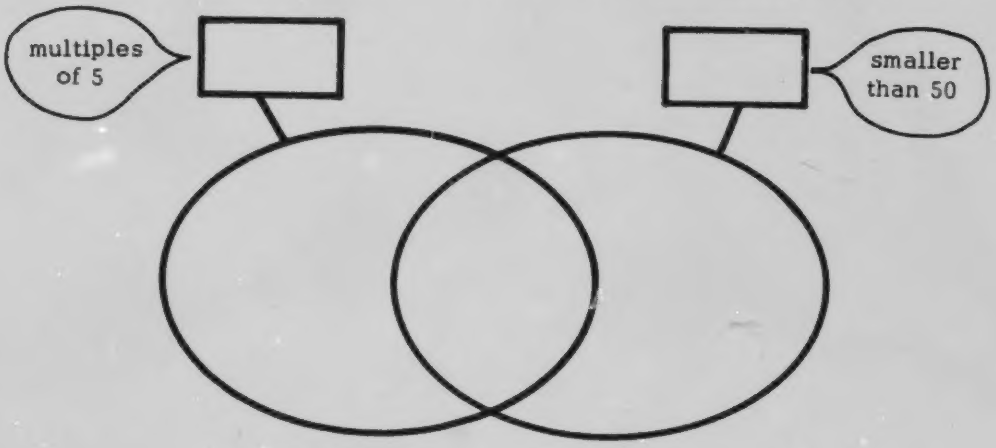
Student: Paper; colored pencils

NOTE: Before the lesson begins, put just one quarter, one dime, and one penny in your pocket or in a small container such as a change purse, a ring box, a match box, etc.

DESCRIPTION OF LESSON

Exercise 1

Draw two different colored, overlapping strings on the board. The two labels are given here for your information, but your students are to discover them.



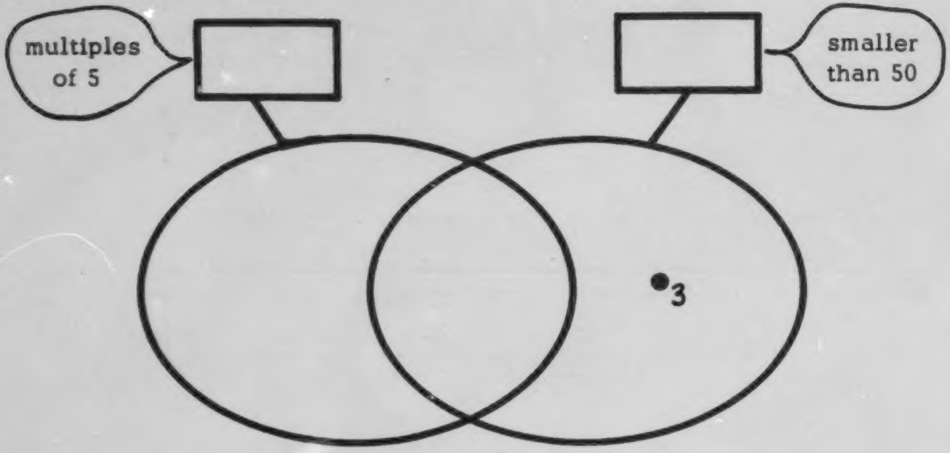
T: There are two strings on the board, but I haven't told you what they are for. I'll say only that they are for some numbers. Let's see if you can figure out how to label them. Someone choose a number.

L 16

S: 3.

T: 3 belongs here.

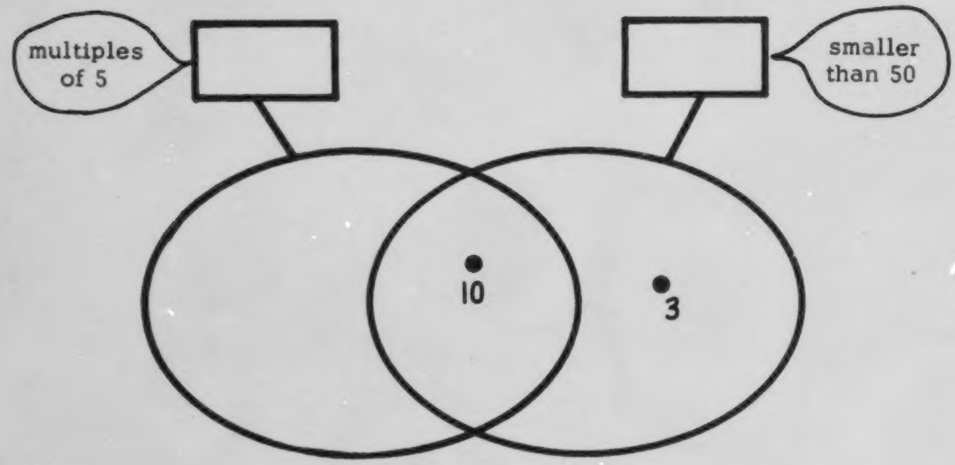
Put 3 in your picture.


multiples of 5
smaller than 50
3

T: Give me another number.

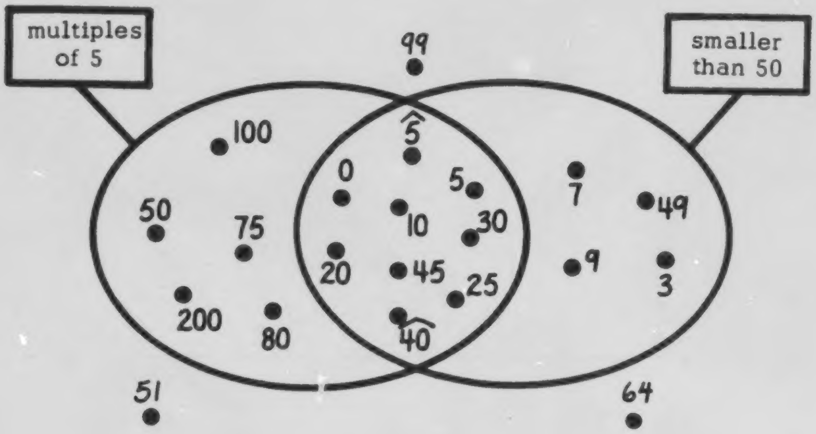
S: 10.

T: 10 belongs here.


multiples of 5
smaller than 50
10
3

Continue until many numbers are in your picture and until most of your students think that they know the labels. The label for the red string should become evident quickly, but the label for the blue string will not be obvious until the numbers selected are close to 50. Label the strings when correctly identified. In the unlikely event that even after twenty or more numbers are in the picture, the class seems completely unable to identify the strings, provide some hints. For example, you might say that the red string is for multiples of some number, or the blue string is for numbers which are smaller than some number.

Perhaps your picture will look similar to this one.



Ask students for other numbers which belong in each of the four regions. Your class should locate 0 and 50, some large numbers, and some negative numbers.

Erase the board.

Exercise 2

T: The red string was for multiples of 5. What are some numbers which are multiples of 2?

S: 0, 2, 4, 6, 8, ...

S: 100, 102, 104, 106, 108, ...

L 16

S: 54 and 86.

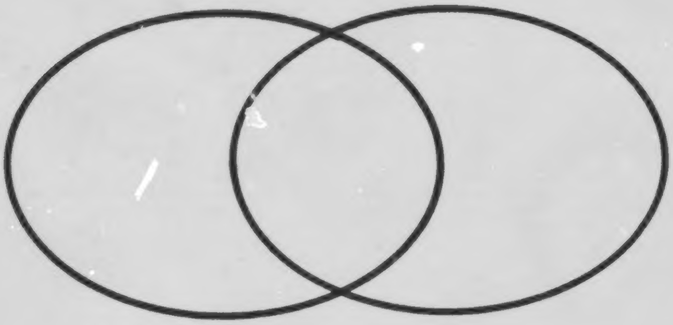
S: $\hat{2}, \hat{4}, \hat{6}, \ldots.$

Continue by asking students to name:

- some multiples of 3[*];
- some multiples of 10.

T: Suppose we want to draw a string picture in which one string is for multiples of 5 and the other string is for multiples of 10. How should we draw the strings?

Most likely a student will draw the strings like this.



Label the strings and distribute paper and colored pencils.

T: Copy this string picture on a piece of paper and put in as many numbers as you can.

Walk around the room looking at the students' pictures. After a few minutes, invite students to put specific numbers from their papers into the picture on the board.

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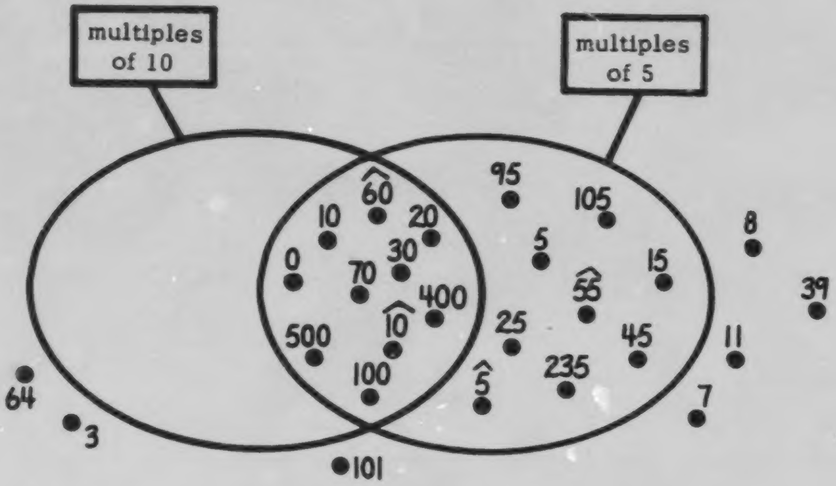
[*] For your information, an integer is a multiple of 3 if and only if the sum of its digits is a multiple of 3.

Perhaps your picture will look like this.


multiples of 10
multiples of 5
60
10
20
30
0
70
400
10
500
100
95
105
5
15
55
25
45
5
235
8
39
11
7
64
3
101

T (pointing to the region on the left inside the red string): Are there any numbers which belong here?

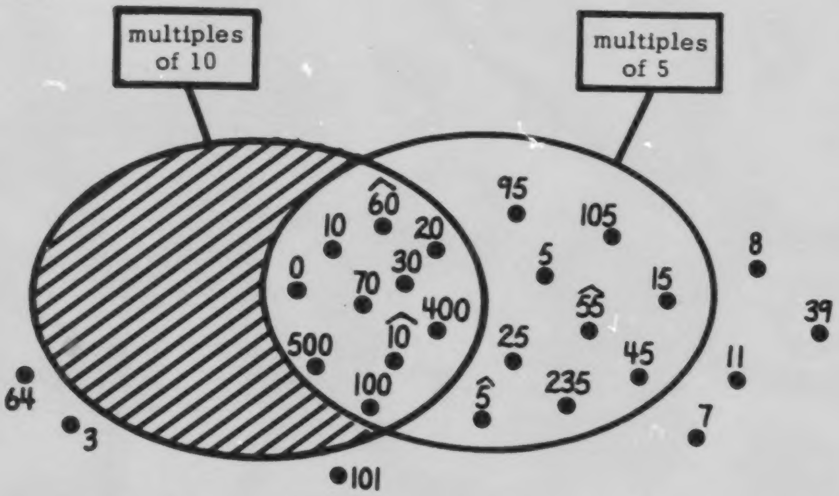
S: No.

T: Why not?

S: Because every multiple of 10 is a multiple of 5.

T: How can I show in the picture that this region is empty?

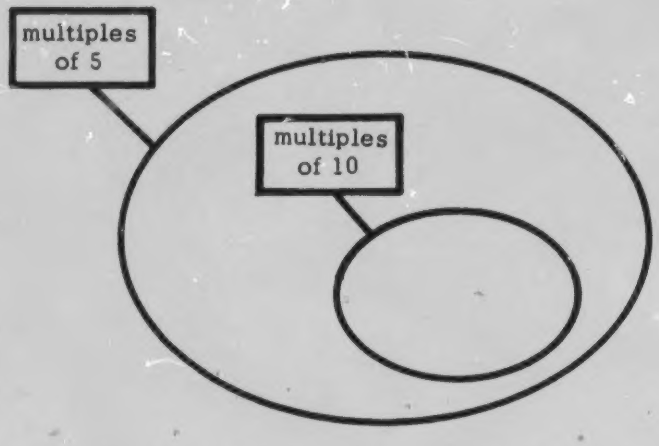
S: Hatch it.


multiples of 10
multiples of 5
60
10
20
30
0
70
400
10
500
100
95
105
5
15
55
25
45
5
235
8
39
11
7
64
3
101

L 16

T: Since this region (hatched region) is empty, is there another way we could have drawn the strings?

If no one suggests drawing the strings as shown below, suggest it yourself.



Locate some of the numbers from the first picture in this picture to associate regions in that picture with regions in this picture.

Erase the board before going on to Exercise 3.

Exercise 3

T: I have some money in my pocket (or this small container). I will give you some clues — see if you can discover how much money.

First clue

T: My first clue is that I have three different coins. What kinds of coins could I have?

As students name the various kinds of coins, record them as headings in a chart on the board.

50¢ Half-Dollars	25¢ Quarters	10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount

T: How much money could be in my pocket?

S: 85¢. You could have a half-dollar, a quarter, and a dime.

S: 40¢. You could have a quarter, a dime, and a nickel.

As students make suggestions, record them in the chart. Encourage the class to find all the possibilities. Each time an amount of money is suggested, be sure the student says which coins make that amount of money. Your completed chart should have ten possibilities listed, although most likely they will not be suggested in the systematic way shown below.

50¢ Half-Dollars	25¢ Quarters	10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount
1	1	1	0	0	85¢
1	1	0	1	0	80¢
1	1	0	0	1	76¢
1	0	1	1	0	65¢
1	0	1	0	1	61¢
1	0	0	1	1	56¢
0	1	1	1	0	40¢
0	1	1	0	1	36¢
0	1	0	1	1	31¢
0	0	1	1	1	16¢

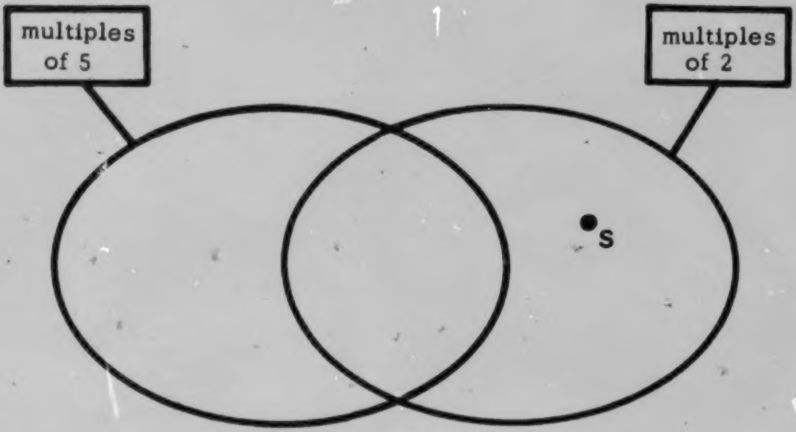
L 16

Leave the chart on the board.

Second clue

T: For my next clue, I'll show you where the number of cents in my pocket (in this container) belongs in a string picture.

Draw this string picture on the board.



T: How much money could I have in my pocket? Remember, it must be one of these amounts (point to the chart) and it is here in this string picture (point to "s").

S: 76¢.

T: 76¢ is in the chart. Is 76 a multiple of 2? (Yes)
Is 76 a multiple of 5? (No)
So I could have 76¢.

Continue in this manner until all the possibilities in the chart have been considered. This clue leaves four possibilities.

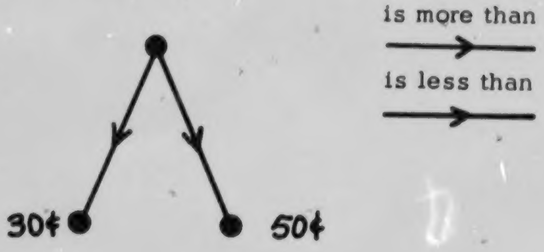
50¢ Half-Dollars	25¢ Quarters	10¢ Dimes	5¢ Nickels	1¢ Pennies	Amount
~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~85¢~~
~~1~~	~~1~~	~~0~~	~~1~~	~~0~~	~~80¢~~
1	1	0	0	1	76¢
~~1~~	~~0~~	~~1~~	~~1~~	~~0~~	~~65¢~~
~~1~~	~~0~~	~~1~~	~~0~~	~~1~~	~~61¢~~
1	0	0	1	1	56¢
~~0~~	~~1~~	~~1~~	~~1~~	~~0~~	~~40¢~~
0	1	1	0	1	36¢
~~0~~	~~1~~	~~0~~	~~1~~	~~1~~	~~31¢~~
0	0	1	1	1	16¢

T: Now you know that I have one of these four amounts of money in my pocket. I'll give you one more clue. When you know for sure how much money is in my pocket (in this container), write it on a piece of paper.

Third clue

T: The amount of money in my pocket is in this picture.

Draw this arrow picture on the board.



Check some of the students' papers. After most students have discovered the correct amount, let someone announce that you have 36¢ in your pocket (container) and show them the coins.

L 17

L17 Telling Stories with Strings

CAPSULE LESSON SUMMARY

Present several different string pictures and for each one tell stories that the picture could be about. Observe some important details of a story as shown by the picture. In a situation where the strings are for some kinds of numbers, put numbers in the picture until it is known what the strings are for.

MATERIALS

Teacher: Colored chalk
Student: None

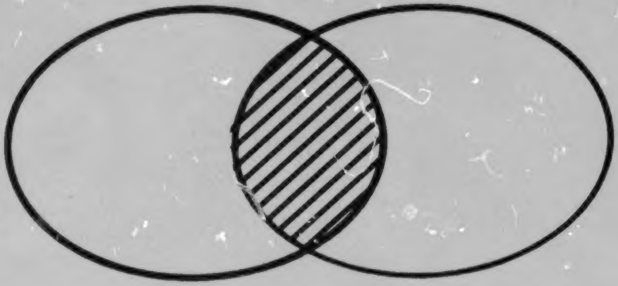
DESCRIPTION OF LESSON

Each exercise presents a string picture and students are invited to invent stories that the picture could be for. After one student tells a story, the other class members give their comments about whether the picture could be for that story. Direct the discussion of a story by asking questions that will help students become more familiar with the string language and the use of hatching. If students are initially relunctant to tell their own stories or their imaginations "run dry", you may wish to present a story of your own. But allow the exercises to move quickly; do not spend too long with one picture.

For the first two exercises, we present dialogue for the discussion of one possible story and we give summaries of some other possible stories. These situations should not limit your discussions. Certainly your students will suggest different stories and indeed some good questions may come from the students. For each picture, encourage the class to discuss more than one story in order to reinforce the idea that one string picture can represent many different situations.

Exercise 1

Draw this string picture on the board.



T: What could the red string and the blue string be for?

S: The red string could be for cats and the blue string could be for dogs.

Allow the class to react before asking any specific questions. Then, for the following questions, invite students to point to the correct regions.

T: I have a fluffy Angora cat. Where would my cat be in this picture?

Ms. Weaver, the principal, has an Irish Setter dog. Where would her dog be in this picture?

Where would Eli the Elephant be in this picture?

Point to each region in turn (save the hatched region in the middle for last) as you ask:

T: What else could be here?

Let several students give suggestions.

T (pointing to the middle region): What does the hatching mean?

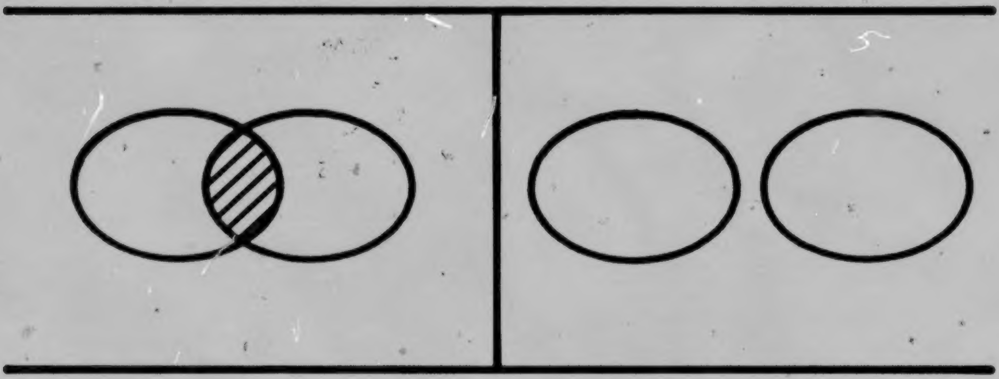
S: There is nothing in that region?

T: Is it correct for our story about dogs and cats?

S: Yes, there cannot be something that is both a dog and a cat.

T: Can you suggest another way to draw a string picture for the same story, but without hatching any region?

After a minute or two of attempts, the class should agree that nonintersecting strings could be for the same situation. Draw the picture next to the original picture and locate some of the same objects as before in this picture. Also, check it against any other story that is suggested.



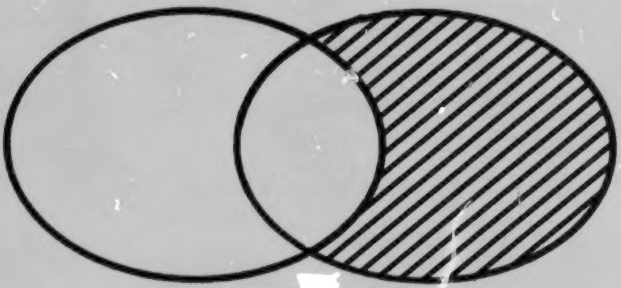
Some other stories that could be told about this picture are summarized below.

- The red string is for bikes and the blue string is for cars.
- The red string is for odd numbers and the blue string is for even numbers.
- The red string is for boys and the blue string is for girls.
- The red string is for yellow A-Block pieces and the blue string is for not yellow A-Block pieces.

Erase the pictures on the board before going on to Exercise 2.

Exercise 2

Draw this string picture on the board.



T: What could the red string and the blue string be for?

S: The red string could be for cars and the blue string could be for blue cars.

Allow the class to react before asking specific questions. Then, for the following questions, invite students to point to the correct regions.

T: I have a green Toyota. Where would my car be in this picture?

Mr. Adams, the assistant principal, has a blue Dodge.
Where would his car be in this picture?

Where would our school bus be in this picture?

Point to each region in turn (save the hatched region for last) as you ask:

T: What else could be here?

Let several students give suggestions. Encourage some students to locate their family cars in the picture.

T (pointing to the hatched region): What does this hatching mean?

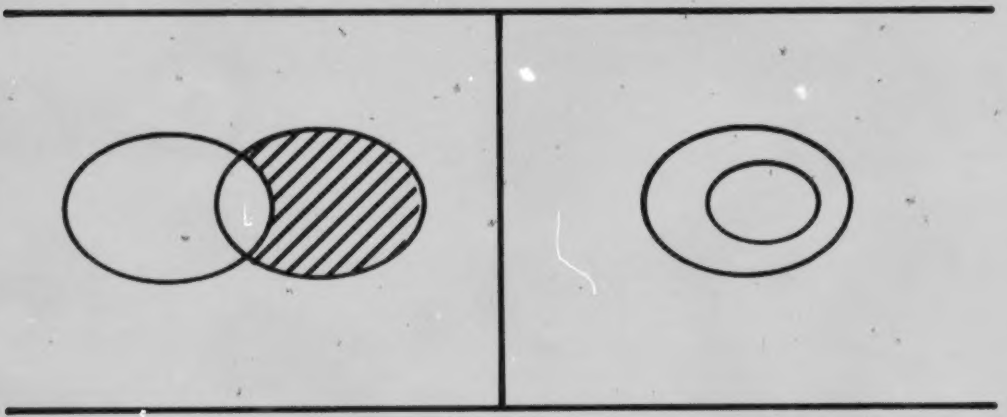
S: Nothing goes there.

T: Is it correct for our story about cars and blue cars?

S: Yes, blue cars must be in the middle because they are cars. Nothing can be a blue car and be outside the red string.

T: Can you suggest another way to draw a string picture for the same story, but without hatching any region?

It may take students a few minutes to discover that they could draw a picture with the blue string completely inside the red string. If necessary, lead the class to this observation and draw the picture next to the original picture. Locate some of the same objects as before in this picture. Check both pictures against another story that is suggested.



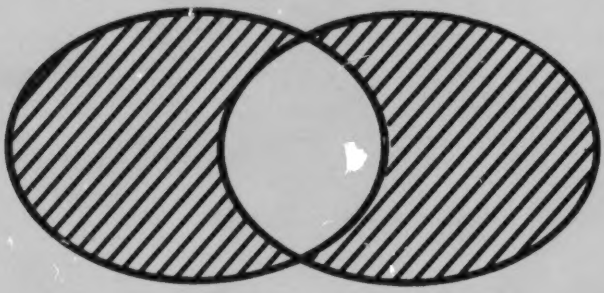
Some other stories that could be told about this picture are summarized below.

- The red string is for snakes and the blue string is for rattlesnakes.
- The red string is for even numbers and the blue string is for multiples of 4.
- The red string is for not circle A-Block pieces and the blue string is for triangles.

Erase the pictures on the board before continuing with Exercise 3.

Exercise 3

Draw this string picture on the board.



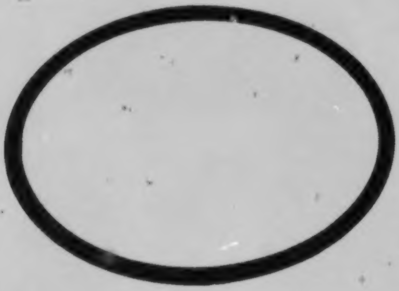
T: What could the red string and the blue string be for?

S: The red string is for multiples of 2 and the blue string is for even numbers. They are the same.

Lead the class to observe that the hatching in this picture indicates that the red string and the blue string are for the same things.

T: How could we draw a string picture for the same story without hatching any region?

The students should remark that you only need to draw one string, or that you could draw the red and blue strings as shown below.



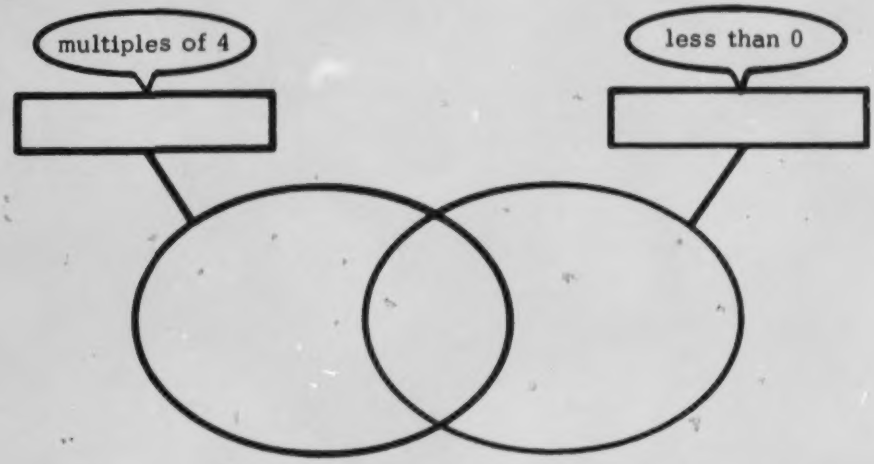
118

L 17

Exercise 4

Draw a string picture (as in the next illustration) on the board and tell the class that this time the strings are for some kinds of numbers, but you won't tell them yet what are the labels. They can give you numbers to put in the picture and when they think they know what the strings are for, they can make a guess.

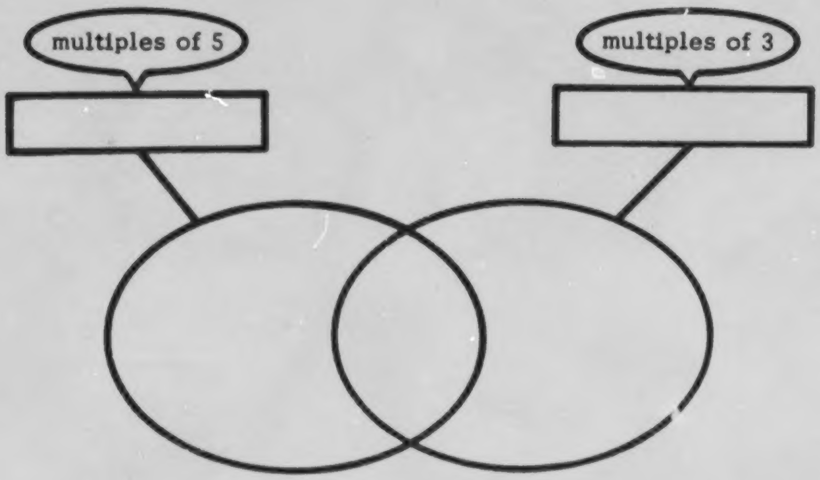
The bubbles indicate what the strings are for; do not tell the class this information.



If your students tend too quickly to just guess at what the strings are for, you could make this activity into a game. On a person's turn, he or she may either give you a number to put in the picture or guess what the strings are for. However, once a person makes a guess, if it is not correct (that is, one or both of the string labels are not correct), then that person is out of the game.

Proceed as for Exercise 1 of Lesson L16 Multiples #2 until the string labels have been discovered.

If time remains and your students enjoy this activity, repeat it with the following situation. (See the next illustration.)


multiples of 5
multiples of 3

At the end of this activity, you might ask the students what they can tell you about the numbers in the middle region. They are the multiples of 15; however, your students may describe them in terms of counting by fifteens.

APPENDIX: The String Game

The String Game is used in many versions throughout the CSMP Elementary School Program. It gives students an opportunity, in a game-like atmosphere, to become familiar with the language of strings, while at the same time it involves them in the kind of reasoning that will be developed and reinforced in various contexts throughout the program.

The String Game is first played with A-Blocks (shapes), but later on in the Intermediate Grades variations which make use of numbers appear. In this appendix we provide you with the necessary information and examples to enable you to play the game using A-Blocks, and we suggest some of the various possibilities open to you when you play the game with shapes. Although only a few lessons in UPG Part II are scheduled for the String Game, we hope you will not feel restricted to playing the game only at these times. It has been found that it is most beneficial and enjoyable for students if you make a regular practice of playing the game about once every two weeks, or whenever you have an extra 10-15 minutes during the course of the day.

Equipment

PLAYING BOARDS

The equipment for this game may be most easily managed if you have a magnetic[*] chalkboard available. Many of the permanently mounted chalkboards in classrooms are magnetic, and you should test yours using one of the demonstration Minicomputer checkers. If your permanent chalkboards are not magnetic, try any portable chalkboard (grid board, etc.) that the school has available. If you do not have a magnetic chalkboard available, you can use your regular chalkboard.

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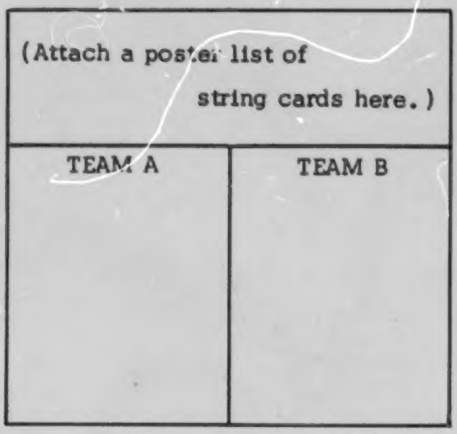
[*] Strictly speaking the chalkboard itself is not magnetic, but rather magnet-sensitive; i.e., magnets adhere to it. However, for simplicity we call such boards "magnetic".

TEAM BOARD

The Team Board is divided into three regions as illustrated below.



a) Magnetic: If you do locate a magnetic chalkboard, you will also need a magnetic or metallic Team Board. With a large magnetic chalkboard, say at least three chalkboard squares, the Team Board could be drawn on the chalkboard. However, if you have a relatively small (portable) magnetic chalkboard, then you will need to obtain a sheet of metal (minimum size: 60 cm by 80 cm) or locate a convenient metallic surface in the classroom such as the side of a file cabinet on which to put the Team Board. In this case, draw the markings for the three regions of the Team Board on a large sheet of (easel pad) paper and tape this paper to your metallic surface.

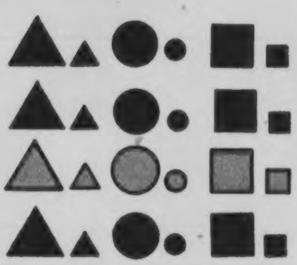
b) Non-magnetic: If you do not have a magnetic chalkboard available for the playing board, then your Team Board could be a large piece of poster board (minimum size: 60 cm by 80 cm). Make the markings for the three regions of the Team Board with a heavy felt-tip pen.

GAME PIECES and STRING CARDS

One set of game pieces and string cards are needed for each version of the game. A list of the string cards (A-Blocks String Game Poster) is to be attached to the upper section of the Team Board - it is a constant reminder during the game of the possible labels for the strings.

Game Pieces

String Game
with
A-Blocks



String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

Game pieces, string cards, and the poster list of the string cards can be found in the A-Blocks String Game Kit.

a) Magnetic: You can magnetize the game pieces (A-Blocks) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in the A-Blocks String Game Kit, or it is available in many hobby and notions departments of stores.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written thereon.

b) Non-magnetic: Game pieces can be attached to the Team Board by means of loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front so that what is written on the card is not obscured. With this type of equipment, you should prepare yourself for running repairs by having sufficient masking tape available so that if a loop of tape loses its stickiness, it can be replaced on the spot. An alternative is to use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape. This material will maintain its stickiness over a much longer period of time.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card face-down. Locate the Team Board conveniently near to the playing board. Share the game pieces between the two sections of the Team Board labeled "TEAM A" and "TEAM B" by randomly placing one-half of the pieces in one section and the remainder in the other section. Divide the class into two teams using whatever method is acceptable to your class; e.g., boys vs. girls; rows 1, 2, 3 vs. rows 4, 5, 6; etc. Call one team "Team A" and the other "Team B". Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This removes from the first players the necessity of operating on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the face-down string cards) in the string picture. The winning team is the one that first places all of its game pieces correctly and identifies the face-down cards correctly, after playing according to the rules.

Rules of the Game

1) The game is played in silence. This is to avoid one student telling another where to play a certain piece, or what the strings are for. Each student should have the opportunity to analyze the game alone. Infringement of this rule by anyone is penalized by the talker's team losing its next turn.

2) The teams alternate, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.

3) You are the judge. If the piece is correctly placed, say "yes". The piece remains in the string picture and the player immediately has a second turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say "no". The player returns the piece to the team's unplayed collection and play passes to the other team.

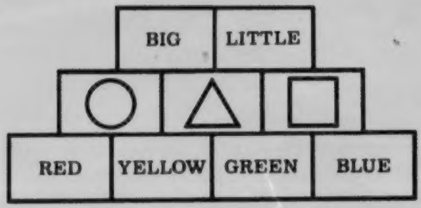
As an aid in judging, prepare a "crib-sheet" showing the correct position of each game piece or at least reminding you of what is on the face-down cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Either move an incorrectly placed piece whose position you had approved to its correct region or replace in the string picture a correctly placed piece that had been rejected because you had disapproved its position.

4) When a team has correctly placed all of its pieces, the player who placed the last piece may thereupon attempt to identify each of the string cards. If these are all correct, the team has won. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.

5) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues on its turn to attempt to identify the strings, while the other team works to place its game pieces.

Version A: The String Game with A-Blocks

This simplest version of the game uses twenty-four A-Blocks as game pieces and only nine string cards.

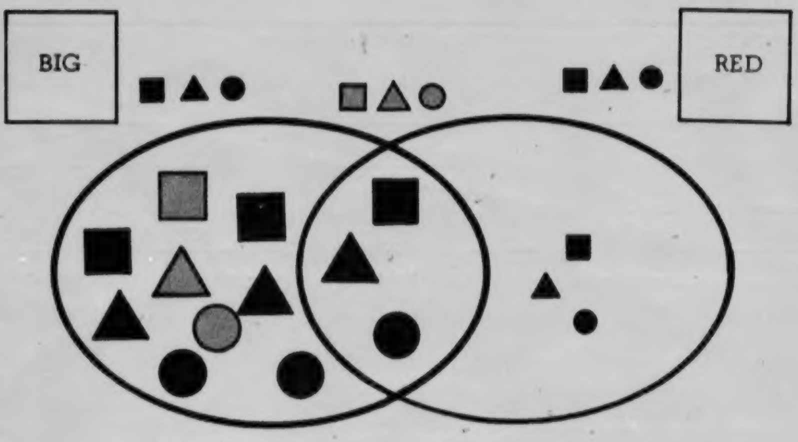


This list of string cards should be posted on the board or attached to the game board.

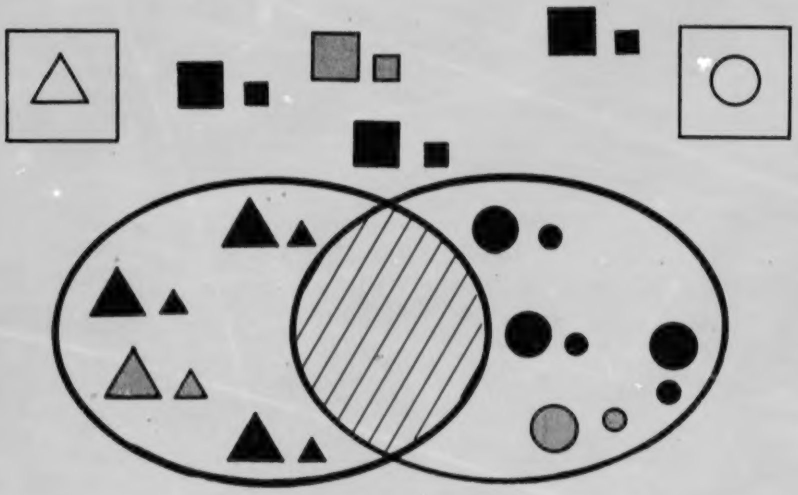
We provide here several example "crib-sheets" for variations of the game with two and with three strings.

Variation #1: Two Strings

Example 1: No Empty Regions

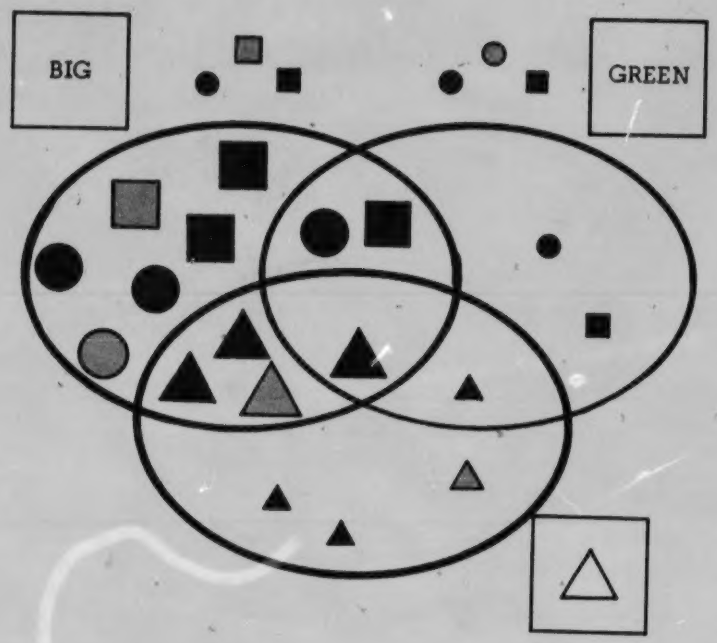


Example 2: One Empty Region

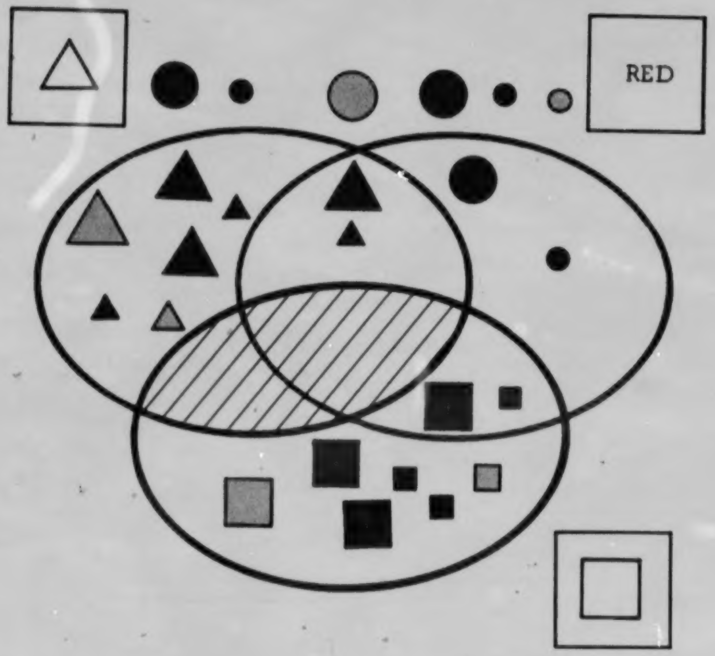


Variation #2 : Three Strings

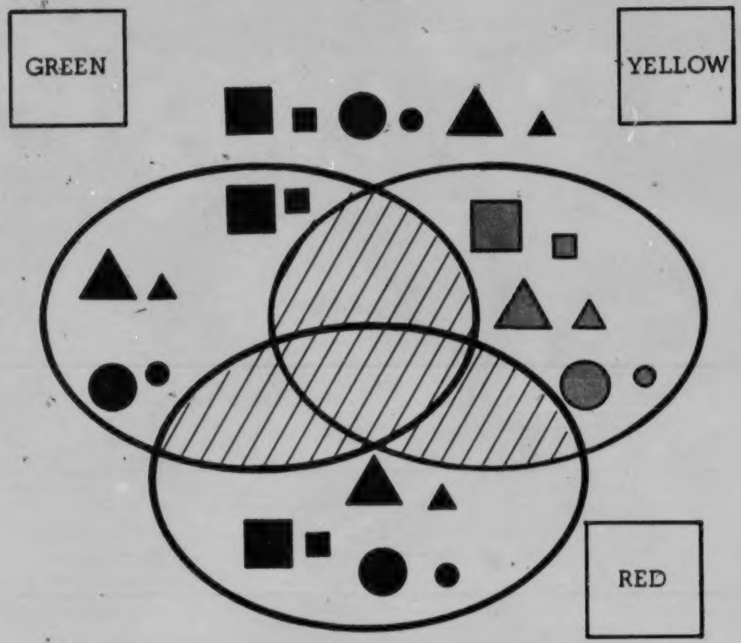
Example 1 : No Empty Regions



Example 2 : Two Empty Regions



Example 3: Four Empty Regions



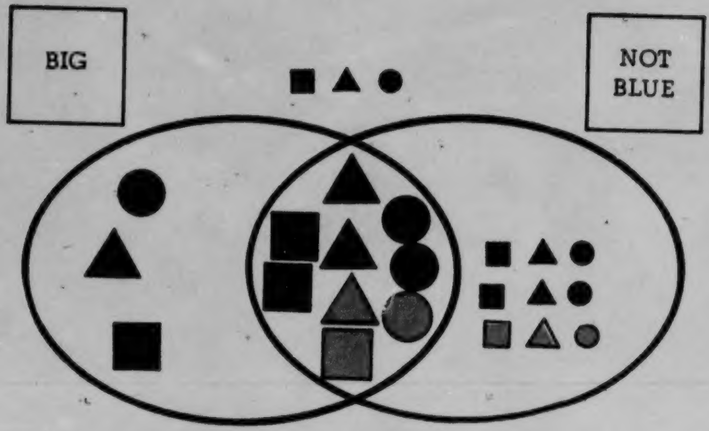
Version B: The String Game with A-Blocks, using "Not" Cards

A more complicated version of the game with A-Blocks uses all sixteen of the string cards. A list of all sixteen string cards should be posted or attached to the Team Board.

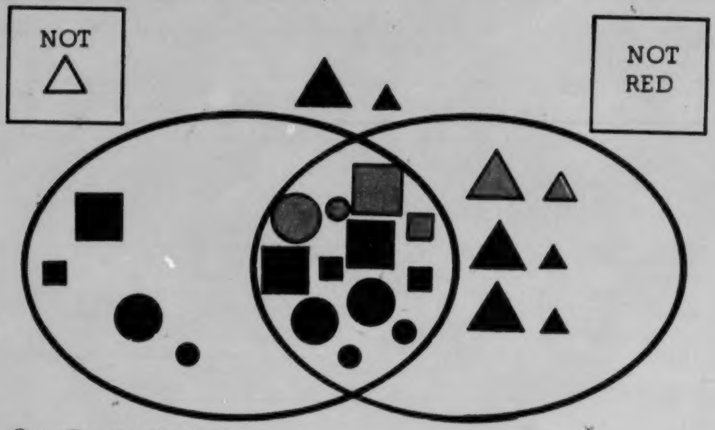
NOTE: Since "NOT BIG" and "LITTLE" describe exactly the same A-Blocks (and similarly for "NOT LITTLE" and "BIG"), such a "not" card is redundant. If a student questions why we do not include these possibilities, let the class discuss why they are unnecessary.

We provide here several example "crib-sheets" for this version of the String Game played with two strings.

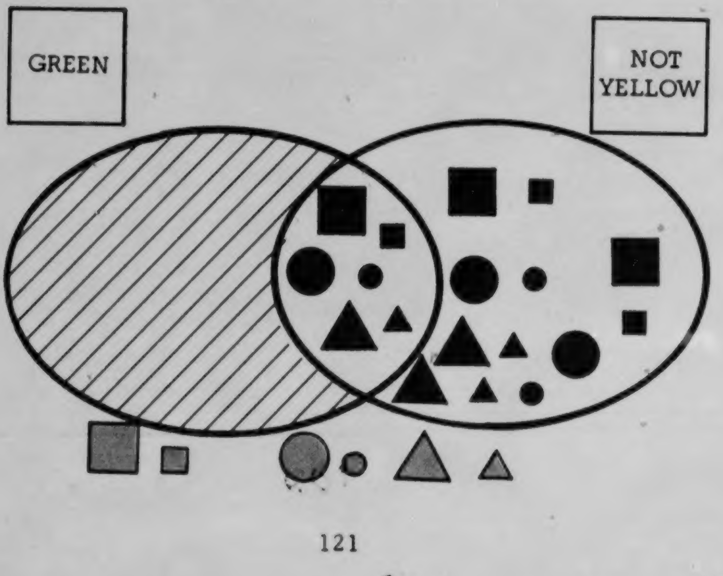
Example 1: No Empty Regions


BIG
NOT BLUE

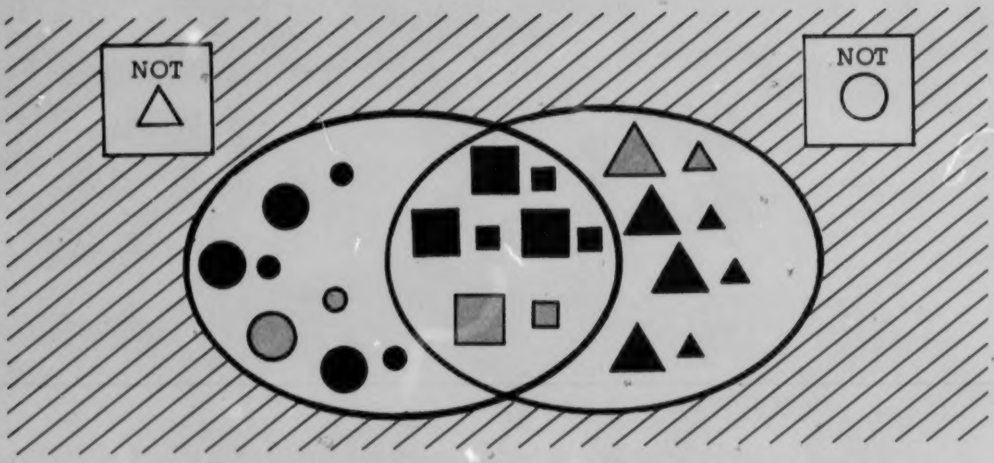
Example 2: No Empty Regions



Example 3: One Empty Region


GREEN
NOT YELLOW

Example 4: One Empty Region



Of course, by hatching the "outside" region of the diagram we mean to indicate that no game pieces can be placed there correctly. Strictly speaking, that region is not empty because, for example, the number 50 is in the outside region.

You should be warned that the "NOT" version of the game played with three strings is almost impossible to judge without a "crib-sheet" and is correspondingly difficult to play. Hence you would be well-advised to avoid it until such time as you think the two-string version is no longer challenging enough for the majority of your students.

Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part II

Geometry and Measurement

CEMREL, Inc.

TABLE OF CONTENTS

INTRODUCTION v

NOTE ON GRIDS vii

G-LESSONS

G1 Estimation of Area 1
Worksheets G1* and ** 6
G2 Length #1 7
G3 Taxi-Geometry #1 12
Worksheet G3 18
G4 Millions of Dots / Tangram Puzzles 19
G5 Length #2 25
Worksheets G5*, **, ***, and **** 31
G6 Taxi-Geometry #2 32
Worksheet G6 39
G7 Fractional Parts of Shapes #1 40
Worksheets G7*, **, ***, and **** 49
G8 Area Puzzle 50
Worksheet G8 51
G9 Length #3 52
Worksheets G9*, **, ***, and **** 57
G10 Area Problems with Tangram Pieces 58
Worksheets G10* and ** 64
G11 Fractional Parts of Shapes #2 65
Worksheets G11*, **, ***, and **** 72
G12 Length #4 73
Worksheet G12 76
G13 Mazes 77
Worksheets G13* and ** 78

132

GEOMETRY AND MEASUREMENT

INTRODUCTION

Geometry is useful and interesting, an integral part of mathematics; geometric thinking permeates mathematics and relates to most of the applications of mathematics. Geometric concepts provide a vehicle for exploring the rich connections between arithmetic concepts (such as numbers and calculations) and physical concepts (such as length, area, symmetry, and shape). Even as children experience arithmetic notions in their daily lives, they encounter geometric notions. Common opinion recognizes that each child should begin developing a number sense early. There is no sound argument to postpone the development of geometric sense. These are compelling reasons why the systematic development of geometric thinking should be a substantial component of an elementary school curriculum.

A benefit of early development of geometric thinking is that it provides alternative opportunities for "slow learners." Children develop number concepts at individual rates. Relative lateness of a child's numerical development often is signified by the child's trouble remembering number facts. A common response to this problem is assignment of arithmetic drill, which serves only to confirm the diagnosis of slowness. However, numerical development may not be an accurate indication of other development, in particular, of geometric development. Often the labels "slow" and "fast" are incorrect when alternative activities are presented.

We are careful to allow students a great deal of time to explore quantitative relationships among physical objects before studying arithmetic. In the same spirit, students should be allowed to explore geometrical relationships through free play and guided experiences. Before studying geometrical properties of lines and shapes, they should have free play with models and with drawing instruments in order to create new figures and patterns. Before considering parallelism and perpendicularity as relations on geometric lines, they should have free play in creating patterns with parallel and perpendicular lines.

Before learning length and area formulas, they should have informal experiences to establish and sharpen intuitive notions of area and length. These early experiences are aesthetically rewarding and just as stimulating and suggestive of underlying concepts as early experiences leading to arithmetic. Such experiences are a crucial first step in the development of a geometric sense.

As with all educational development occurring through informal experiences, the payoffs come unexpectedly and spasmodically. In the development of geometric thinking, the suggested activities vary widely, reach into so many other facets of the program, and continue for so long a span that it would be nonsense to set a priori milestones of accomplishments.

Thus, we must not rush the students through the free play activities that precede the more substantive lessons. These preliminary activities have implicit significance for geometric thinking and also provide aesthetic opportunities and explicit practice. We must restrain our inclination to view these activities as mere play. On the other hand, some activities appear to have important consequences or clearly to relate geometry to other parts of the program. Here the inclination might be to spend more time on these activities than is suggested. Again this should be resisted, remembering that similar activities will provide further experience later in slightly different contexts.

The key word is "experience" not "mastery".

But what sorts of experiences will lead students to think geometrically? The standard curriculum usually consists of measurement activities, recognition of some special geometric figures, and association of these figures with their technical names; in short, a good deal of vocabulary but little opportunity for geometric thinking.

A glance at the lessons of this strand indicates that the CSMP Upper Primary Grades curriculum includes measurement activities (leading to the use of metric units) as well as topics not usually in traditional programs at this level.

Measurement activities eventually lead to notions of distance. Traditionally one waits for a good deal of maturity before studying properties of figures, such as circles, that are defined in terms of distance. The reason is that Euclidean distance (distance "as the crow flies") requires sophisticated notions of "exact" measurement in terms of the real number system in order to be able to measure every line segment. But in the CSMP curriculum very young children (even kindergarten) begin thinking about distance in the simplified setting of "taxi-geometry." Taxi-distance (as a taxicab travels) between points is measured in terms of whole numbers by an easy counting process. Experiences in taxi geometry lead to some creative situations, yet do not require special technical skills or vocabulary.

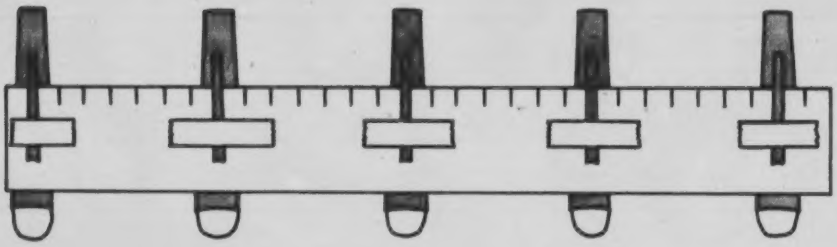
Putting the emphasis on experience rather than on mastery frees us to engage in imaginative activities with children that we ordinarily would not attempt. For example, even when considering such commonplace topics as area and length, we should feel free to devise activities that help students recognize patterns, develop cognitive strategies, and relate the topics to others in the program, while suppressing the urgency to get the "basic" formulas and to master "basic" skills. What is basic here, at this level, is that children are thinking rather than doing tasks on command. The skills and mastery will come eventually, in their own time and place in the curriculum.

NOTE ON GRIDS

A demonstration grid board is needed for the taxi-geometry lessons and for the LENGTH AND AREA workbook lessons. This should be a large square grid board at least 12 squares by 12 squares (and grid lines spaced 6 to 8 centimeters apart) on which you can draw with colored chalk or with colored pens. A portable, commercially made, permanent grid chalkboard is ideal. However, if such a grid board is not available to you, try one of these alternatives.

- Printed grids on large sheets of paper* are provided with full classroom sets of materials for CSMP Mathematics for the Upper Primary Grades, Part II. Tape one of these sheets to the board, use thick felt-tip pens for drawings, and discard the sheet after use; or mount one of the sheets on cardboard, cover it with clear contact paper to provide a protective surface, and use erasable wax crayons for drawings.

- For each lesson, carefully draw a grid on the chalkboard. One can make a tool for drawing such grids efficiently. Use a music staff drawer with chalk in the first, third, and fifth positions; or proceed as follows: clip five pen caps to a ruler at centimeter intervals and secure them with tape. Pen caps are usually of a diameter to hold a piece of chalk securely.



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* These printed grids are packaged under the title "Robot Walk Grid". The title is from lessons in the CSMP Kindergarten curriculum and has nothing to do with the lessons in this strand.

G1 Estimation of Area

CAPSULE LESSON SUMMARY

Compare the areas of many shapes to the area of a given rectangle. Find or estimate the areas of the rectangle and the other shapes using a small square as the unit of measure.

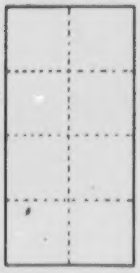
MATERIALS

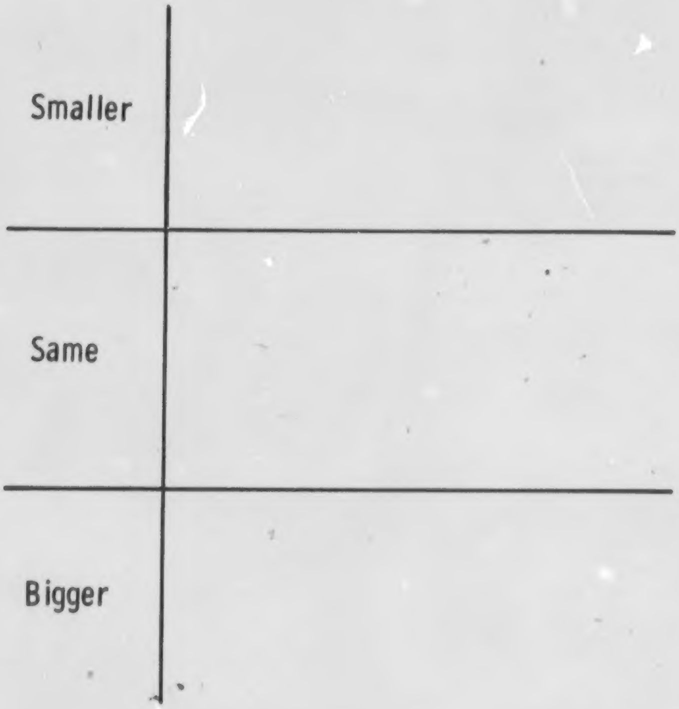
Teacher: Shapes from Geometry Poster #1 for UPG-I
Student: Pencil; Worksheets G1* and **

NOTE: Before this lesson begins, locate the shapes which were prepared for use in Lesson G9 from UPG-I. If necessary, obtain a new copy of Geometry Poster #1 for UPG-I and cut out the shapes.

DESCRIPTION OF LESSON

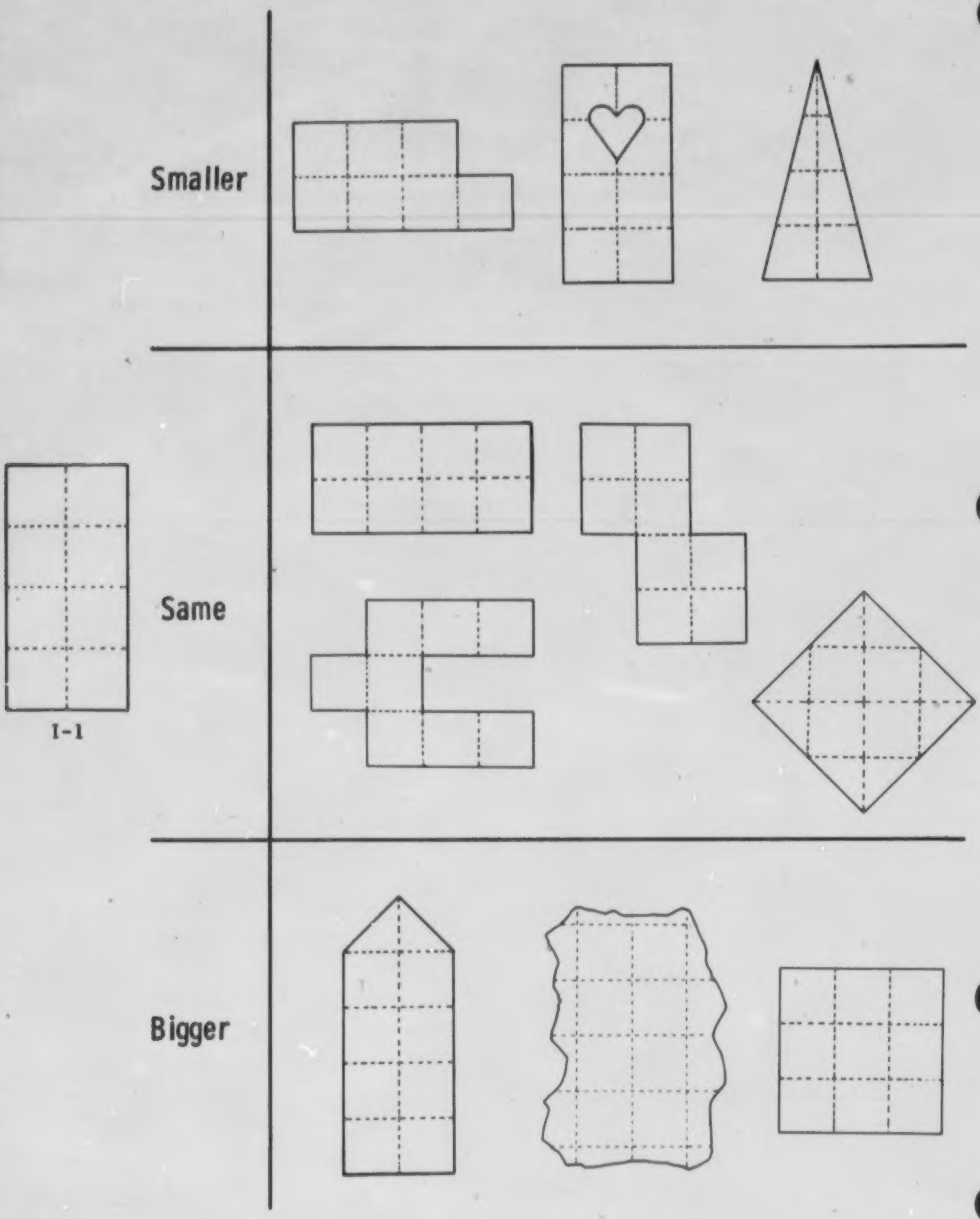
Tape one of the rectangular shapes (see the next illustration) to the board and draw a table large enough to hold the other shapes from Geometry Poster #1 for UPG-I used in Exercise 2 of G9 Comparison of Area from UPG-I, page 54.


I-1



G 1

Quickly compare the sizes of the other shapes to this rectangle by discussing with the class where each shape goes in the table. At the end of this activity, you should have the various shapes taped on the board and correctly placed in the table.



Hold up the small square.

T: We are going to measure these shapes with this little square. The area of this little square is 1 square. What is the area of this rectangle (point to the rectangle at the side of the table)?

S: 8 squares.

Write "8 squares" under the rectangle on the board.

T: Are there any other shapes on the board which have area 8 squares?

S: Yes, all those shapes which have the same area as the rectangle.

Point to all the shapes in the "SAME" section of the table and write "8 squares" under each one.

Point to the first shape (see the previous illustration) in the "SMALLER" section of the table.

T: What is the area of this shape? Is it easy to find its area?

S: Yes, just count the squares. The area is 7 squares.

Write "7 squares" under the shape as you repeat the explanation.

T: Are there other shapes in this table whose areas you can find?

Let the students discuss the other shapes. If a student correctly gives the area of a shape, ask for an explanation. After the class agrees on the correct area of a shape, write it underneath.

G 1

For two of the shapes, it will not be possible to say exactly what is the area. In these cases, encourage estimation. For example,

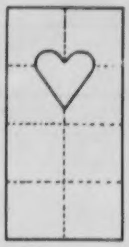
T: It may not be possible to find the area of this shape (point to the rectangle with a heart-shaped hole in it) exactly. What can we say about its area?

S: It is smaller than 8 squares.

T: Yes, and it is bigger than . . . ?

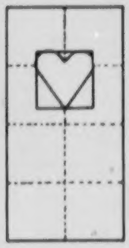
S: 4 squares.

Many answers to this last question are acceptable. Ask for an explanation and if you and the class agree, write the estimate under the shape.



Between 4 and 8 squares

Perhaps the best (whole number) estimate in this case is between 7 squares and 8 squares, because the heart cut-out is smaller than 1 square.

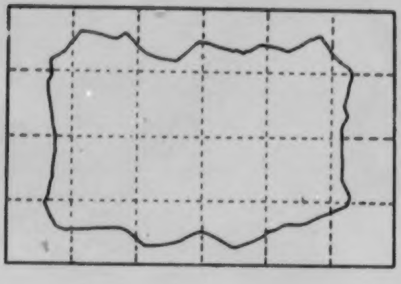


Between 7 and 8 squares

Do not insist on this close an estimate unless a student provides it. Take the best estimate that is suggested and explained as your estimate.

140

One way to discuss the irregular shape shown below is to enclose it in a larger rectangle.



T: How can we estimate the area of this shape now?

S: It is larger than 8 squares.

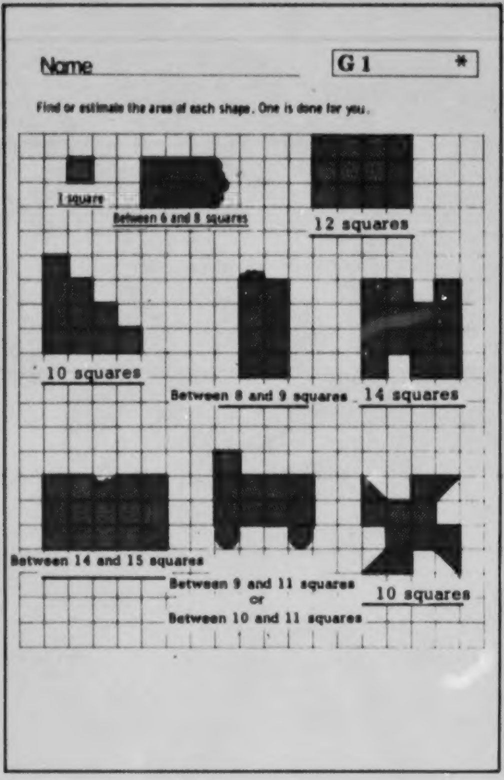
T: Yes, and it is smaller than . . . ?

S: 24 squares.

As before, better estimates may be suggested. Take the best estimate that is suggested and explained as the estimate you write underneath the shape.

Distribute Worksheets G1* and **. Explain that for each shape the students should write its area on the blank underneath or estimate its area as best they can.

NOTE: Save the shapes from Geometry Poster #1 for use in G8 Area Puzzle.



Name

G 1 *

Find or estimate the area of each shape. One is done for you.

1 square

Between 6 and 8 squares

12 squares

10 squares

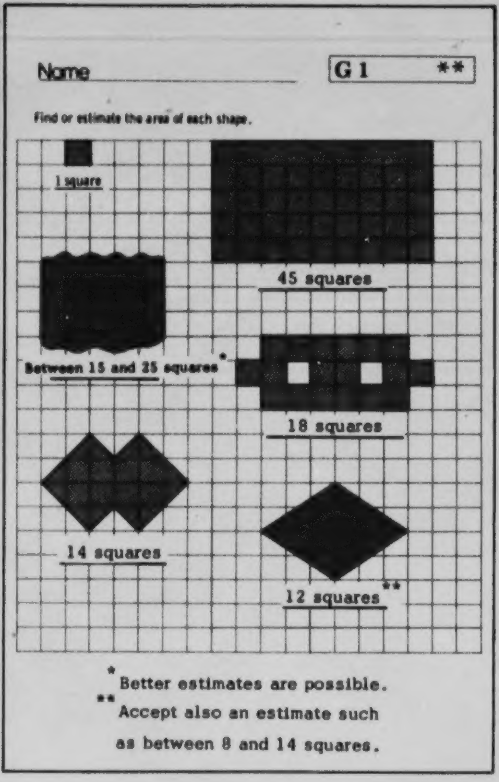
Between 8 and 9 squares

14 squares

Between 14 and 15 squares

Between 9 and 11 squares
or
Between 10 and 11 squares

10 squares



Name

G 1 **

Find or estimate the area of each shape.

1 square

45 squares

Between 15 and 25 squares*

18 squares

14 squares

12 squares**

* Better estimates are possible.

** Accept also an estimate such as between 8 and 14 squares.

G2 Length #1

CAPSULE LESSON SUMMARY

Find two objects in the room that are about the same length. Measure the objects in both orange units (decimeters) and centimeters to compare the lengths. Find an object that is about 100 cm long, a second object that is about 10 cm long, and a third object that is about 1 cm long. Use a centimeter tape measure to find the lengths of a selection of other objects in the room.

MATERIALS

Teacher: Centimeter tape measure
Student: Paper; pencil; centimeter tape measure

DESCRIPTION OF LESSON

T: I would like to find two objects in this classroom which are about the same length.

Allow several students to make suggestions and choose one that is agreed upon by the class and which has accessible objects to measure. Suppose, for example, a student suggests the side edge of the teacher's desk and the bottom edge of the bulletin board.

T: How can we check that the side of my desk and the bulletin board have about the same length?

S: Measure them with a tape measure.

Display a tape measure and review the fact that on one side of the tape an orange rod fits between two orange marks. If you have not used Cuisenaire rods, simply refer to this as "the orange length."

T: Let's measure my desk and the bulletin board in orange rods.

G 2

Invite students to make these measurements and record the results on the board. For example,

Teacher's Desk (side): between 7or and 8or

Bulletin Board: between 7or and 8or

T: It appears that the side of my desk and the bulletin board are about the same length. How could we get a more accurate measurement and perhaps be able to say which one is longer?

Accept suggestions which indicate measuring with smaller units. Another natural suggestion would be to put a mark on the tape measure indicating the length of one of the objects and then comparing this with the length of the other. If this suggestion is made, invite students to do it.

Suppose, for example, that the class determines that the bulletin board is longer than the desk.

T: Let's use the other side of the tape measure and measure in centimeters.

If appropriate, review that one centimeter is the length of a white (Cuisenaire) rod.

Write "centimeter" and its abbreviation "cm" on the board. Ask several students to read the word aloud.

Invite students to measure the lengths of your desk and the bulletin board in centimeters and to record the results on the board. For example,

Teacher's Desk (side): 73 cm

Bulletin Board: 76 cm

T: Now I would like to find something in this classroom which is about 10 orange rods long. How many centimeters is 10 orange rods?

S: 100 centimeters.

T: This tape measure is 10 orange rods or 100 centimeters long. What do you see in the room that is about the same length?

Accept several suggestions and choose one that is accessible. Suppose, for example, a student suggests the front edge of the teacher's desk.

T: Do you think the front edge of my desk is longer or shorter than 100 cm?

Allow several students to comment and then invite one student to make the measurement. If it is longer than 100 cm, you will need to discuss how to mark 100 cm and then measure the excess. Record the result on the board.

Teacher's Desk (front): 114 cm

T: If we were to measure the front edge of my desk in orange rods, what would we get?

S: Between 11 orange rods and 12 orange rods.

If necessary, ask a student to make the measurement in orange rods.

T: Now, I would like to find something in this classroom which is about 10 centimeters long. How many orange rods is 10 centimeters?

S: 1 orange rod.

Show the class 10 centimeters on the tape measure and then accept suggestions of objects in the room which are about this same length. Suppose a student suggests his pencil.

T: Hold up your pencil, Mark. Do you think Mark's pencil is longer or shorter than 10 centimeters?

G 2

Allow several students to comment and then ask a student to measure the pencil. If necessary, suggest that the student measure to the nearest centimeter. For example, if the pencil is between 10 cm and 11 cm, ask which length is closer. Record the result on the board.

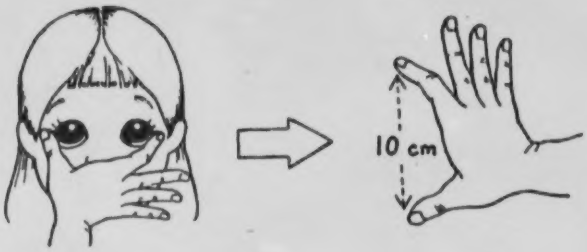
Mark's pencil: 11cm

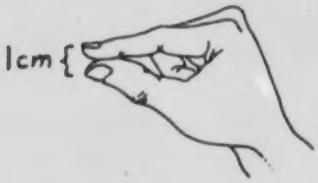
T: Now I would like to find something in the room which is about 1 cm long.

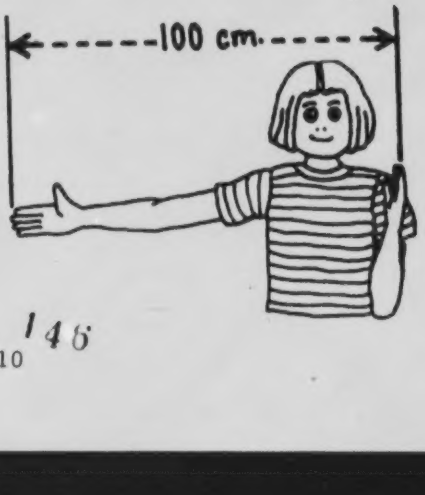
Accept several suggestions and let the students measure the suggested objects. This problem may be more difficult because of the relative difference in length of two very short objects. Also, sometimes it is hard to describe what it is you wish to measure; for example, a student may suggest the diameter of a button.

Erase the board. Ask the entire class to follow these instructions. Observe their gestures and then give a suggestion yourself — as illustrated in the next picture.

T: Hold up two fingers so that they are about 10 centimeters apart.
Hold up two fingers so that they are about 1 centimeter apart.
Hold up your two hands so that they are about 100 centimeters apart.







Distribute tape measures and paper. Ask the students to copy a list of objects that you write on the board, to measure these objects to the nearest centimeter, and to write their results on the paper. List four or five objects which are easily accessible, for example,

eraser
paper clip
height of my desk
height of my chair
width of doorway
crayon
my longest finger

Choose objects which were not measured in a previous lesson.

G 3

G3 Taxi-Geometry #1

CAPSULE LESSON SUMMARY

Find the lengths (taxi-measure) of shortest paths between corner points of a grid. Find all the corner points that are 5 blocks from a given point along a shortest path and observe that there are twenty such points. Locate a point knowing that it is 3 blocks east and 2 blocks north of the given point.

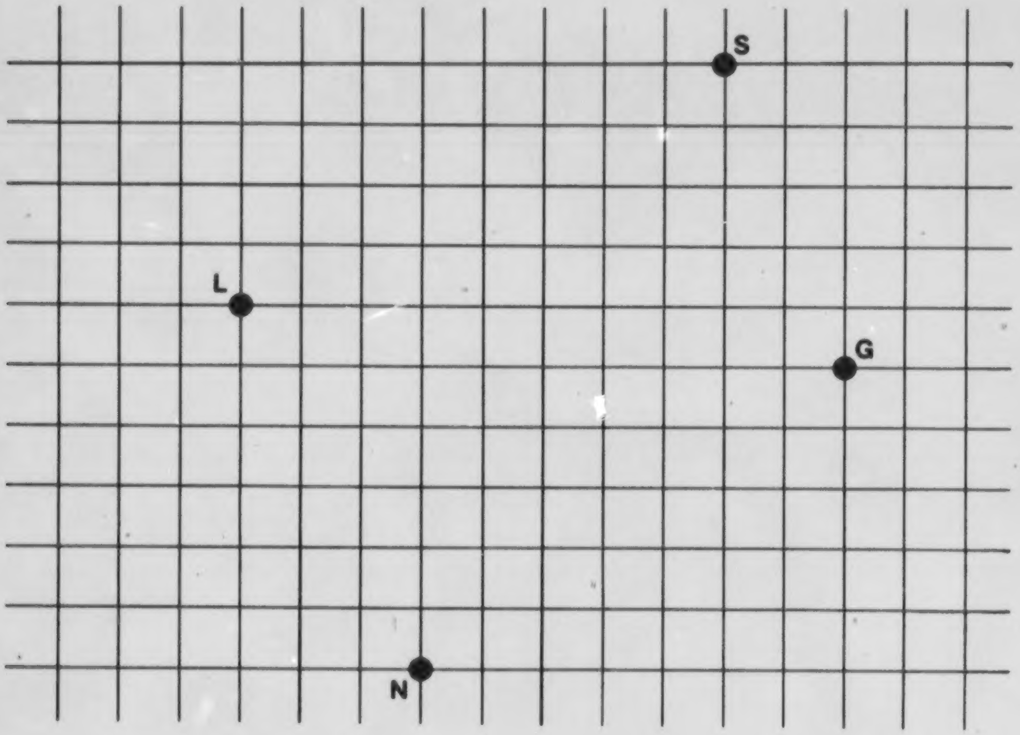
MATERIALS

Teacher: Grid board[*]; colored chalk
Student : Worksheet G3; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Display your grid board. Draw and label four dots as shown below.



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[*] See the NOTE ON GRIDS in the introduction to this strand.

148

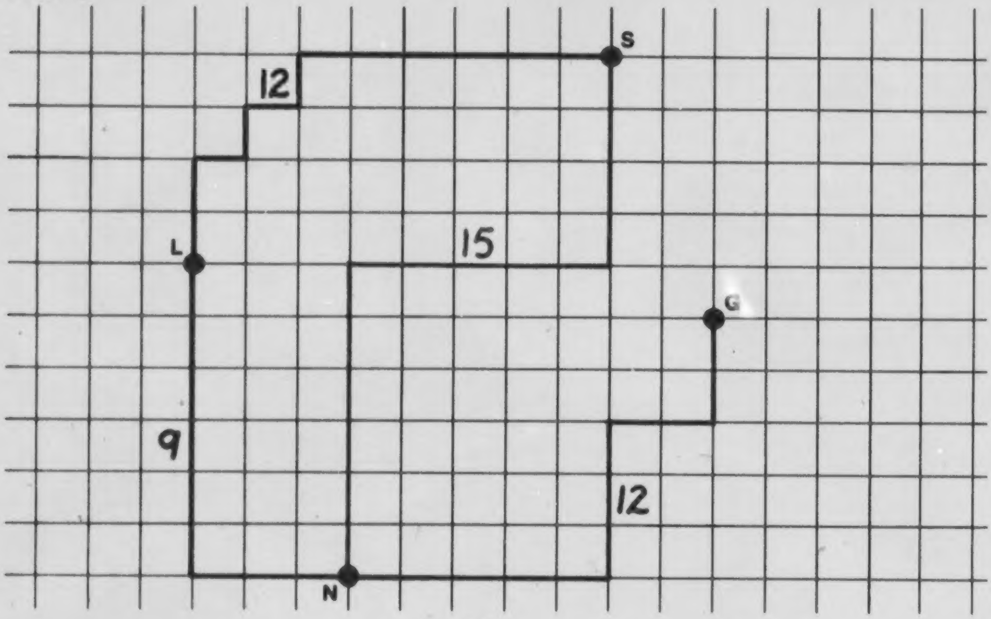
T: This is the map of the city where Nora lives. Here is Nora's house (point to "N"), and here is her grandmother's house (point to "G"), and here is Nora's school (point to "S"), and here is the library (point to "L"). Who can tell us something about our friend Nora?

Let students briefly recall some of the previous activities involving Nora. Encourage them to discuss long and short walks and counting the number of blocks in a path. If possible during this discussion, ask students to guess which place, L, S or G, is closest to Nora's house.

T: Suppose it is raining and Nora wants to take a shortest path from her house (N) to school (S). How many blocks is a shortest path from N to S? Who can show us a way for Nora to walk?

Call on a volunteer to trace a path from N to S. Ask the class if anyone can find a shorter path. When the students agree on a shortest path, draw it on the grid board. Ask a student to count the blocks in this shortest path. (Answer: 15 blocks)

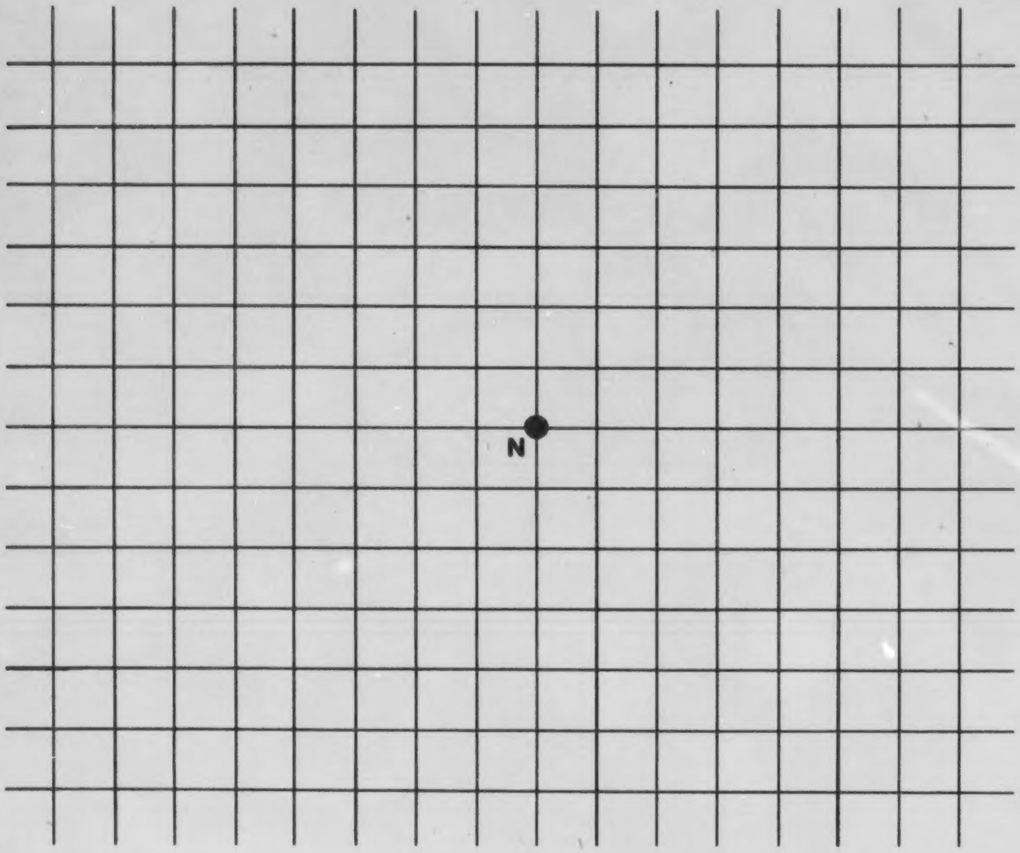
Repeat the questioning to get a shortest path from N to G, and a shortest path from N to L, and a shortest path from S to L. Possible responses are illustrated below.



G 3

Exercise 2

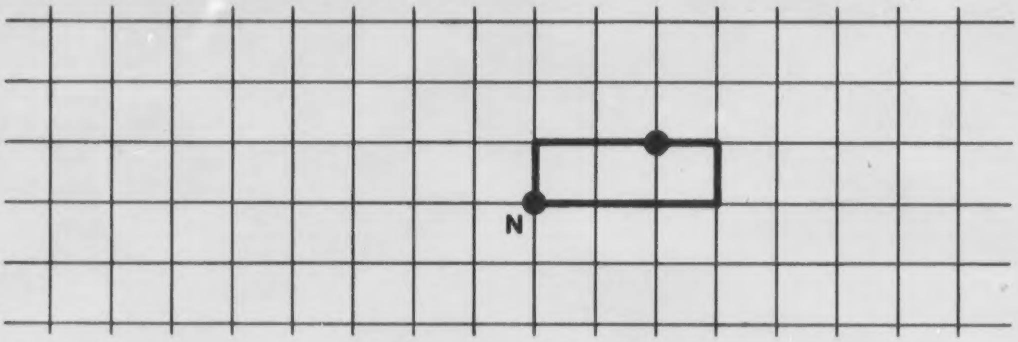
Erase the grid board. Draw a dot near the center of the grid and label it "N".



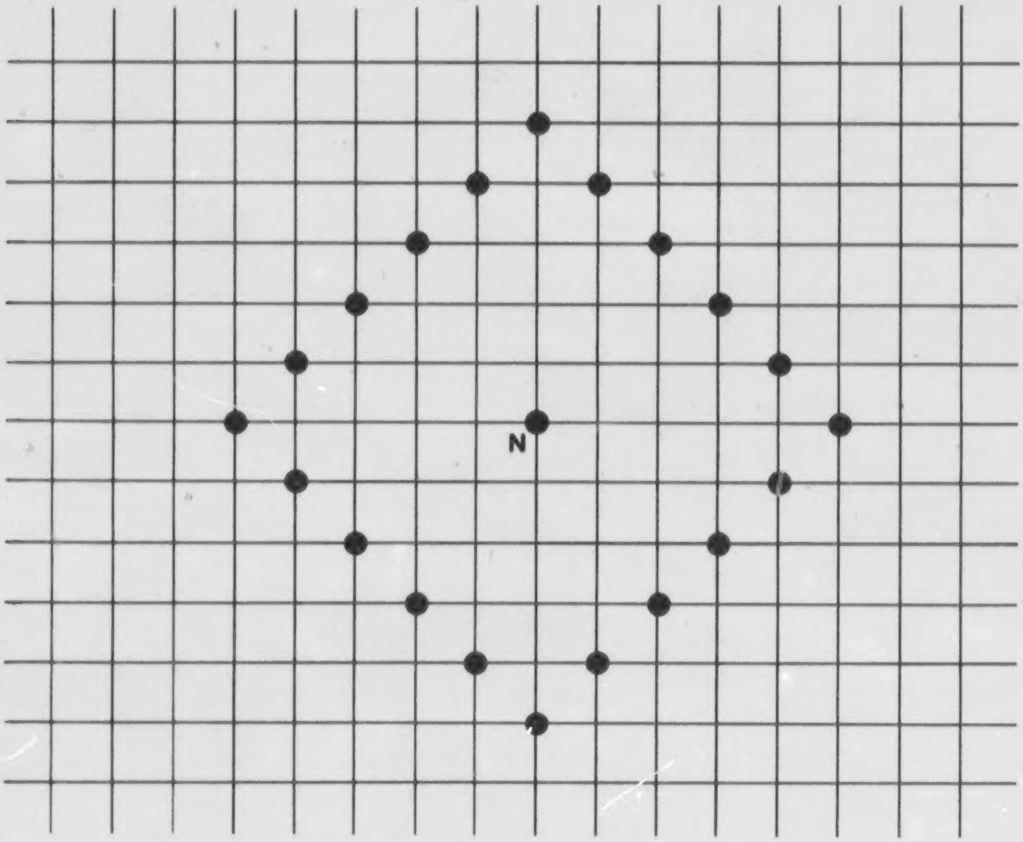
T: Nora has a friend, Paula. I'm not going to show you where Paula lives, but I will tell you that a shortest path from Nora's house to Paula's house is 5 blocks. Where could Paula's house be?

There are twenty points where Paula's house could be (see the illustration on page 15). Call on students to locate some of these points, each time tracing a shortest 5-block path from N to a point where Paula's house could be. Draw red dots at correctly located points. You may have to remind students that a shortest walk from N to Paula's house is 5 blocks. A student may suggest, for example, the following 5-block path (the red path in the next illustration).

In this case, demonstrate that the red path is not a shortest path. There is a 2-block path (in blue) from N to this point.



After several points have been located, some students should recognize a pattern. When this happens, encourage them to describe the pattern and perhaps to locate several more points following the pattern. A completed picture should have twenty red dots as shown here.



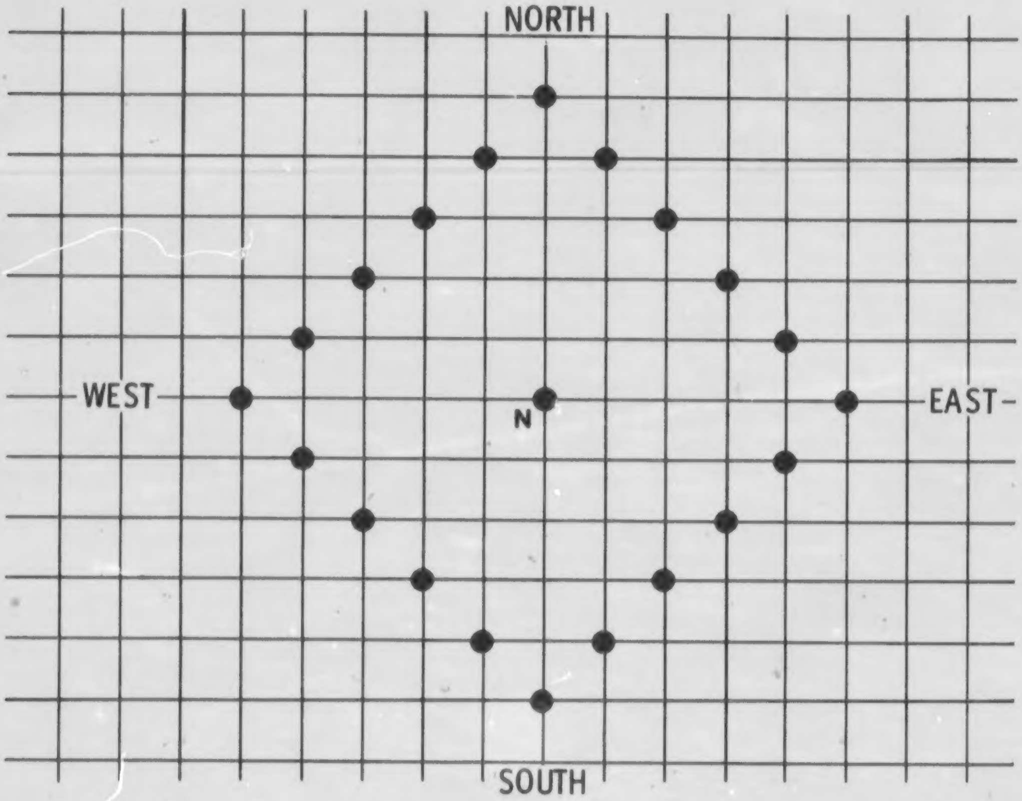
G 3

T: How many places did we find where Paula's house could be?

S: Twenty.

T: Just one of them is where Paula's house is. Do you know which directions are North, South, East, and West on a map?

Ask students to indicate each of these directions or if necessary, explain them yourself and write "north", "south", "east", and "west" on the grid board.



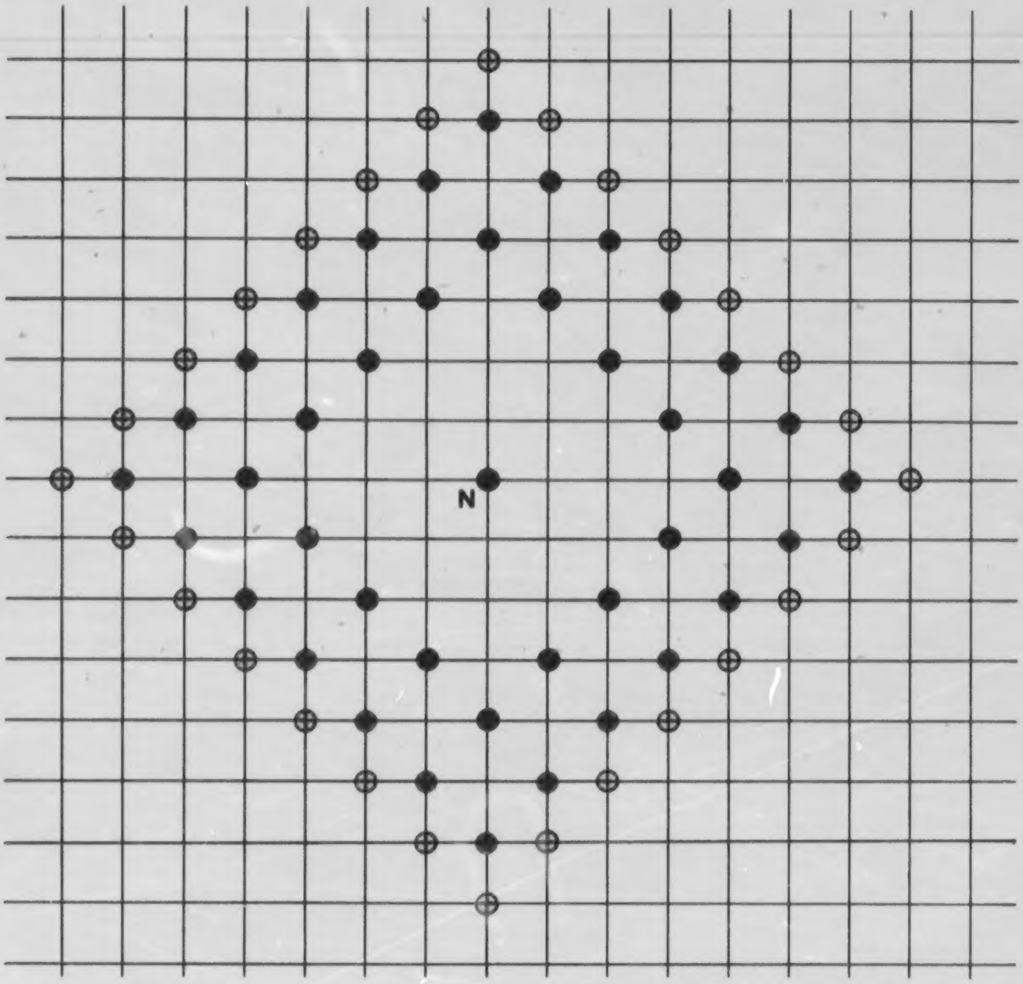
T: Now I can tell you how to find Paula's house. A shortest path from Nora's house to Paula's house is 3 blocks east and 2 blocks north.

Write "3E, 2N" on the board. Let a student locate Paula's house and label the dot "P".

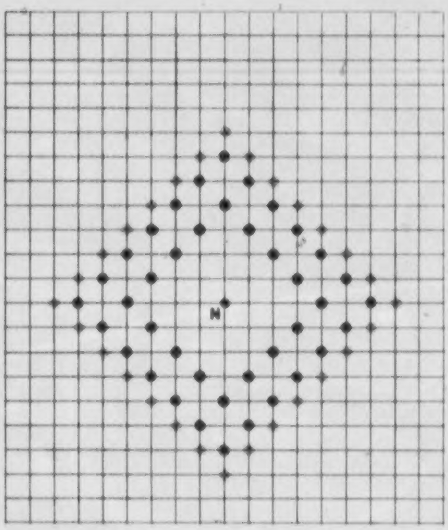
Distribute Worksheet G3 and colored pencils.

T: Here is Nora's house, the dot labeled "N". Nora's friend Maria lives 6 blocks away from Nora's house. A shortest walk from Nora's house to Maria's house is 6 blocks. On your paper draw blue dots at all the places where Maria's house could be.

Students who finish this problem quickly could be asked to find all the places which are 4 blocks or 7 blocks (along a shortest path) from Nora's house. Use different color dots for these places. The picture below has blue dots for places 6 blocks from N, green dots for places 4 blocks from N, and yellow dots for places 7 blocks from N.



Name ____________________ G 3



Blue: places six blocks from N.
Green: places four blocks from N.
Yellow: places seven blocks from N.

G4 Millions of Dots/Tangram Puzzles

CAPSULE LESSON SUMMARY

Observe the similarity between two pictures where one is simply a reduction of the other. Note the number of dots in a given picture and then look at other pictures which are reductions of this picture. Find out how many dots would be in all the copies of a reduced picture needed to cover the given picture. Individually work on the Tangram booklet to make certain designs with all the Tangram pieces.

MATERIALS

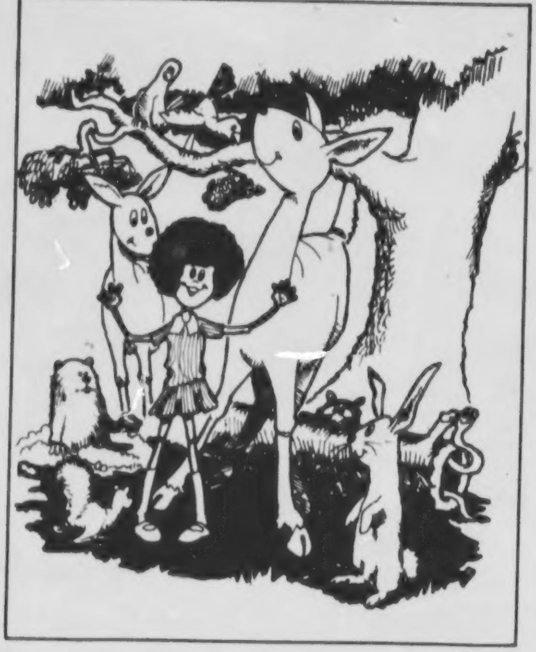
Teacher: Geometry Poster #1 for UPG-II
Student: Tangram; Tangram booklet; pencil

NOTE: Before this lesson begins, cut out the pictures on Geometry Poster #1.

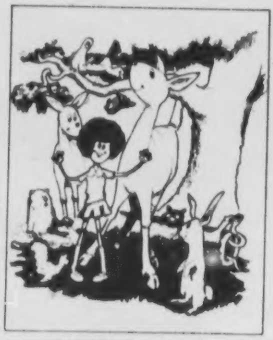
DESCRIPTION OF LESSON

Exercise 1

Tape Picture A and the reduced copy of Picture A to the board.



PICTURE A (1)



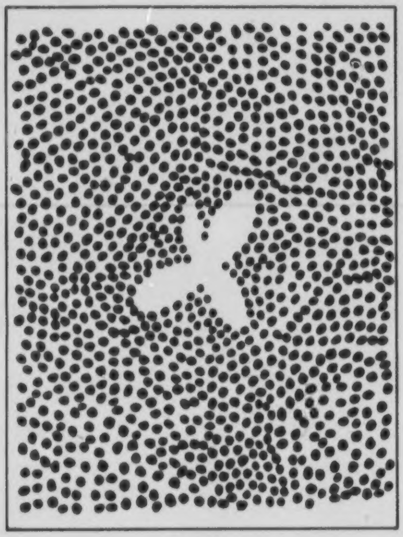
PICTURE A (2)

G 4

T: What do you think about these two pictures?

Let students react. Try to bring out of the discussion the fact that the two pictures are the same except that one is a reduced copy of the other. You might observe, for example, that there are nine animals in the big picture and also nine animals in the small one.

Hold up Picture B (1).



PICTURE B (1)

T: How many dots do you think there are in this picture?

Accept several estimates from the students. In fact, there are exactly 1,000 dots in the picture. If the guesses are wild, you might suggest counting the dots in some small part of the picture and then ask for an estimate for the entire picture.

T: There are exactly 1,000 dots in this picture.

Tape Picture B (1) to the board and write "1,000 dots" under it. Then hold up Picture B (2) and let the class comment. They should notice that it is just a reduced copy of Picture B (1).

156

T: How many dots are there in this picture?

S: 1,000.

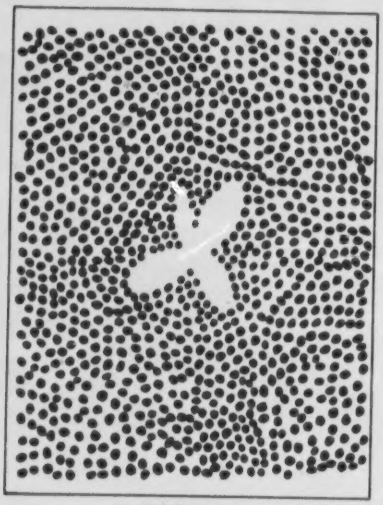
Perhaps this answer will not be obvious to everyone in the class. If necessary, refer back to Picture A and its reduced copy to discuss that there are the same number of animals in both the big and small picture.

Hold Picture B (2) in front of Picture B (1).

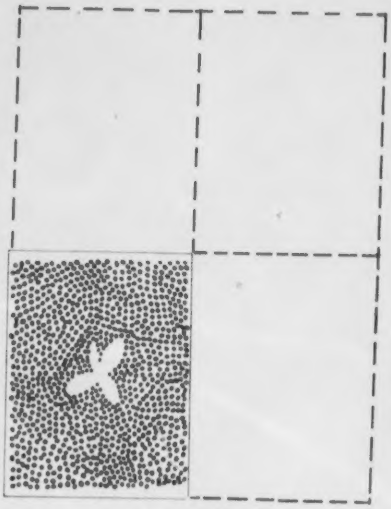
T: How many of these smaller pictures would we need to cover the big picture?

S: Four.

Ask a student to demonstrate how four copies of Picture B (2) would cover Picture B (1). Then tape Picture B (2) to the board and draw the dotted lines illustrated below to make it clear.



1,000 dots



G 4

T: Suppose we had four copies of this picture here. Together they would make a new big picture. How many dots would be in the new big picture?

S: 4,000.

Write "4,000 dots" under the outline for the new big picture. Then hold up Picture B (3) and let the class comment. They should notice that it is an even smaller copy of the same picture.

T: How many dots in this picture?

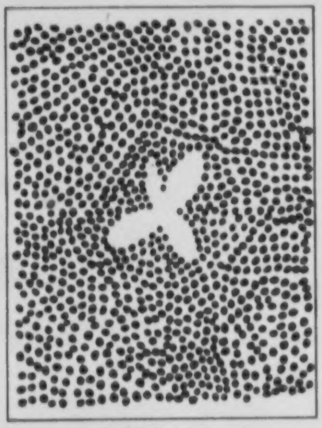
S: 1,000; just like the big one.

Hold Picture B (3) in front of Picture B (1).

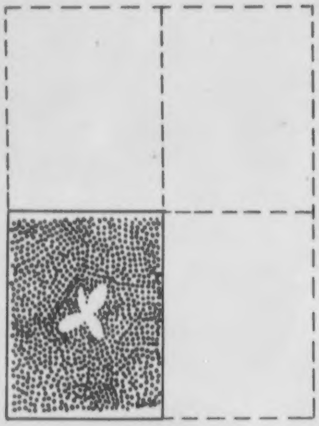
T: How many of these small pictures would we need to cover the big picture?

S: Sixteen.

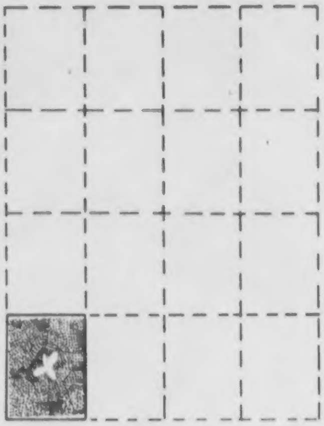
Again, ask a student to demonstrate how sixteen copies of Picture B (3) would cover Picture B (1). Then tape Picture B (3) to the board and draw dotted lines as in the next illustration.



1,000 dots



4,000 dots



T: Suppose we had sixteen copies of this picture here. Together they would make another new big picture. How many dots would be in this new big picture?

S: 16,000.

Write "16,000 dots" under the outline for this new big picture. Then hold up Picture B (4).

T: This is a very small copy of our big picture. How many dots are there in this little picture?

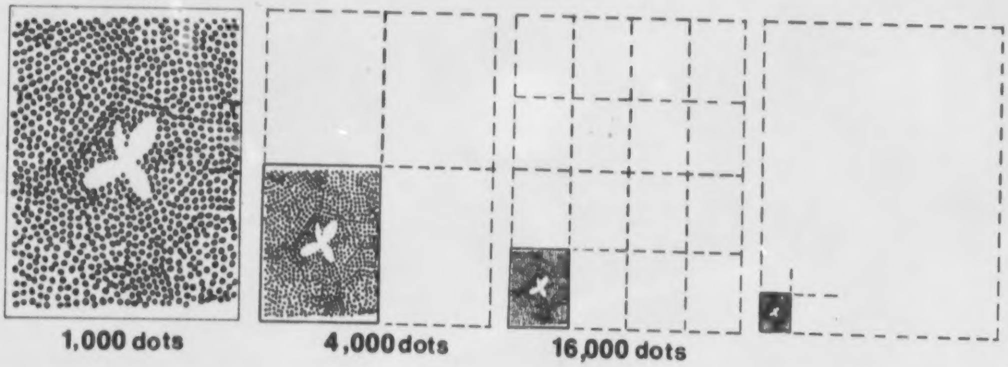
S: 1,000.

Hold up Picture B (4) in front of Picture B (1).

T: How many of these small pictures would we need to cover the big picture?

Let the class think about this problem for a few minutes. Suggest that students who think they know the answer (64) whisper it to you and, if they are correct, ask them to write their answer on a piece of paper. Ask them also to write the number of dots in that many copies of the little picture. When several members of the class have found the answer, invite a student to answer aloud.

Tape Picture B (4) to the board and prepare to draw dotted lines to indicate all the copies you would need to cover Picture B (1).



G4

Encourage students to discuss how they found the answer.

S: Four copies of the little picture cover the next size picture (B(3)), so I counted by fours sixteen times.

S: Sixteen copies of the little picture cover this size picture (B(2)), and four copies of this picture (B(2)) cover the big picture, so I added: 16 + 16 + 16 + 16 = 64.

As the students discuss their methods of solution draw in the rest of the dotted lines and observe that you need eight rows of eight copies. This picture might suggest other methods of calculating "8 x 8". For example,

8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 64;

4 x 4 = 16 and 2 x 16 = 32 and 2 x 32 = 64.

T: These dots are very small, but we could make them even smaller. Try to imagine how many dots we could have on a piece of paper this size (indicate the size of picture B(1)) if we made them so tiny you would need a microscope to see them. Perhaps there would be so many no one could ever count them.

Exercise 2

Distribute Tangram pieces and the students' copies of the Tangram booklet. Review the instructions for this booklet (see Exercise 2 of Lesson G7 in UPG-I, page 40). Tell the students that they should work on any pages which are not checked with a smiling face or star. Solutions for the Tangram puzzles are at the end of Lesson G7 in UPG-I, pages 41-48.

If students become frustrated with some pages, instruct them to go on to other pages and return to those pages later. If some students think Tangram pieces are missing from their sets, suggest they check by putting all their pieces on the cover square. Students who complete the booklet before the end of the period could be asked to draw some of their own Tangram puzzles.

At the end of the lesson, collect the Tangram pieces as they will be used again in other lessons.

G5 Length #2

CAPSULE LESSON SUMMARY

Find the lengths of zigzag paths between two points by measuring in centimeters the lengths of the parts and adding them together. Find shorter paths between the points and compare the lengths of these zigzag paths to the lengths of the shorter paths.

MATERIALS

Teacher: Centimeter tape measure; Geometry Poster #2 for UPG-II

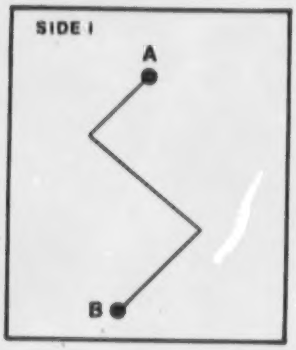
Student: Centimeter tape measure or 30 cm ruler; pencil; Worksheets G5*, **, ***, and ****

NOTE: Cut Geometry Poster #2 in half along the center line.

DESCRIPTION OF LESSON

Exercise 1

Tape side 1 of Geometry Poster #2 to the board.



T: Here (point to "A") there is a bug and here (point to "B") is some food. The bug walks along this zigzag path (trace the path from A to B) to get the food. I would like to know how long is this path? How can we find its length?

S: Measure it.

T: Yes, and how should we measure it?

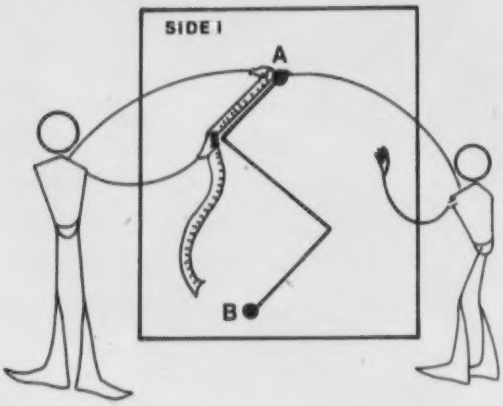
G 5

S: With a tape measure.

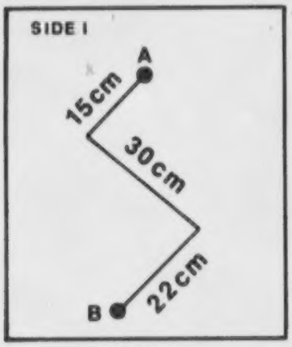
Display a tape measure and let the class discuss how to do the measuring. Direct the discussion so that the class realizes the three pieces of the zigzag path may be measured separately, and then their three lengths added to find the total length of the path.

T: We will measure the path in centimeters (review that the marks on one side of the tape measure are for centimeters), but before we do it, can you guess how long this path is?

Record several students' guesses on the board. Then call on three different students to assist you in measuring the three pieces of the zigzag path. With each student, emphasize that the end mark on the tape is put at one end of the piece (segment) and you read the number of centimeters at the other end.



After each piece is measured, write the length next to the segment.



T (tracing the appropriate segments): We found that the bug walked 15 cm along this part of the path, and then 30 cm along this part, and then 22 cm along this part. How can we find how long the entire path is?

S: Add 15, 30, and 22.

Write the addition problem on the board and collectively do the calculation.

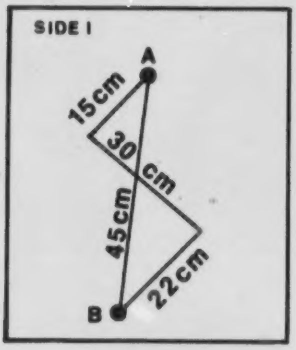
$$\begin{array}{r} 15\,\text{cm} \\ 30\,\text{cm} \\ +\ 22\,\text{cm} \\ \hline 67\,\text{cm} \end{array}$$

T: So, this path is 67 cm long.

Compare the length of the path with the students' guesses and determine which guess is closest.

T: Is there a shorter path the bug could take to get the food?

Allow students to make suggestions until the shortest path, a straight line segment, from A to B is mentioned. Then draw this path on the poster and ask a student to measure it.



T: This path is only 45 cm long. How much shorter is it than the first zigzag path?

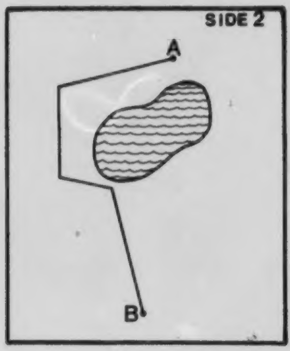
S: 22 cm shorter.

G 5

The subtraction problem may be difficult for some students, but do not spend too much time on it. One way to observe that the difference is 22 cm is to note that the first two pieces of the zigzag are 45 cm together and the third piece is the extra 22 cm.

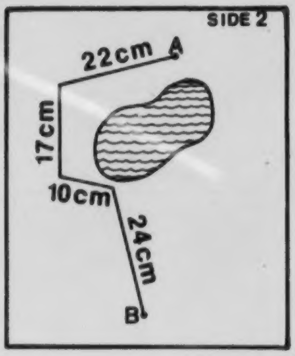
Exercise 2

Tape side 2 of Geometry Poster #2 to the board.



T: Here (point to "A") is the bug and here (point to "B") is its food. There is a puddle of water between the bug and its food and the bug cannot swim, so it walks along this zigzag path (trace the path from A to B) to get to the food. About how long is this path, do you think?

Record several students' guesses on the board and then, as in Exercise 1, call on different students to assist you in actually measuring each piece of the zigzag.



164

T: How do we find the total length of this path?

S: Add 22, 17, 10, and 24.

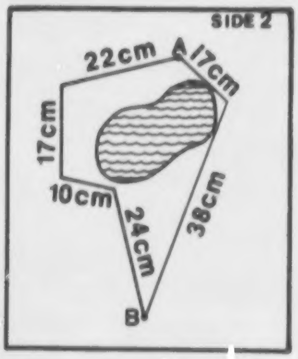
Write the problem on the board and collectively do the calculation.

$$\begin{array}{r} 22\text{ cm} \\ 17\text{ cm} \\ 10\text{ cm} \\ +\ 24\text{ cm} \\ \hline 73\text{ cm} \end{array}$$

Compare the length of the path with the students' guesses to determine which one is the closest.

T: Who can show us a shorter path the bug could take to get to the food? Remember, the bug cannot swim so does not go through the puddle.

Allow students to make suggestions and choose one that the class agrees will be shorter. Draw the path on the poster and ask students to assist you in measuring it. A possibility, one which comes close to the shortest path, is shown in the next illustration.



T: How long is this new path? Do the calculation on your paper.

S: 55 cm.

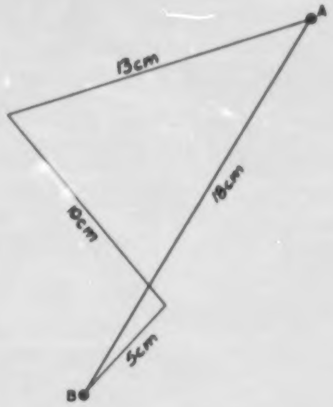
G 5

T: It certainly is shorter than the first path.

If you feel it would be appropriate for your class, ask them to determine how much shorter the new path is than the original path. (18 cm)

Worksheets G5*, **, ***, and **** are available for independent work. It would be helpful if students had straight edges with which to draw. Otherwise, suggest that they draw straight line segments with the edge of a crayon box or book as a guide. Then they should measure with the tape measure. Some students may need further instruction on measuring.

Name ____________ G 5 *


A
13cm
18cm
10cm
5cm
B

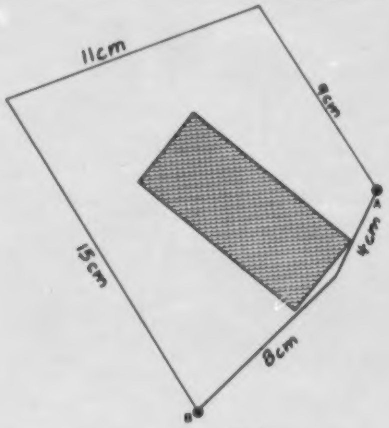
How long is this zigzag path from A to B? 28 cm

Try to find a shorter path from A to B. Draw it.

How long is your path? 18 cm

Other solutions are possible.

Name ____________ G 5 **


11cm
9cm
A
4cm
15cm
8cm
B

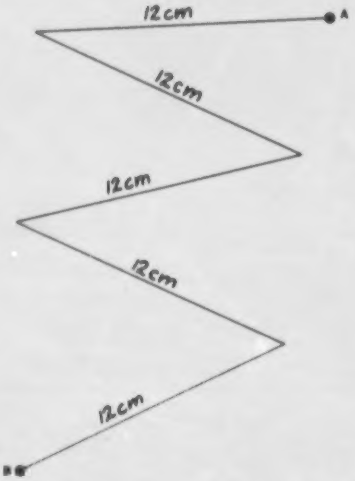
How long is this zigzag path from A to B? 35 cm

Try to find a shorter zigzag path. Draw it.

How long is your path? 12 cm

Other solutions are possible.

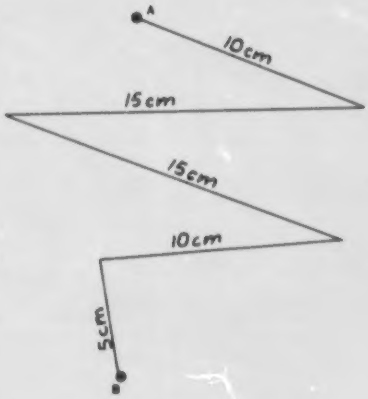
Name ____________ G 5 ***


12cm
A
12cm
12cm
12cm
12cm
B

How long is this zigzag path from A to B? 60 cm

Name ____________ G 5 ****

Draw a zigzag path from A to B which is longer than 50 cm.


A
10cm
15cm
15cm
10cm
5cm
B

How long is your path? 55 cm

Many solutions are possible.

G6 Taxi-Geometry #2

CAPSULE LESSON SUMMARY

Locate several points on a grid given their positions in terms of number of blocks north, south, east, or west of a given point. Find points which are 6 blocks (along a shortest path) from one given point and 3 blocks (along a shortest path) from another given point. Find many places which are the same distance (taxi-measure) from two given points.

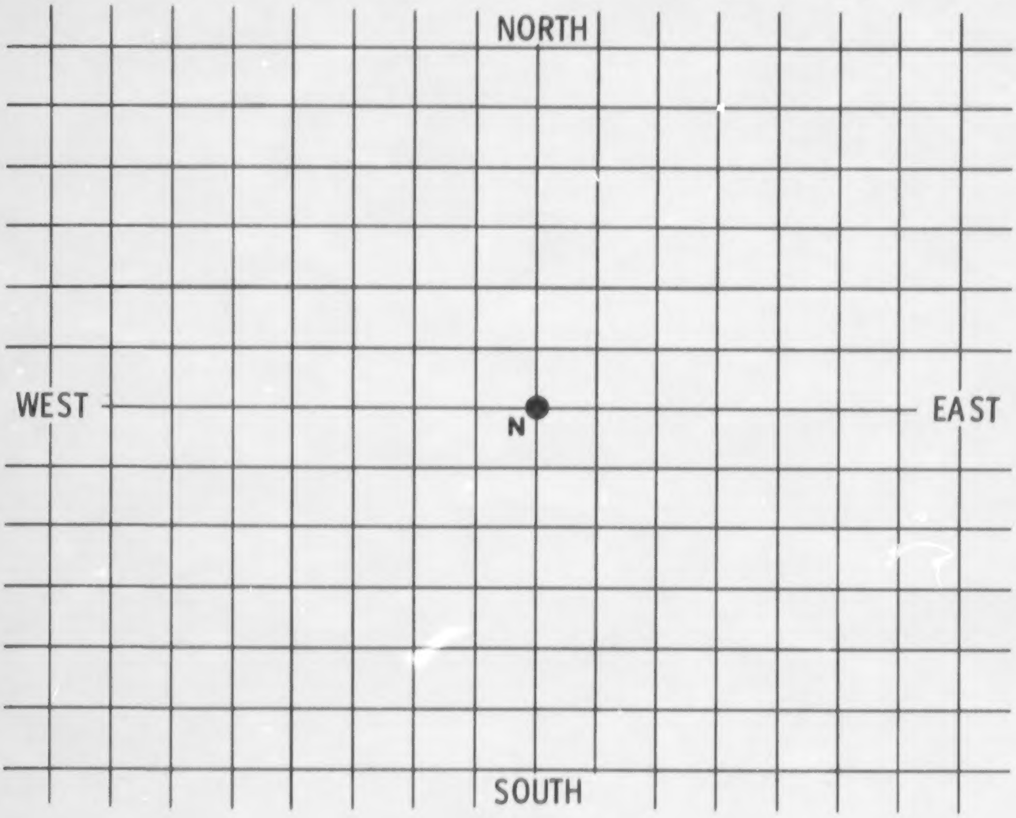
MATERIALS

Teacher: Grid board[*]; colored pencils
Student: Worksheet G6; colored pencils

DESCRIPTION OF LESSON

Exercise 1

Display your grid board. Write "North", "South", "East", and "West" on the appropriate edges. Draw a dot (centrally located) and label it "N".



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[*] See the NOTE ON GRIDS in the introduction to this strand.

T: This is a map of the city where our friend Nora lives. When you read a map, usually this direction is north (point up toward "North"), this direction is south (point down toward "South"), this direction is east (point right toward "East"), and this direction is west (point left toward "West"). Here is Nora's house (point to "N"). Do you remember where Nora's friend Paula lives?

S: 5 blocks from Nora's house.

T: Yes, a shortest walk from Nora's house to Paula's house is 5 blocks. But we found lots of places like that. Do you remember the directions?

S: 3 blocks east and 2 blocks north.

You need not insist on this recall; but if no student gives the correct directions, draw and label "P" the dot for Paula's house and then ask for the directions.

Call on students to locate several other points from directions; for example,

- starting at N go 6 blocks west and 2 blocks north;
- starting at N go 3 blocks south and 3 blocks west;
- starting at N go 5 blocks north and 4 blocks east;
- starting at N go 4 blocks east and 5 blocks north.

Point to some intersections and ask students to give directions to those points.

Exercise 2

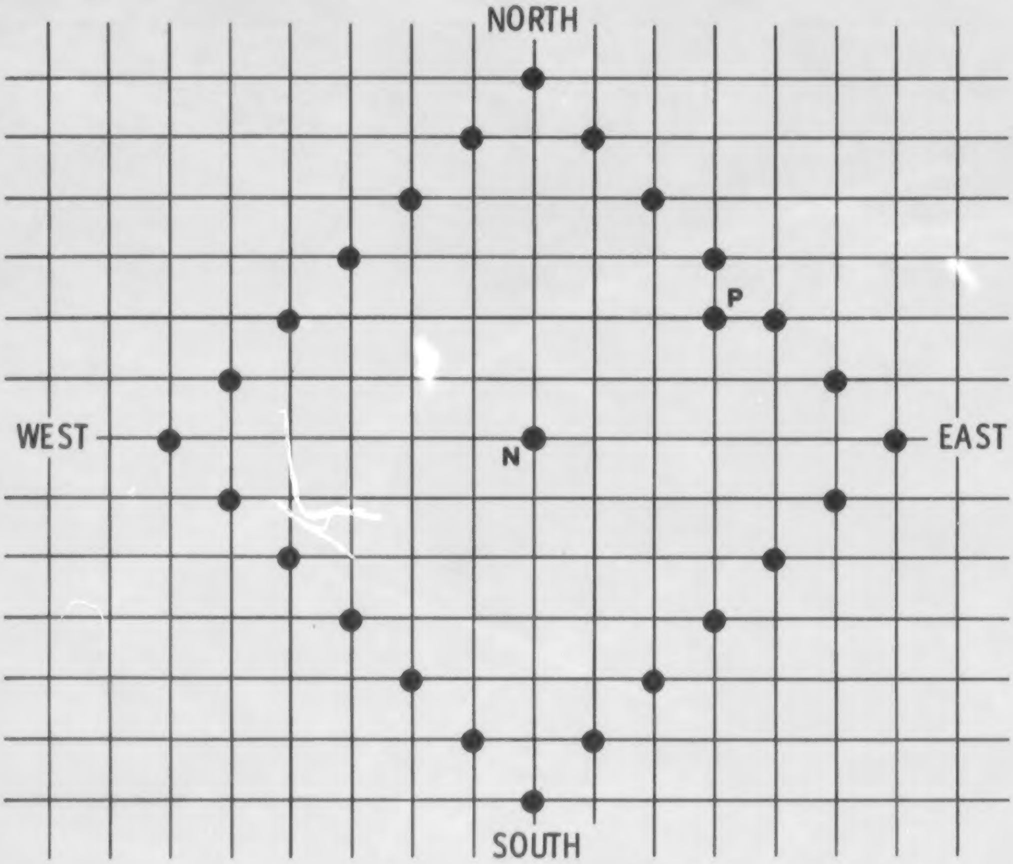
Distribute Worksheet G6 and colored pencils. Ask students to look at the side where there is a dot (N) for Nora's house and a dot (P) for Paula's house.

T: Nora and Paula have another friend Maria. A shortest walk from Nora's house to Maria's house is 6 blocks. On your paper draw red dots at the places where Maria's house could be.

G 6

Walk around and help students who have trouble getting started. As the class is working, invite some students to draw the red dots where Maria's house could be on the demonstration grid board. Let one student draw several dots at a time.

A completed drawing is shown below.



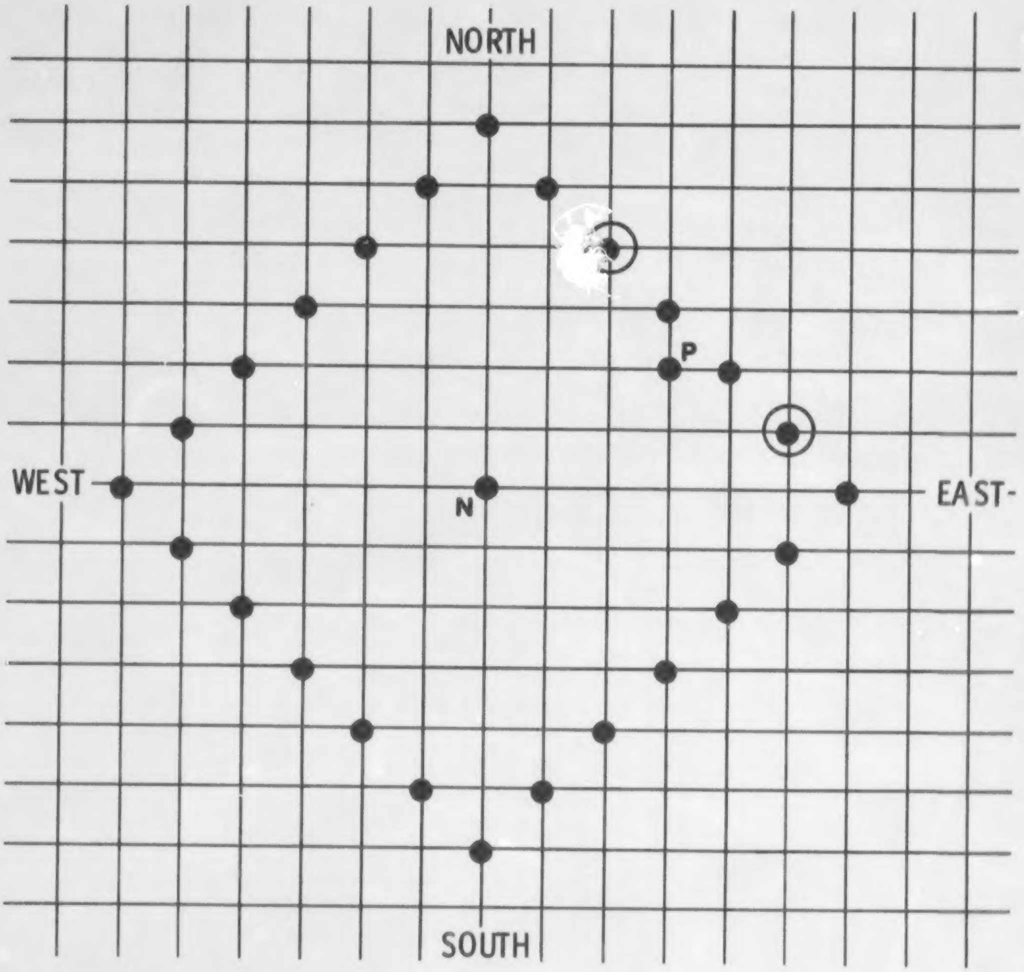
T: How many places are there that Maria's house could be?

S: Twenty-four.

T: I will give you some more information. A shortest walk from Paula's house to Maria's house is 3 blocks. Do you know where Maria's house is now?

170

Let students suggest places and each time check that a shortest walk from P to that place is 3 blocks. There are still two possibilities, circled in the picture below. Circle these dots on your grid board and erase the other red dots as they are ruled out.



T: If you walk from Maria's house to Nora's house, you can go 2 blocks west and 4 blocks south. Do you know where Maria's house is now?

Call on a student to point to the correct dot. Label it "M".

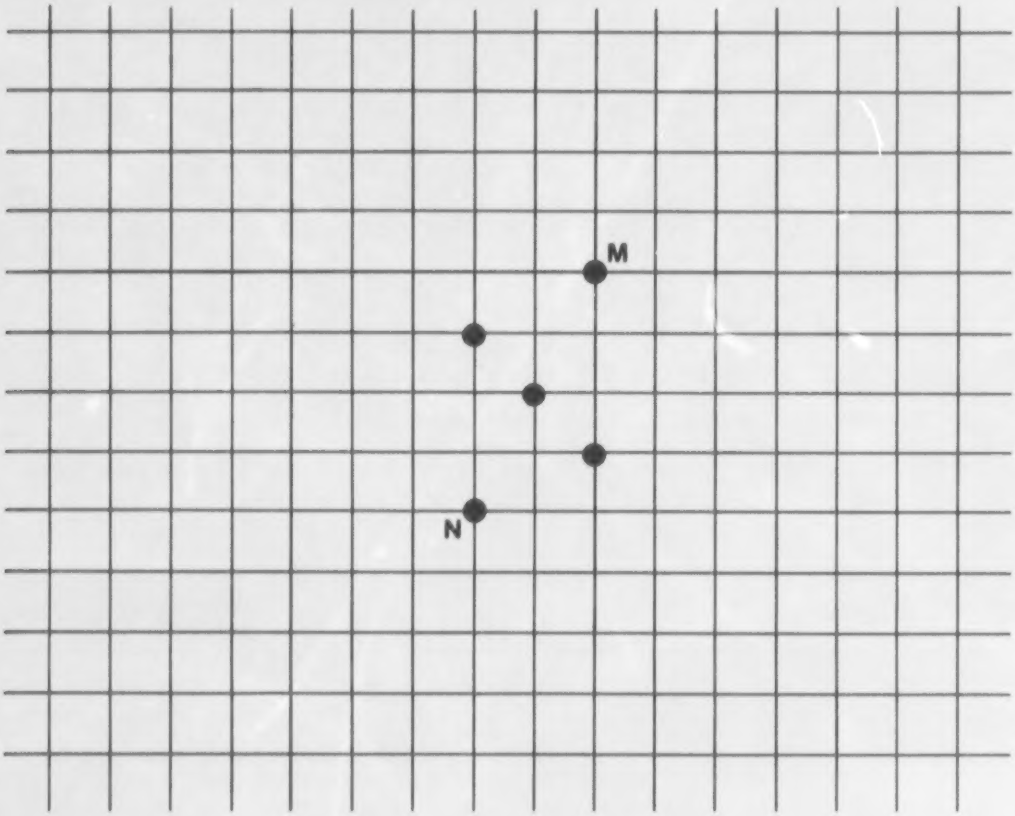
G 6

Exercise 3

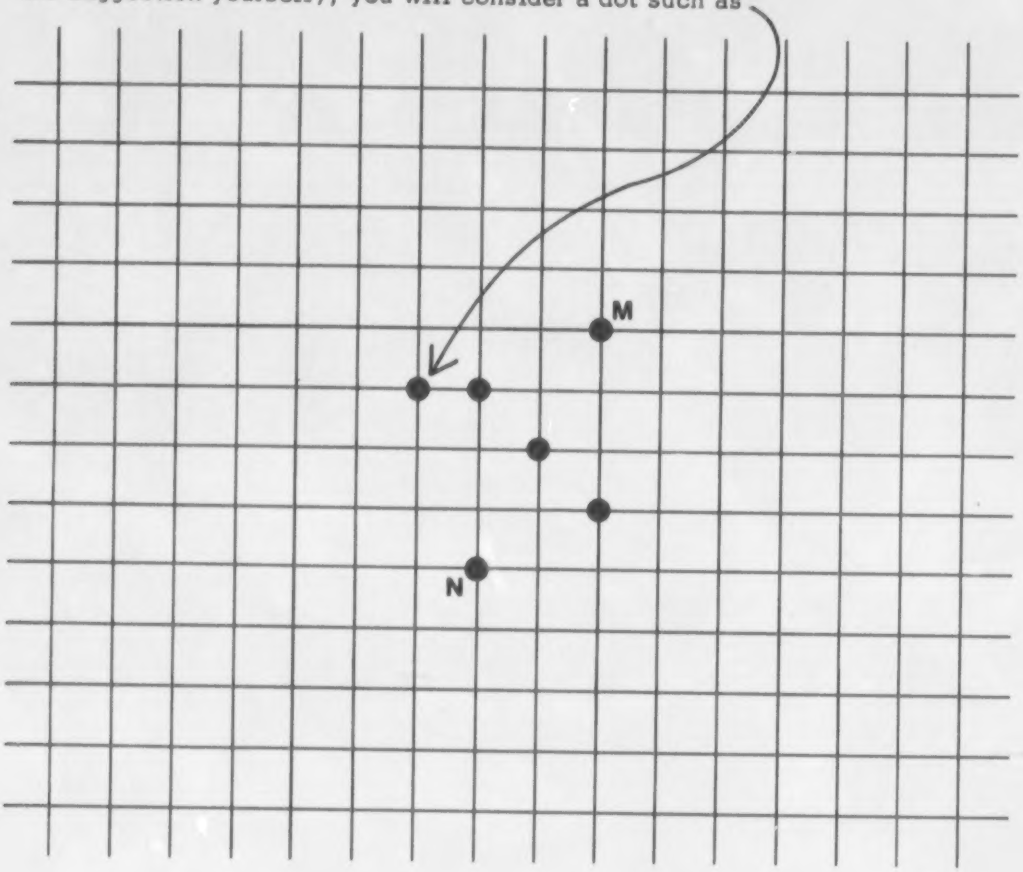
Ask the students to turn over the worksheet and to note that there is a dot (N) for Nora's house and a dot (M) for Maria's house. Erase all but these two dots on your demonstration grid board.

T: Nora and Maria would like to find a place to meet so they both have to walk the same number of blocks. On your papers try to find some places where they could meet. Remember, they should both have to walk the same distance.

Let the students work independently on this problem. If a student finds just one place, suggest that you know there are more places where the girls could meet. After a while, call on students to locate dots on the demonstration grid board and to color them red showing where the girls could meet. Most students will probably find these three red dots.



Try to encourage students to find other places the girls could meet, and ask the class to check each new place. Eventually (or perhaps you will have to make the suggestion yourself), you will consider a dot such as



T: What is a shortest walk Nora can make to this place?

S: 4 blocks.

T: What is a shortest walk Maria can make to this place?

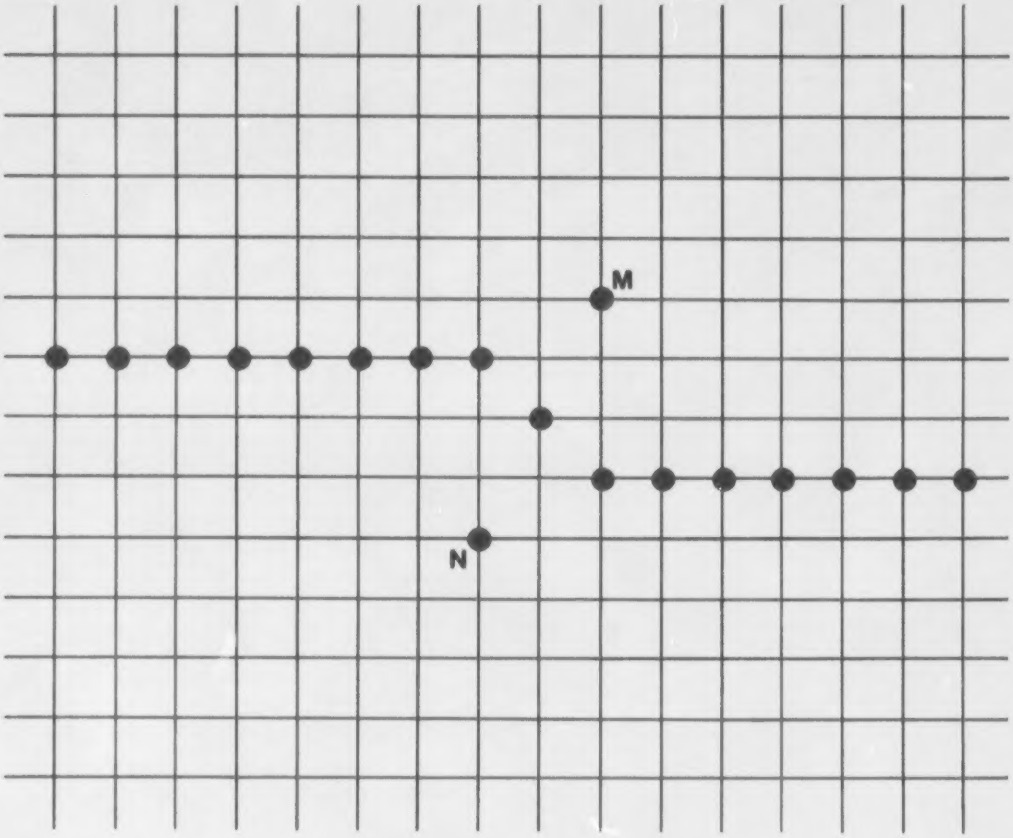
S: 4 blocks.

T: Very good. They both have to walk the same number of blocks. Can you find some more places where they could meet?

G 6

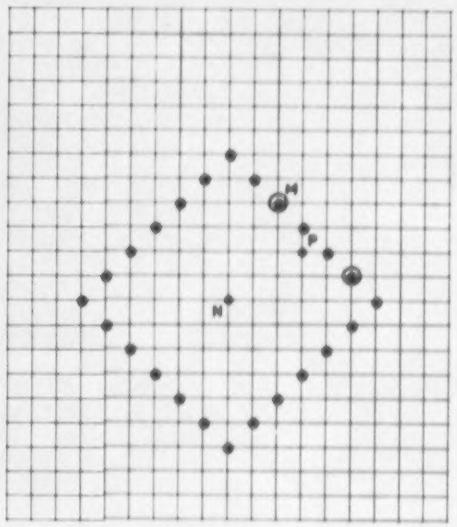
Let the students continue to work independently. Some students may discover a pattern; but you should not expect everyone to find a pattern. Those who do not see any pattern will continue to use trial and error methods of finding new places.

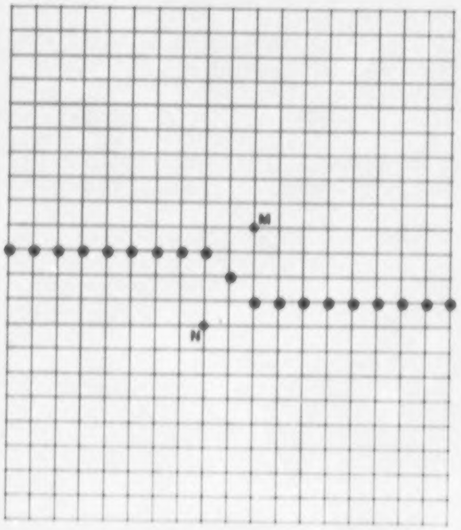
A complete picture is shown below (limited of course by the size of the grid board).



174

Name ____________ **G 6**





G 7

G7 Fractional Parts of Shapes #1

CAPSULE LESSON SUMMARY

Use the number of squares in a shape on a grid to divide it into halves, fourths, eighths, and sixteenths. Find the number of squares in one-half, one-fourth, one-eight, and one-sixteenth of the shape. Do a similar activity with a shape whose symmetry lends itself to dividing it into thirds, sixths, and twelfths.

MATERIALS

Teacher: Colored chalk; Geometry Poster #3 for UPG-II
Student: Colored pencils; Worksheets G7*, **, ***, and ****

NOTE: Before the lesson begins, cut out the shapes from Geometry Poster #3. The shapes will be used in Exercises 2 and 3.

DESCRIPTION OF LESSON

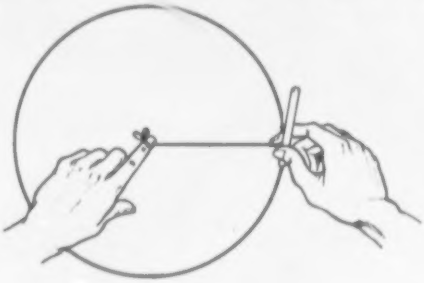
T: Do you remember the twins Jack and Jill who live next door to Booker's Bakery?

The students should recall the Jack and Jill often buy cookies and cakes at Booker's Bakery and share them with their friends.

Exercise 1

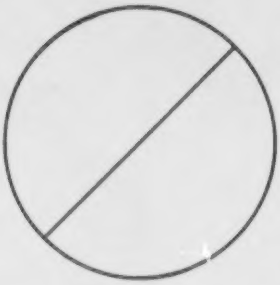
NOTE: This exercise is a review of material covered in some geometry lessons in UPG-I and should move quickly.

Very carefully draw a circle with a radius of about 20 cm on the board. One way to do this is to use a piece of string about 25 cm long with loops tied at both ends. Put a piece of chalk through one loop and secure it tightly. Put an index finger through the other loop and draw the circle as shown below.

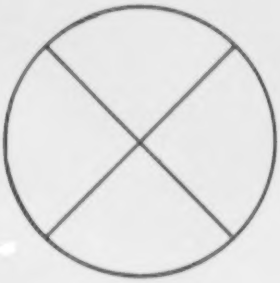


T: Jack and Jill want to share a cake (point to the circle) fairly between the two of them. How could they cut the cake?

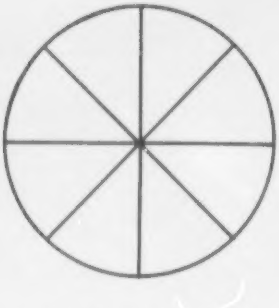
Draw a dividing line segment when a correct suggestion is made.



T: But before they eat any of the cake, two friends come over. How could they cut the cake so that the four of them could share equally the cake?



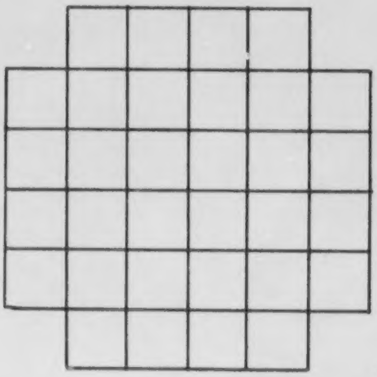
T: No sooner do they cut the cake, when four more friends come over. How could they cut the cake so that the eight of them could share equally the cake?



Erase the board before going on to Exercise 2.

Exercise 2

Tape this shape to the board.



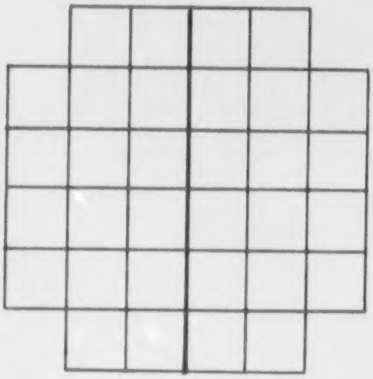
T: One day Jack and Jill buy a strange shaped cake from Booker's Bakery. (Point to the shape on the board.) The cake has vanilla icing with chocolate stripes. How many squares do the chocolate stripes divide the cake into?

Invite a student to count them at the board.

S: Thirty-two.

T: Yes and they are all the same size. Jack and Jill would like to share the cake fairly between them. How could they cut the cake?

Let students suggest several divisions and then indicate one on the shape; for example,



T: How many squares would Jill get? . . . Jack?

S: They would each get sixteen squares.

Record this number story on the board as you say:

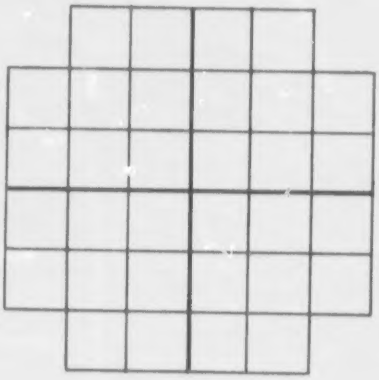
$$\frac{1}{2} \times 32 = 16$$

T: There are thirty-two squares in the whole cake and there are sixteen squares in one-half of the cake.

Continue the story.

> But before Jack and Jill take their shares of the cake, their parents come home and would like to have some cake also. How could they cut the cake so that the four of them can share equally the cake?

Let students make suggestions. Choose a natural division and indicate it on the shape.



T: How many squares would each of the four people get?

S: Eight.

G 7

Record this new number story on the board as you say:

$$\frac{1}{2} \times 32 = 16$$

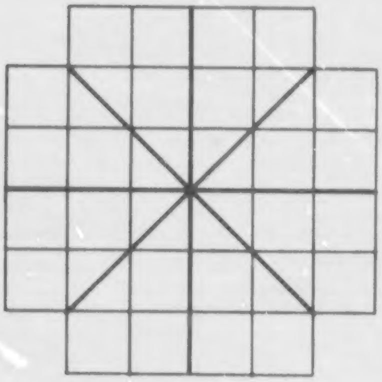
$$\frac{1}{4} \times 32 = 8$$

T: There are thirty-two squares in the whole cake and there are eight squares in one-fourth of the cake.

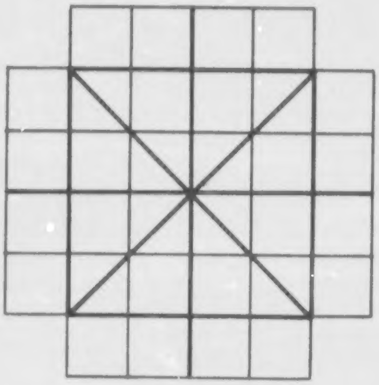
Continue the storyline:

- with four neighbors coming over (eight people altogether); and then,
- with their eight children coming over looking for their parents (sixteen people altogether).

The next illustration shows one way that each of the divisions could be made.



eight pieces



sixteen pieces

You should have this list of number stories on the board.

$$\frac{1}{2} \times 32 = 16$$

$$\frac{1}{4} \times 32 = 8$$

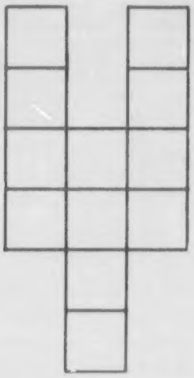
$$\frac{1}{8} \times 32 = 4$$

$$\frac{1}{16} \times 32 = 2$$

Exercise 3

T: Jack and Jill and some of their friends formed a Saturday club. Jack and Jill ordered a cake for the next meeting. It looks like this.

Tape this shape to the board. Remove the shape from Exercise 2 and erase the number stories if you need the board space.



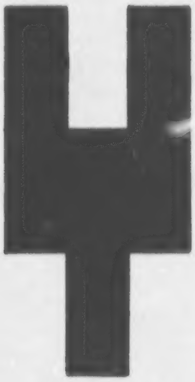
T: This cake has vanilla icing and chocolate stripes like the other one. How many squares do the stripes divide this cake into?

S: Twelve.

T: At first only Jack and Jill are at the meeting. How could they cut the cake to share it fairly between the two of them?

G 7

Allow students to suggest a way to share the cake. Perhaps you could let them color Jack's share one color and Jill's share another color. For example,



T: How many squares would each of the twins get?

S: Six.

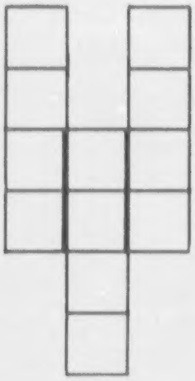
$$\frac{1}{2} \times 12 = 6$$

T: Soon a third member of the club comes to the meeting.

Remove the first copy of the shape and put up a clean copy.

T: How could they cut the cake so that each of the three would get the same amount?

Very likely your students will see the symmetry of the shape and suggest this division.



T: How many squares of cake would each of the three children receive?

S: Four.

Write this number story close to the shape as you say:

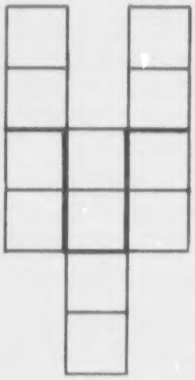
$$\frac{1}{3} \times 12 = 4$$

T: There are twelve squares in the whole cake and there are four squares in one-third of the cake.

Continue the story.

T: The three members decide that they really ought to wait a little longer before eating the cake. After about ten minutes, three more members arrive. How could they share the cake equally among the six of them?

Encourage the idea of cutting each of the three pieces in half.



T: How many squares of cake would each of the six children receive?

S: Two.

G 7

Record a number story for this division on the board.

$$\frac{1}{2} \times 12 = 6$$

$$\frac{1}{3} \times 12 = 4$$

$$\frac{1}{6} \times 12 = 2$$

T: What if all twelve members come to the meeting? How much cake would each of them receive?

S: One square each.

T: $\frac{1}{12} \times 12 = \ldots$?

S: 1.

Record this number story in the list near the shape.

$$\frac{1}{2} \times 12 = 6$$

$$\frac{1}{3} \times 12 = 4$$

$$\frac{1}{6} \times 12 = 2$$

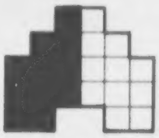
$$\frac{1}{12} \times 12 = 1$$

Worksheets G7*, **, ***, and **** are available for individual work.

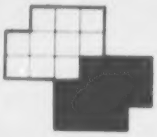
Name ______________ G 7 *

Color one-half of each shape and complete the number story.

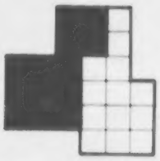
$\frac{1}{2} \times 16 =$ 8



$\frac{1}{2} \times 22 =$ 11



$\frac{1}{2} \times 20 =$ 10

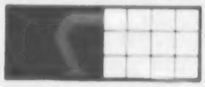


$\frac{1}{2} \times 26 =$ 13

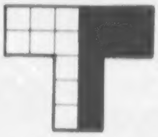
Other colorings are possible.

Name ______________ G 7 **

Color one-half of each shape and complete the number story.

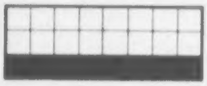


$\frac{1}{2} \times 24 =$ 12

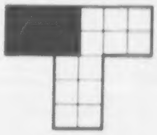


$\frac{1}{2} \times 18 =$ 9

Color one-third of each shape and complete the number story.



$\frac{1}{3} \times 24 =$ 8



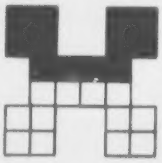
$\frac{1}{3} \times 18 =$ 6

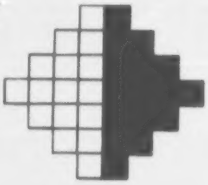
Other colorings are possible.

Name ______________ G 7 ***

Color one-half of each shape red.

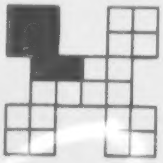
$\frac{1}{2}$

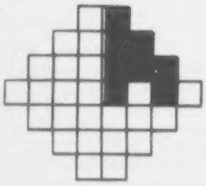




Color one-fourth of each shape blue.

$\frac{1}{4}$

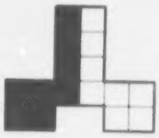




Other colorings are possible.

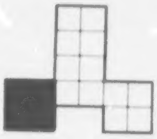
Name ______________ G 7 ****

Color one-half of this shape.



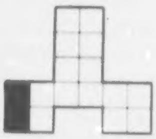
$\frac{1}{2}$

Color one-fourth of this shape.



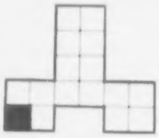
$\frac{1}{4}$

Color one-eighth of this shape.



$\frac{1}{8}$

Color one-sixteenth of this shape.



$\frac{1}{16}$

Other colorings are possible.

G 8

G8 Area Puzzle

CAPSULE LESSON SUMMARY

Compare the areas of various shapes in a puzzle to the areas of two given shapes. Color shapes with the same area the same color and find that the puzzle makes a design.

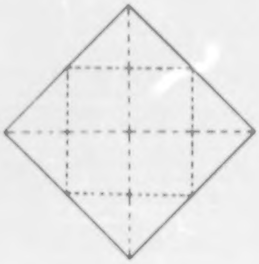
MATERIALS

Teacher: Shapes from Geometry Poster #1 for UPG-I
Student: Worksheet G8; colored pencils

NOTE: The shapes were originally used in Lesson G9 from UPG-I and then again in Lesson G1 Estimation of Area. If necessary, obtain a new copy of Geometry Poster #1 for UPG-I and cut out the shapes.

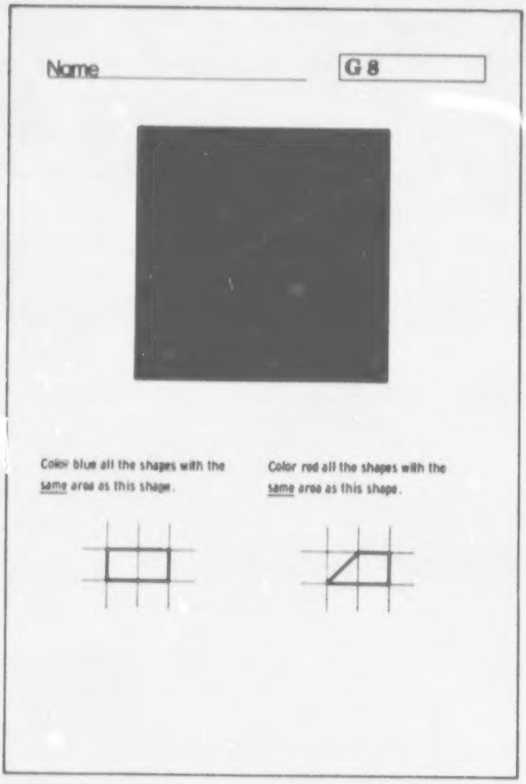
DESCRIPTION OF LESSON

Begin the lesson by reviewing briefly the first part of G1 Estimation of Area. Display the shapes from Geometry Poster #1 for UPG-I and compare their areas to the square.



Discuss which shapes have the same area, which have smaller area, and which have bigger area than this square.

Provide each student with Worksheet G8 and colored pencils. Read the directions to the class and then let the students work independently on this puzzle. You may find that some students need help getting started. Also, you may have to remind students that they are looking for shapes with the same area but not necessarily the same shape.



Name

G 8

Color blue all the shapes with the same area as this shape.

Color red all the shapes with the same area as this shape.

G 9

G9 Length #3

CAPSULE LESSON SUMMARY

Find the lengths of zigzag paths between two points with an obstacle between the points by measuring in centimeters the lengths of the parts and adding them together. Find shorter paths between the points avoiding the obstacles and compare the lengths of the original paths to the lengths of the shorter paths.

MATERIALS

Teacher: Centimeter tape measure; Geometry Poster #4 for UPG-II

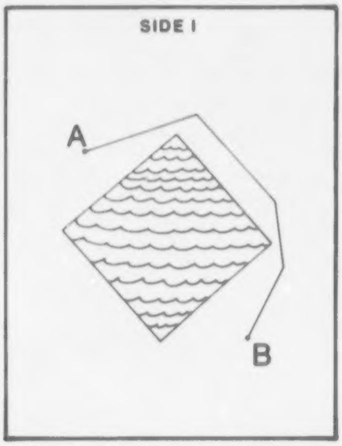
Student: Centimeter tape measure or 30 cm ruler; pencil; Worksheets G9*, **, ***, and ****

NOTE: Cut Geometry Poster #4 in half along the center line.

DESCRIPTION OF LESSON

Exercise 1

Tape side 1 of Geometry Poster #4 to the board.



T: Here (point to "A") is our friendly bug and here (point to "B") is some food. There is a pool of water between the bug and the food, so the bug walks around it along this zigzag path (trace the indicated path from A to B). How long is the path that the bug takes to get to the food? How can we find its length?

S: Measure it.

T: How should we measure it?

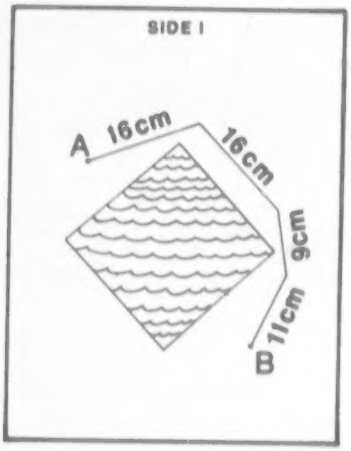
S: With a tape measure.

Display a tape measure and let the class discuss how to do the measuring. Students should observe that they can measure the four pieces of the zigzag separately and then add the four lengths to find the total length of the path.

T: We will measure the path in centimeters (review that the marks on one side of the tape measure are for centimeters), but before we do the measuring, can you guess how long this path is?

Record several guesses on the board. Then, call on three students to assist you in measuring the three pieces of the zigzag. For each piece, emphasize that you put the end mark of the tape measure at one end of the piece (segment) and you read the number of centimeters at the other end.

After measuring each piece, write the length next to the segment.



T: Now, we know the length of the four parts of this path. How can we find the length of the entire path?

G 9

S: Calculate "16 + 16 + 9 + 11".

Write the addition problem on the board and collectively do the calculation.

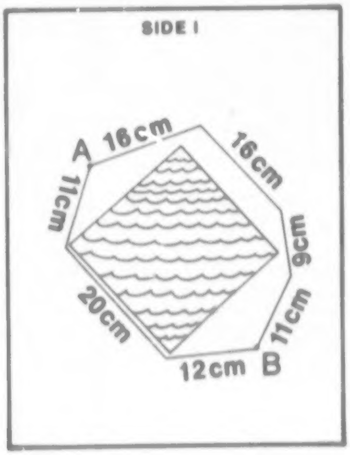
$$\begin{array}{r} 16\text{ cm} \\ 16\text{ cm} \\ 9\text{ cm} \\ +\ 11\text{ cm} \\ \hline 52\text{ cm} \end{array}$$

T: This path is 52 cm long.

Compare the length of the path with the students' guesses and determine which guess is closest.

T: Is there a shorter zigzag path the bug could take to get the food? Remember, the bug cannot swim.

Allow students to make suggestions and to trace paths they think will be shorter. Let the class decide which of the suggested paths they believe is shortest and then draw that zigzag path on the poster. It is not necessary that the class finds a shortest zigzag path from A to B; however, the following illustration shows in red one that is close to a shortest path.



With student assistance, determine the length of this new path by adding the lengths of its parts. For our example, the length is (11 cm + 20 cm + 12 cm) 43 cm. Compare the length of the new path with that of the original.

T: A zigzag path of length 43 cm is certainly shorter than the zigzag path of length 52 cm. How much shorter is it?

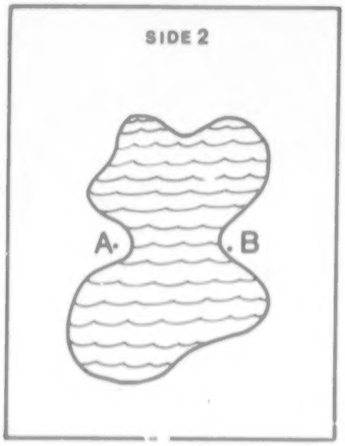
S: 9 cm.

With the class, observe that this last question is answered by solving a subtraction problem. Write the problem on the board and let the class use any method they wish to solve it.

52 − 43 = 9

Exercise 2

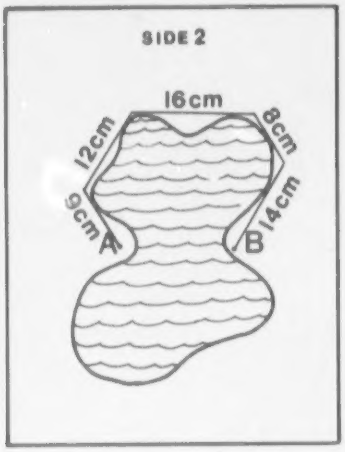
Tape side 2 of Geometry Poster #4 to the board.



T: Here (point to "A") is the bug and here (point to "B") is the food. Let's try to find a zigzag path from A to B for the bug to take to get the food. The bug would like as short a path as possible.

G 9

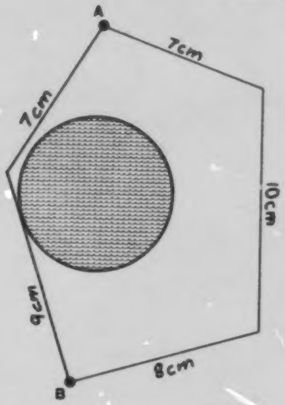
Let students make suggestions and trace paths they think will be as short as possible. The class should decide which of the suggested paths is shortest and then draw that zigzag path on the poster. Again, it is not necessary that the class finds a shortest zigzag path from A to B; however, the following illustration shows one that is close to a shortest path.



Direct the class in determining the length of your zigzag path. You may wish to ask the students to do the necessary addition calculation on their own before doing it collectively. In this example, the zigzag path is (9 cm + 12 cm + 16 cm + 8 cm + 14 cm) 59 cm long.

Worksheets G9*, **, ***, and **** are available for independent work. Provide students with centimeter tape measures or 30 cm rulers. If necessary, suggest that they draw straight lines with the edge of a book or some other straight edge.

Name ________________ **G 9** *



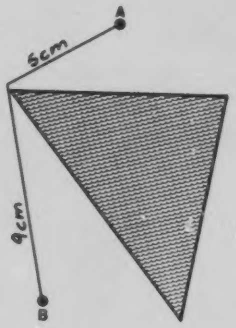
How long is this zigzag path from A to B? 25 cm

Try to find a shorter zigzag path. Draw it.

How long is your zigzag path? 16 cm

Many solutions are possible.

Name ________________ **G 9** **

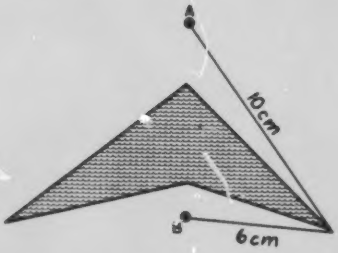


Draw a zigzag path from A to B. Try to make your zigzag path as short as possible.

How long is your zigzag path? 14 cm

Accept other solutions.

Name ________________ **G 9** ***



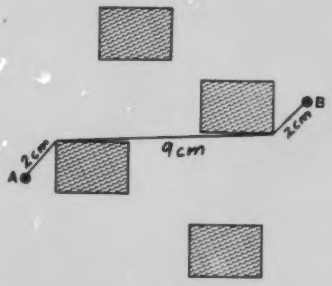
Draw a zigzag path from A to B. Try to make your zigzag path shorter than 18cm.

How long is your zigzag path? 16 cm

Accept other solutions which are shorter than 18 cm.

Name ________________ **G 9** ****

Draw a zigzag path from A to B. Make your zigzag path as short as possible.



How long is your zigzag path? 13 cm

Accept other solutions.

G 10

G10 Area Problems with Tangram Pieces

CAPSULE LESSON SUMMARY

Examine the Tangram pieces and determine their relative sizes. Find the area of each piece first using the little square as the unit, and then using the little triangle as the unit, and finally using the big triangle as the unit. Build shapes with the tangram pieces and find their areas in terms of the various units.

MATERIALS

Teacher: Tangram
Student: Tangram; pencil; Worksheets G10* and **

DESCRIPTION OF LESSON

Exercise 1

Provide each student with a Tangram. Examine the pieces of a Tangram with the class and review that there are seven pieces; two small triangles, one middle-sized triangle, two big triangles, one square, and one (non-rectangular) parallelogram. Let the students decide what name they wish to give to the parallelogram; for example, they may see it as a diamond or as a crooked (or lopsided or flattened) diamond.

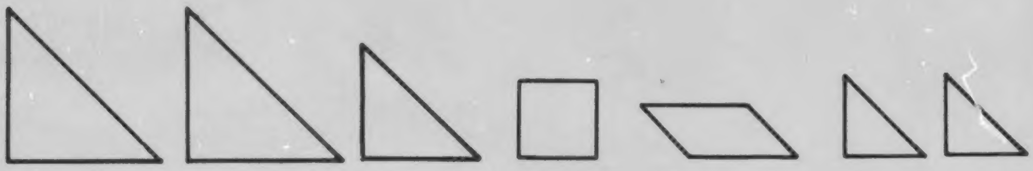
T: Which piece(s) is the largest?

S: The big triangles.

T: Which piece(s) is the smallest?

S: The small triangles.

Ask the students to line up the Tangram pieces on their desks with the largest pieces on the left and then the other pieces in order by size to the smallest pieces on the right.



NOTE: Other arrangements of the three middle pieces are correct since they are all the same size.

T: Which piece did you put after the large triangles?

S: The square.

S: The middle-sized triangle.

S: The crooked diamond.

The discussion here should lead to the observation that these three pieces are all the same size and any ordering of them in the middle is correct.

T: Suppose we choose the square as a unit; that is, the area of the square is 1.

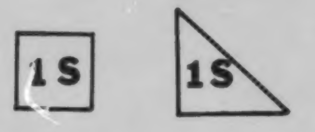
Draw the square on the board and write "1S" inside.

T (holding up the middle-sized triangle): What is the area of this triangle?

S: Also 1S, because it's the same size as the square.

G 10

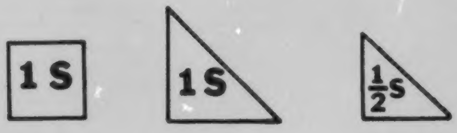
Draw a picture on the board to represent this information.



T (holding up a little triangle): What is the area of this triangle?

S: $\frac{1}{2}$S, because it takes two little triangles to cover the square.

Record the information in a picture on the board.

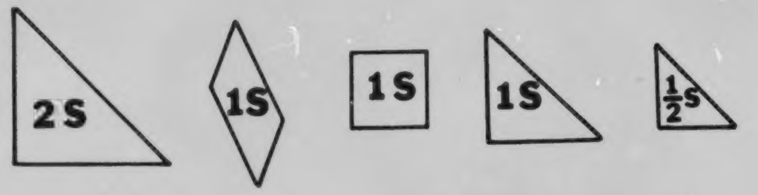


T (holding up a big triangle): What is the area of this triangle?

Give the students time to think about this question as it may cause a little difficulty and require experimenting with the Tangram pieces.

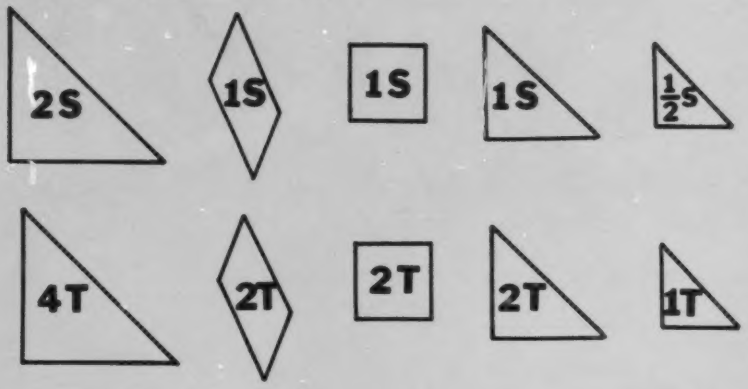
S: 2S, because two middle-sized triangles would cover the big triangle and a middle-sized triangle has area 1S.

Record this information on the board. Then ask about the parallelogram and record its area in a picture on the board.

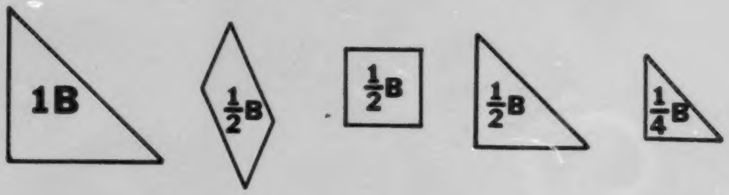


Repeat this line of questioning with the little triangle as the unit. The class should observe that in this case the unit is half the size as before and that the areas, consequently, are twice as many units as before.

A good way to display this fact is to redraw each shape under the previous pictures and label them accordingly.



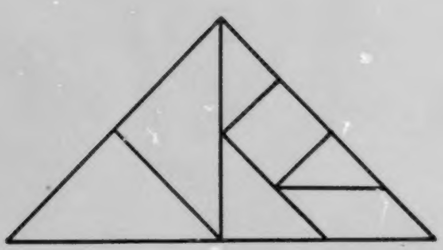
If your class is doing well with this activity, repeat the questioning with the big triangle as the unit.



NOTE: Here "B" is used when the big triangle is the unit.

Exercise 2

Ask the students to use all seven Tangram pieces to build a triangle on their desks. Allow several minutes for this activity. If after a few minutes, some students are having great difficulty, you might get them started by placing two or three key pieces and asking them to go on by building around these pieces.

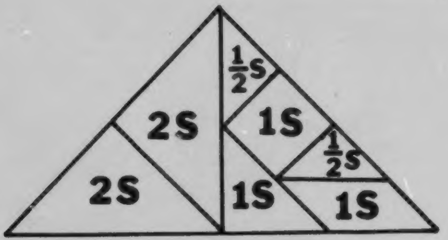


G 10

When most students have completed the task, sketch this drawing on the board so all the students can build the triangle at their desks.

T: If we use the square as unit (1S), what is the area of this triangle?

S: 8S; because each big triangle has area 2S, the middle-size triangle and the crooked diamond each have area 1S like the square, and the two small triangles together have area 1S. I added these areas and got 2 + 2 + 1 + 1 + 1 + 1 = 8.



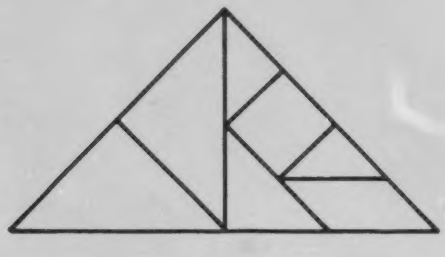
T: If we use the small triangle as unit (1T), what is the area of this triangle?

S: 16T, because it would take twice as many small triangles as squares to cover this triangle.

T: (Optional) If we use the big triangle as unit (1B), what is the area of this triangle?

S: 4B, because it would take half as many big triangles as squares to cover this triangle.

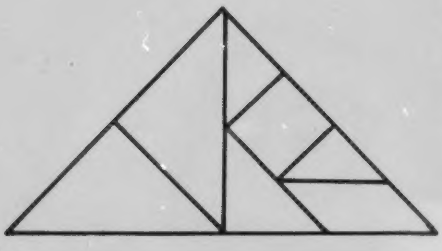
Write this information on the board.



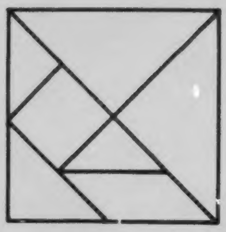
Area
8S
16T
4B

Next ask the students to use all seven Tangram pieces to build a square on their desks. Again, allow several minutes for this activity. In this case, however, some students may observe that one only needs to move the two big triangles in the previous triangle configuration. If students have great difficulty, lead them to consider moving just the big triangles.

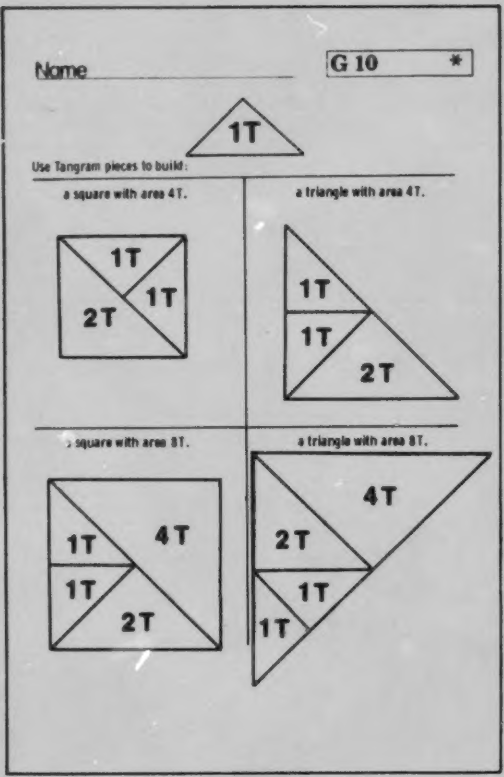
When most students have completed the task, sketch a drawing on the board and ask the questions about area again. The class should quickly observe that the area of this square is the same as the area of the previous triangle.



Area
8S
16T
4B



Distribute Worksheets G10* and **. Solve the first problem collectively to give the students an idea of what they are to do. Include in your instructions that they are to make a drawing of each shape they build.



Name ____________________ **G 10 ***

Use Tangram pieces to build:

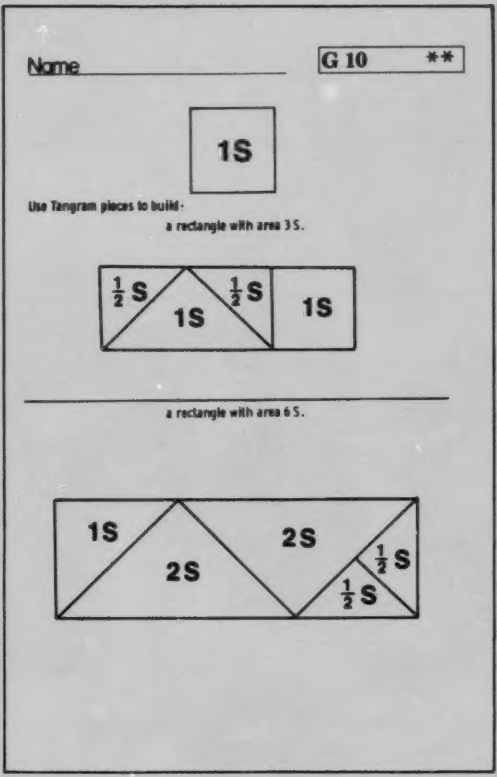
a square with area 4T.

a triangle with area 4T.

a square with area 8T.

a triangle with area 8T.

1T
1T
1T
2T
1T
1T
2T
1T
4T
1T
2T
4T
2T
1T
1T



Name ____________________ **G 10 ****

Use Tangram pieces to build:

a rectangle with area 3S.

a rectangle with area 6S.

1S
$\frac{1}{2}$S
1S
$\frac{1}{2}$S
1S
1S
2S
2S
$\frac{1}{2}$S
$\frac{1}{2}$S

200

G11 Fractional Parts of Shapes #2

CAPSULE LESSON SUMMARY

Divide a shape on a grid into thirds in two different ways and record appropriate number stories. Find the number of squares in one-third, one-sixth, and one-twelfth of another shape.

MATERIALS

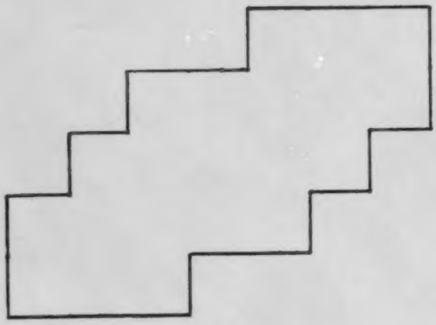
Teacher:	Geometry Poster #5 for UPG-II (two copies); colored markers; tape
Student:	Worksheets G11*, **, ***, and ****; colored pencils

NOTE: Before the lesson begins, cut out the shapes on the two copies of Geometry Poster #5.

DESCRIPTION OF LESSON

Exercise 1

Before the lesson begins, outline shape A on the board.



T: Jack and Jill offer to help Mr. Booker at his bakery one Saturday morning, Saturdays being Mr. Booker's busiest days.

A customer that morning tells Mr. Booker that she wants a cake like one of those on the shelf. (Point to the shape on the board.) The cake is for her son's birthday and since he likes strawberry, lemon, and blueberry icings, she doesn't know which to choose!

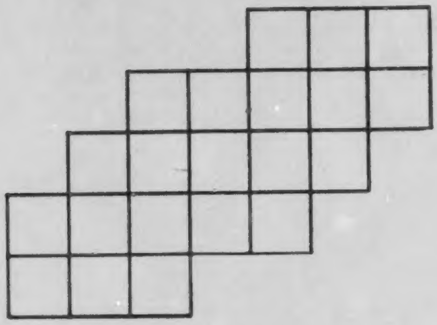
G 11

T: Jack and Jill excitedly ask to let them ice the cake in all three flavors. Mr. Booker and the customer agree to let them.

Let the students discuss ways to ice the cake.

T: Jack decides to section the cake into squares.

Tape a copy of shape A over its outline, already on the board.



T: How many squares are in the whole cake?

Let a student count them at the board.

S: Twenty-one.

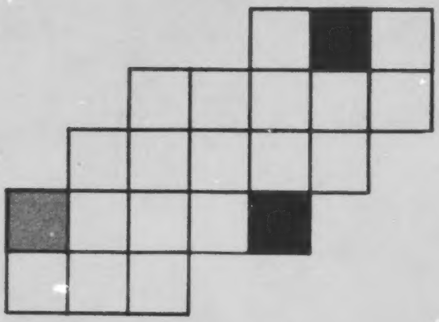
T: Jill thinks that they ought to ice the same amount in each flavor. How could they ice the cake so that there is the same amount for all three flavors?

Perhaps someone will suggest icing seven squares in each flavor. Accept this suggestion and ask the class to check that this would result in the whole cake being iced. Whether or not this observation is made, continue as follows.

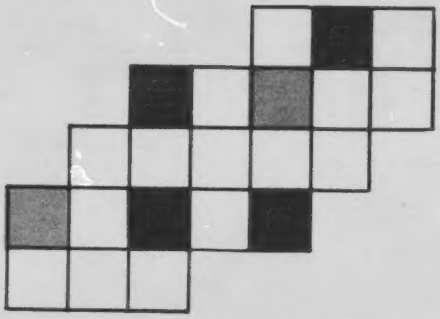
T: I would like three of you to come to the board — one for strawberry, one for blueberry, and one for lemon.

Choose three students and give each of them the appropriate color marker or crayon.

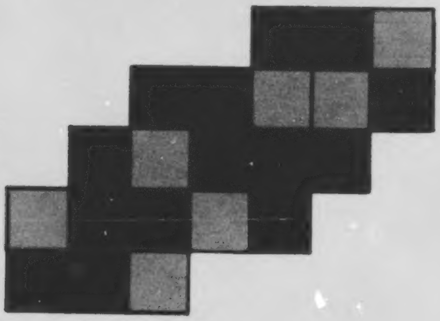
T: The three of you are going to ice the cake. When I say "Go", I want each of you (the three students at the board) to color one square. . . . Go!



T: Color another square.



Continue signaling the students to color another square until every square in the shape has been colored. This is one possible coloring.



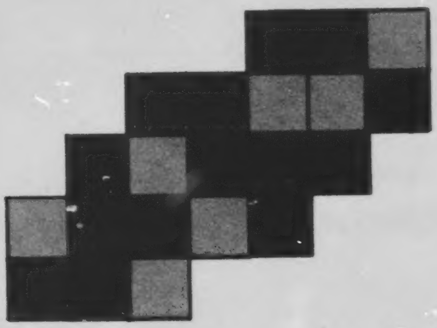
G 11

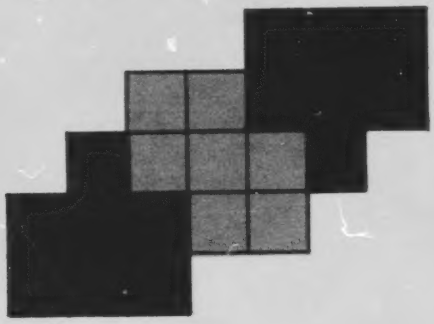
NOTE: The students may simply "scribble" in the squares, ▩ . This keeps the lesson from slowing down because of careful coloring.

T: How many squares have strawberry icing? (Seven)

Point to each red square as the students count them. Do the same for the blue and the yellow squares. Then note that 3 x 7 = 21.

Tape the other copy of shape A to the board and invite three other students to repeat the activity.





Again, count the squares in each color.

T: These two pictures have the squares colored differently. But what is the same about the pictures?

S: The both have red, blue, and yellow squares.

S: Each has seven red, seven blue, and seven yellow squares.

T: How many squares altogether in the cake?

S: Twenty-one.

T: We had three colors. What number is $\frac{1}{3}$ x 21 (read: one-third of twenty-one)?

S: 7.

Record these number stories on the board.

$\frac{1}{3} \times 21 = 7$ $3 \times 7 = 21$

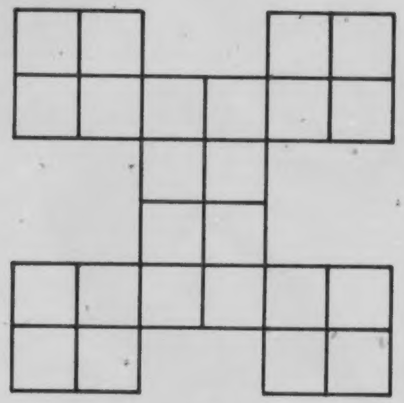
Remove the shapes and erase the board before going on to Exercise 2.

Exercise 2

T: Mr. Booker is pleased with Jack and Jill's work that morning and rewards them with a cake to bring to their Saturday Club meeting.

Shortly before the meeting is to begin, Jack divides the cake into square pieces.

Tape a copy of shape B to the board.



Ask a student to count the squares at the board.

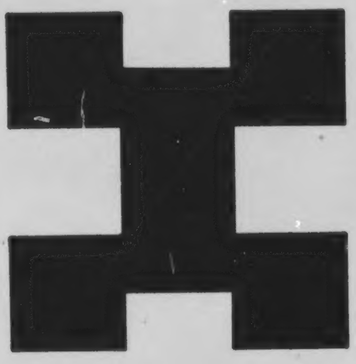
S: Twenty-four.

T: Jill protests, "But we only have twelve members!" Jack replies that it doesn't matter because whoever comes can have more than one piece.

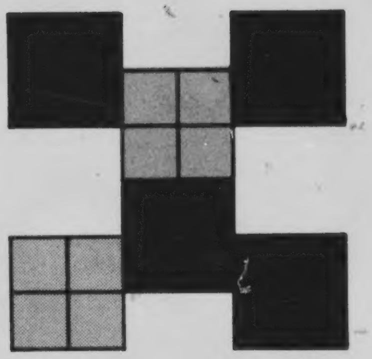
G 11

Ask how many pieces each person receives if the following numbers of members show up for the meeting. Be sure to mention that they share the cake equally among the members present. Invite students to indicate how to section off the cake in each situation. Although colorings are shown below, do colorings only if you feel your students need them. (Two copies of shape B are provided.)

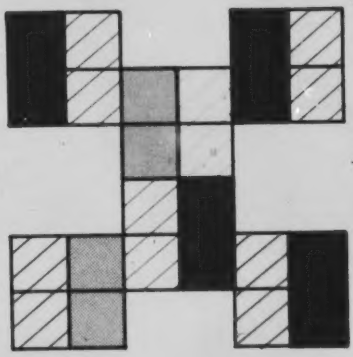
- Three members come to the meeting.



- Six members come to the meeting.



- Twelve members come to the meeting.



Record the corresponding number story for each situation as you consider it. You should have this list on the board.

$$\frac{1}{3} \times 24 = 8$$

$$\frac{1}{6} \times 24 = 4$$

$$\frac{1}{12} \times 24 = 2$$

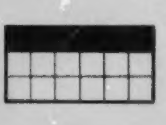
NOTE: If your students are concerned about what to do if five, seven, nine, or eleven members come, suggest each member present get the same number of pieces and decide on a nice way to deal with any extra pieces; for example, give them to the members' younger brothers and sisters.

Worksheets G11*, **, ***, and **** are available for individual work.

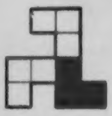
Name

G 11 *

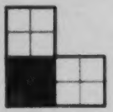
Color one-third of each shape and complete the number story.



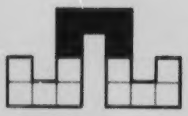
$\frac{1}{3} \times 18 =$ 6



$\frac{1}{3} \times 9 =$ 3



$\frac{1}{3} \times 12 =$ 4



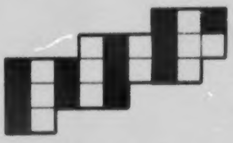
$\frac{1}{3} \times 15 =$ 5

Other colorings are possible.

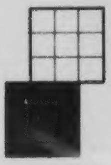
Name

G 11 **

Color one-half of each shape and complete the number story.

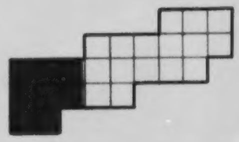


$\frac{1}{2} \times 24 =$ 12

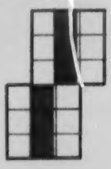


$\frac{1}{2} \times 18 =$ 9

Color one-third of each shape and complete the number story.



$\frac{1}{3} \times 24 =$ 8



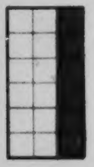
$\frac{1}{3} \times 18 =$ 6

Other colorings are possible.

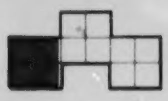
Name

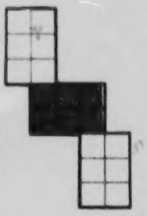
G 11 ***

Color one-third of each shape.

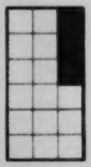


$\frac{1}{3}$

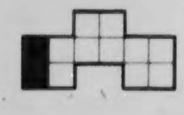


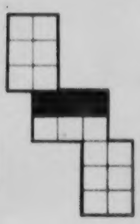


Color one-sixth of each shape.



$\frac{1}{6}$



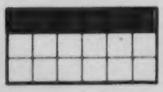


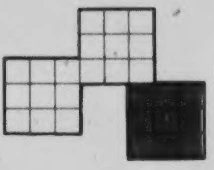
Other colorings are possible.

Name

G 11 ****

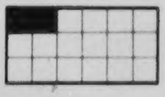
Color one-third of each shape.

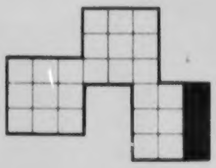




$\frac{1}{3}$

Color one-ninth of each shape.





$\frac{1}{9}$

Other colorings are possible.

G12 Length #4

CAPSULE LESSON SUMMARY

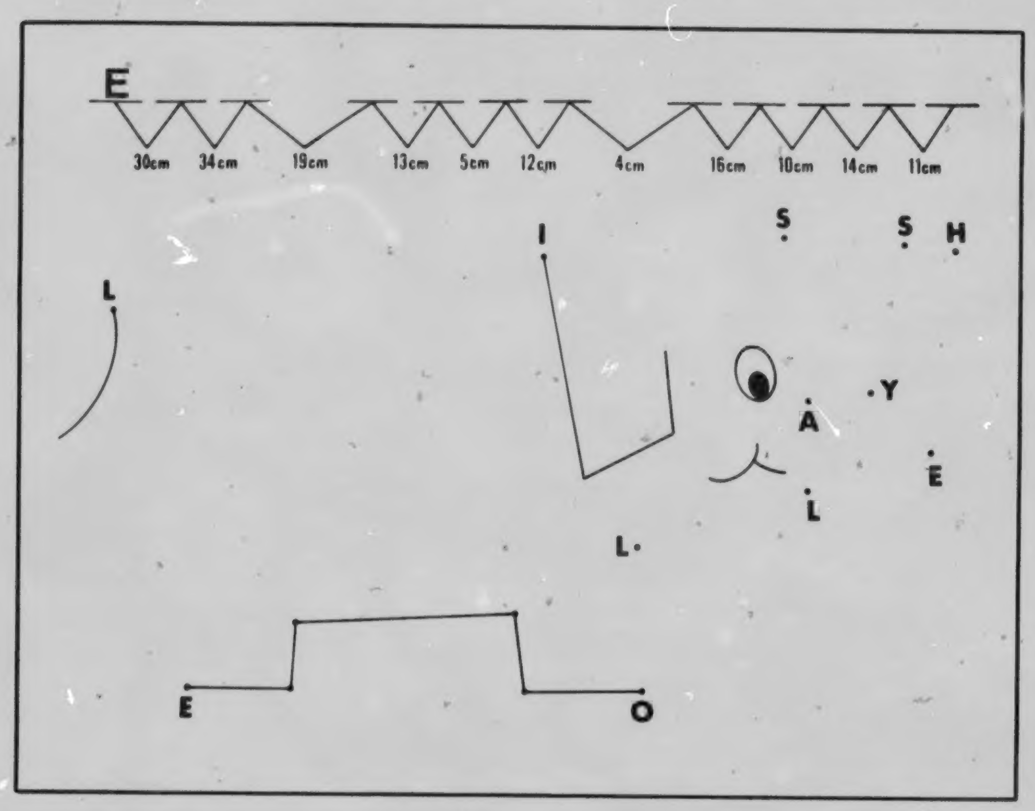
Do a dot-to-dot drawing by making measurements and discover a secret message.

MATERIALS

Teacher:	Centimeter tape measure; straight edge; Geometry Poster #6 for UPG-II; felt-tip pen
Student:	Worksheet G12; pencil; centimeter tape measure or 30 cm ruler

DESCRIPTION OF LESSON

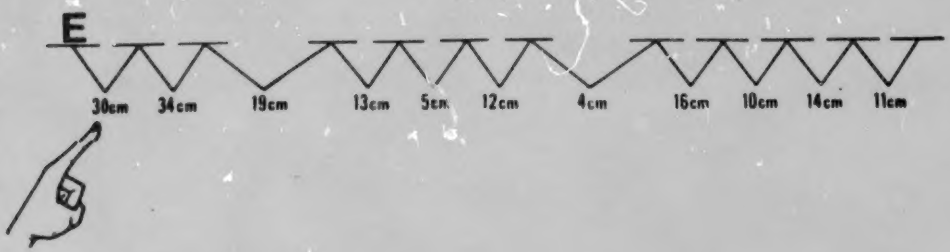
Tape Geometry Poster #6 to the board.


E
30cm
34cm
19cm
13cm
5cm
12cm
4cm
16cm
10cm
14cm
11cm
S
S
H
I
L
Y
A
E
L
L
E
O

G 12

Guide your students to complete the dot-to-dot picture using this method:

- Start the picture at the dot, labeled "E", in the lower left hand corner. Refer to the sequence of measurements given below the blanks for the message to find the distance from E to the second dot. (30 cm)



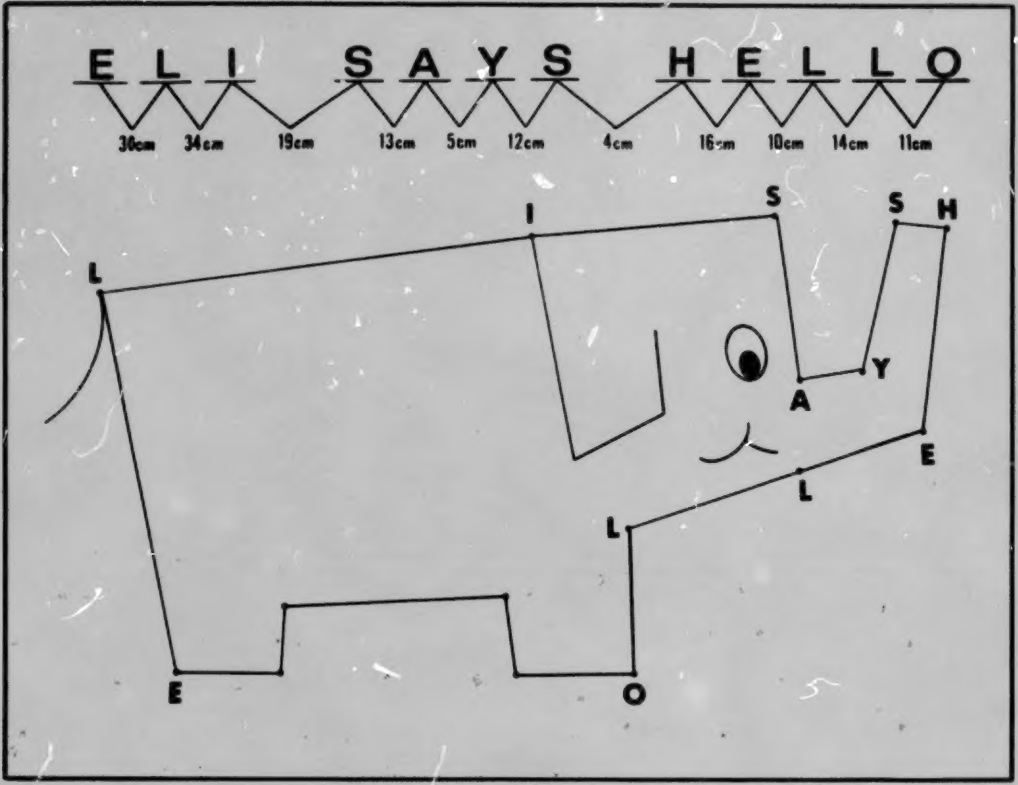
- Measure the distance from the starting dot E to other dots until the dot which is 30 cm from E is found. Connect the two dots with a segment and record the appropriate letter in the second blank. (L)

- Refer to the given sequence of measurements to find the distance from the second dot (labeled "L") to the third dot. (34 cm) Then proceed as before by making measurements. Once the third dot is found, connect it to the second dot and record its corresponding letter in the third blank. (I)

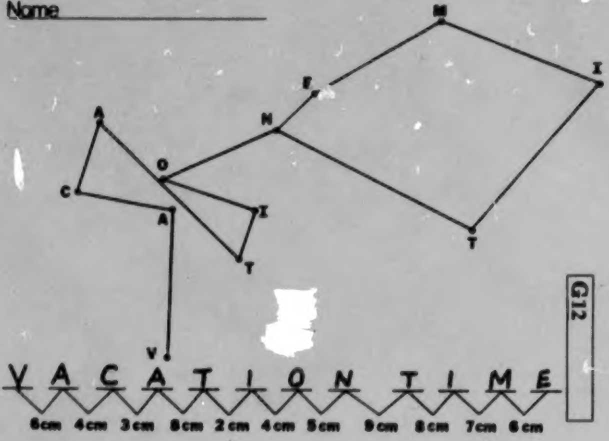
- Continue in this manner until the message is complete.

The completed picture is shown in the next illustration.

Worksheet G12 is available for individual work. The instructions are not on the worksheet, so you should tell the students to proceed in the same way as for the Eli the Elephant picture.


ELI SAYS HELLO
30cm
34cm
19cm
13cm
5cm
12cm
4cm
16cm
10cm
14cm
11cm
I
S
S
H
L
A
Y
E
L
L
E
O

Name ____________________


M
E
I
H
A
O
C
I
A
T
T
V
V A C A T I O N T I M E
6cm 4cm 3cm 8cm 2cm 4cm 5cm 9cm 8cm 7cm 6cm

G12

212

G13 Mazes

CAPSULE LESSON SUMMARY
Find paths through mazes between two given spots in the maze.

MATERIALS
Teacher: None
Student: Worksheets G13* and **; pencil

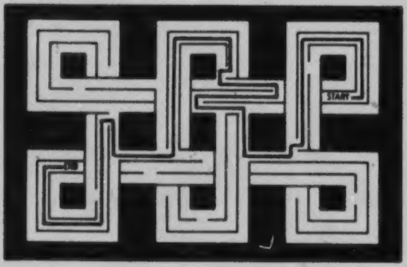
DESCRIPTION OF LESSON

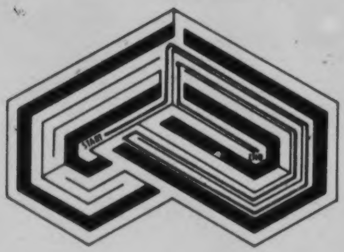
Begin the lesson by leading a class discussion about mazes. Let students comment freely and discuss some of their previous experiences trying to find paths through mazes.

Distribute Worksheets G13* and ** and instruct the students to find the appropriate paths through the mazes. Encourage them to first lightly sketch their paths in pencil before drawing them. This way they will be able to erase if they come to a dead-end. Students who finish both worksheets quickly may be encouraged to draw a maze of their own for you or one of their classmates to solve.

Name ______________________ **G 13 ***

In each maze, find a path from start to end.

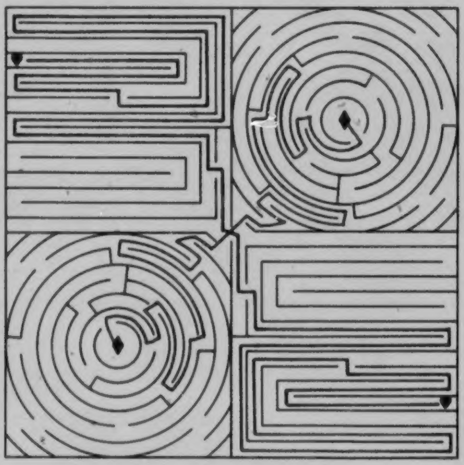




Other solutions are possible.

Name ______________________ **G 13 ****

In this maze, find a path connecting the hearts ♥ and another path connecting the diamonds ♦.



Comprehensive School Mathematics Program

CSMP Mathematics for the Upper Primary Grades

Part II

Workbooks

CEMREL, Inc.

TABLE OF CONTENTS

INTRODUCTION v

WORKBOOK PROGRESS CHART viii

W-LESSONS

W1 A Caravan of Problems #1 (Lesson 1) 1
W2 A Caravan of Problems #1 (Lesson 2) 4
W3 Detective Story #1 17
W4 Length and Area (Lesson 1) 21
Worksheet W4 25
W5 Length and Area (Lesson 2) 26
Worksheet W5 30
W6 Detective Story #2 35
Worksheets W6*, **, ***, and **** 42
W7 A Caravan of Problems #2 (Lesson 1) 43
W8 A Caravan of Problems #2 (Lesson 2) 46
W9 Detective Story #3 57
W10 Nora's Neighborhood 62
W11 Detective Story #4 71
W12 Fishing for Numbers (Lesson 1) 79
W13 Fishing for Numbers (Lesson 2) 82
W14 Detective Story #5 95
W15 Summer School in the Old Days 107
W16 Summer School: 0's Discovery 110
W17 Detective Story #6 114

WORKBOOKS

INTRODUCTION

There are many opportunities for the student to work individually during the course of the lessons described in the other content strands. In the Workbook strand, however, it is this individualized work which becomes the principal mode of the majority of lessons. It has been our goal in this strand to provide the students with opportunities:

- to review many of the ideas they have met in other content strands;
- to apply their acquired knowledge to new situations requiring various kinds of strategic thinking;
- to learn how to read and use mathematics workbooks.

The following workbooks are provided.

A Caravan of Problems #1
Length and Area
A Caravan of Problems #2
Nora's Neighborhood
Fishing for Numbers

These five workbooks are written with problems of varying levels of difficulty all in one booklet. Approximately the first ten pages of each booklet are easy problems, the next ten to twelve pages are average level difficulty, and the last ten pages are more challenging problems. For each workbook, we suggest that all students start work at the easiest level of difficulty (i.e., on page 1) and then work through as many pages as they can handle during the one or two lessons scheduled for that workbook. We estimate that, in an average class, about two-thirds of the students will finish the first ten pages, about one-third will finish the first twenty pages, and a few will finish all or most of the booklet. These proportions will vary from class to class.

This guide contains an answer key for each workbook. The key follows an introduction to the workbook and suggested collective lessons which present the workbook to the whole class or provide a warmup activity on a problem similar to one found in the workbook.

Nine of the seventeen lessons in the strand have the individual work in the workbooks as their main activity. The Storybook SUMMER SCHOOL IN THE OLD DAYS and its accompanying Story-Workbook SUMMER SCHOOL: 0's DISCOVERY provide the topics for two lessons. Representation of numbers is the main focal point of these richly illustrated books. The other six lessons present detective stories where there is a secret number and several clues which lead to the discovery of the number. The clues provide review of many ideas from the other content strands as well as involve students in new situations involving patterns and counting techniques.

Use of the Workbook Strand for Evaluation Purposes[*]

The workbooks provide an excellent instrument to assess the progress of your students on a weekly basis. You may not feel that it is necessary to check every page and problem for each student. You should develop the sort of procedures for checking students' work with which you are most comfortable. These may include checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or just looking carefully at the first few pages to be sure the students have understood the general idea of the problems in that particular book.

A progress chart is provided on pages viii and ix to help you keep track of

1) where each of your students is on any particular workbook day,
2) how much each student has completed under each title,
3) your personal degree of satisfaction with the student's work on each title,

---

[*] See also Role and Use of Workbooks in Section 1 of the General Introduction.

4) which students are showing improvement from one workbook to the next,

and

5) which students are having difficulty with even the easiest problems and would benefit from more individual attention during the lessons.

Here is a list of some of the important points to bear in mind for workbooks.

- All students start at the beginning of each workbook and progress as far as they can.
- All students begin a workbook on the same day, even if some students have not finished work on the preceding workbook.
- Not all students are expected to complete a given workbook. Only the faster students will reach the most challenging problems. The slower students may succeed only to do the easiest problems, although you should not assume that this will automatically be the case--surprises are not at all uncommon in this area.
- You should read the introductory material for each workbook and give the short collective lesson(s) described.

Workbook Progress Chart

Student's Name	A Caravan of Problems #1	Length and Area

Workbook Progress Chart

A Caravan of Problems #2	Nora's Neighborhood	Fishing for Numbers

W1 A Caravan of Problems #1 (Lesson 1)

CAPSULE LESSON SUMMARY

Label the dots in an arrow picture with + 3 and - 2 arrows. Draw all the possible + 6 arrows in the same picture. Review and extend experiences in various content strands in the Workbook A CARAVAN OF PROBLEMS #1.

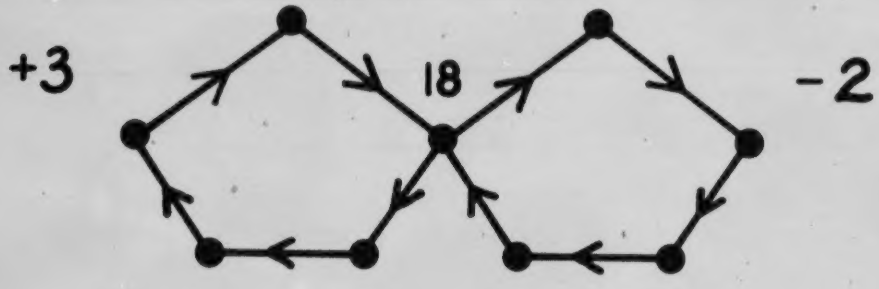
MATERIALS

Teacher: Colored chalk

Student: Workbook A CARAVAN OF PROBLEMS #1; colored pencils

DESCRIPTION OF LESSON

Draw this arrow picture on the board.



T: What are the blue arrows for in this picture?

S: + 3.

T: ... and the red arrows?

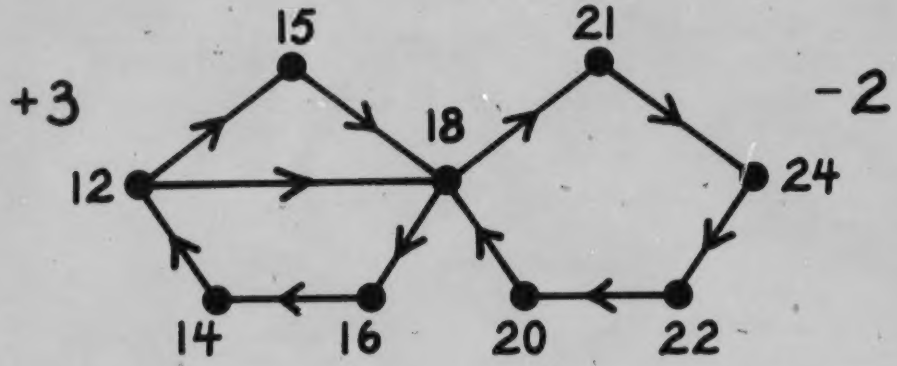
S: - 2.

T (pointing to "18"): Here is the number 18. Which other numbers are in this picture?

Call on students, one at a time, to label the other dots. Each time a dot is labeled, check the appropriate calculation.

W 1

After all the dots are labeled, draw this green arrow.



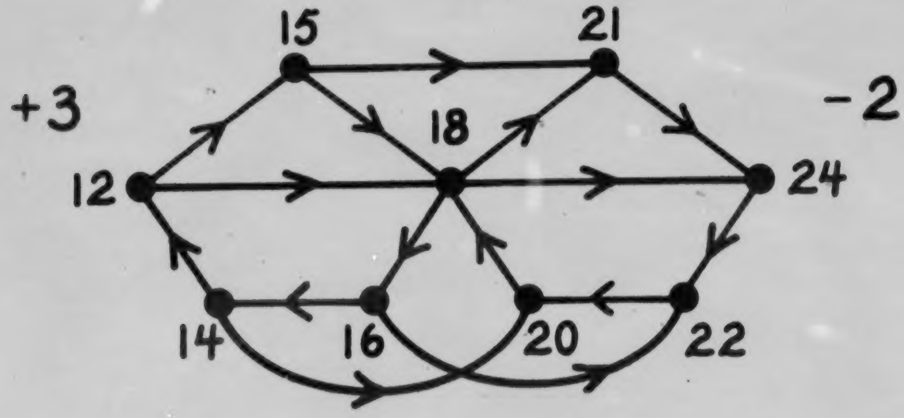
T: What could this green arrow be for?

S: +6, because 12 + 6 = 18.

Encourage the class also to observe that +3 followed by +3 is +6 and write "+6" in green near the picture.

T: There are more places where we can draw green +6 arrows in our picture. Try to find them.

Invite students first to trace +6 arrows and then, if correct, to draw them in green. Each time check the calculations. After a few minutes and perhaps with a little prompting, your students should find four more green arrows.



W 1

Distribute copies of the Workbook A CARAVAN OF PROBLEMS #1 and let students work independently for the rest of the math period. If many students have difficulty with a particular page, you may wish to have a collective discussion about that page.

At the end of the lesson, collect the workbooks for your review. They will be used again in Lesson W2.

W 2

W2 A Caravan of Problems #1 (Lesson 2)

CAPSULE LESSON SUMMARY

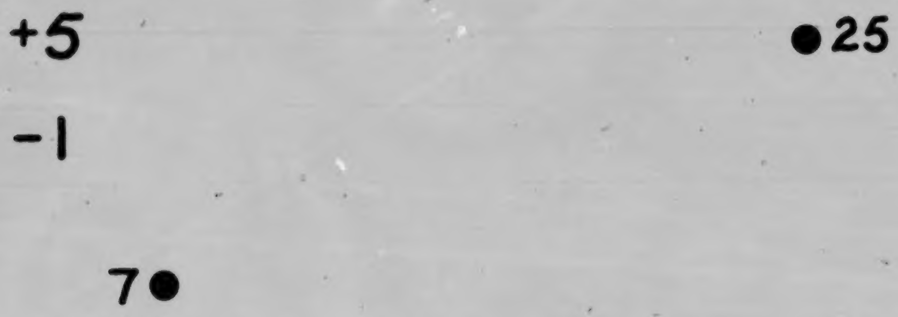
Build an arrow road from 7 to 25 using + 5 and - 1 arrows. Examine a road from 25 to 7 using the return arrows. Continue work in the Workbook A CARAVAN OF PROBLEMS #1. (This is the second of two lessons using this workbook.)

MATERIALS

Teacher:	Colored chalk
Student:	Workbook A CARAVAN OF PROBLEMS #1; colored pencils

DESCRIPTION OF LESSON

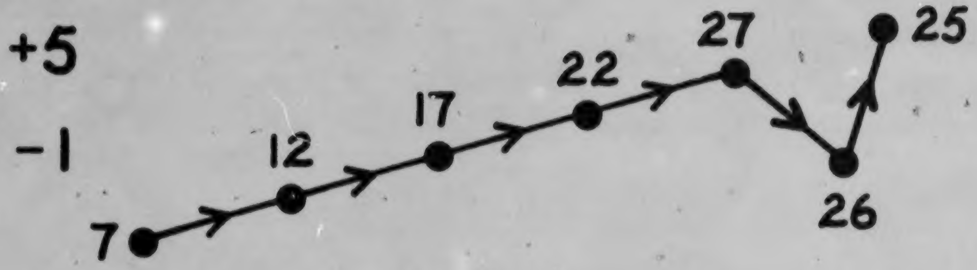
Put this information on the board.



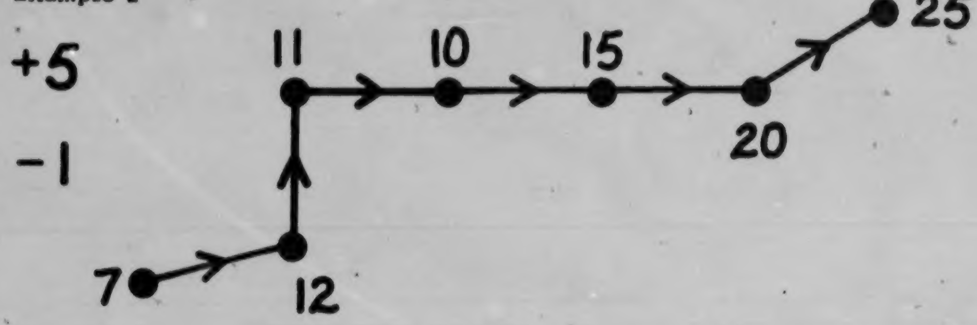
T: Let's build a road from 7 to 25 using blue + 5 arrows and red - 1 arrows. How should we start?

Let the students make suggestions and follow any suggestions they give. There are no "wrong" answers in this situation as long as + 5 or - 1 arrows are suggested. Since we cannot predict the road that your class will build, we provide several examples. The first two of these examples are the shortest possible roads. The third example affords the opportunity of discussing how one could eliminate one blue and five red arrows from 7 to 25. In the first two examples, one could note that at least four blue and two red arrows are needed. Let the class determine the direction of the discussion.

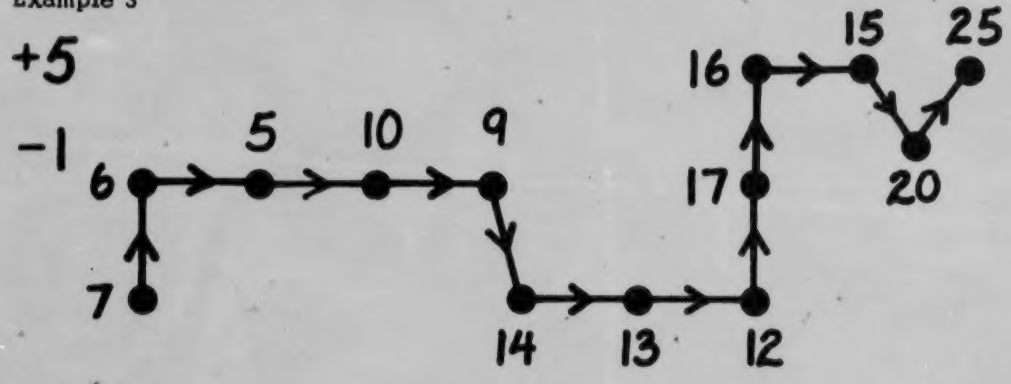
Example 1


+5
-1
7
12
17
22
27
26
25

Example 2


+5
-1
7
12
11
10
15
20
25

Example 3


+5
-1
7
6
5
10
9
14
13
12
17
16
15
20
25

After your students build a road from 7 to 25 using + 5 and - 1 arrows, direct them to consider a return road from 25 to 7.

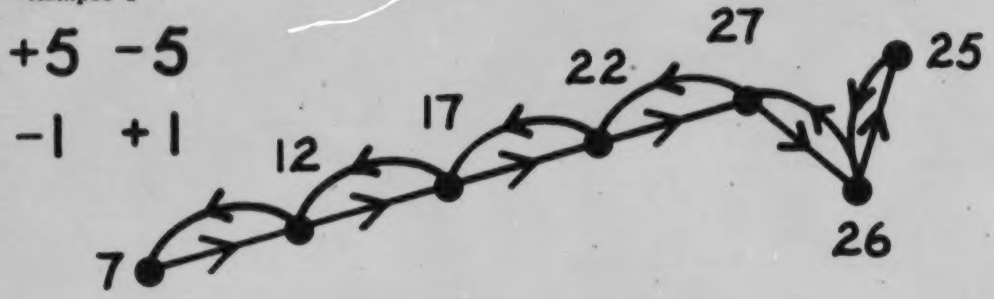
T: Now, let's try to find a road returning from 25 to 7 and landing on the same numbers as we did going from 7 to 25. How could we do this?

W 2

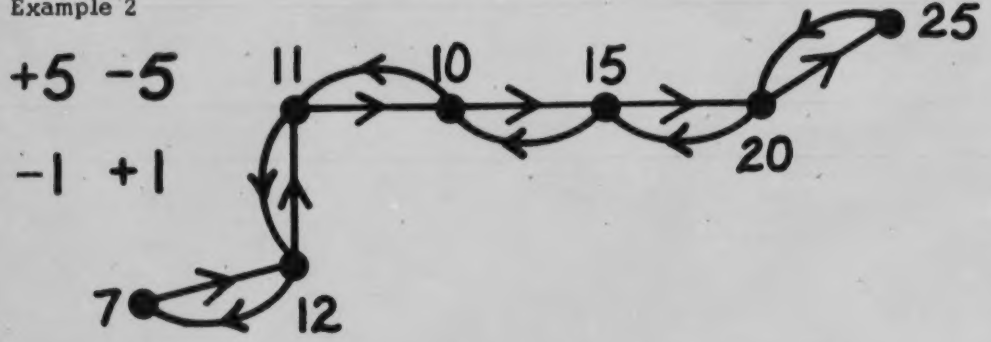
S: Use - 5 arrows and + 1 arrows.

Lead the class to observe that the return arrow of a + 5 arrow is a - 5 arrow and the return arrow of a - 1 arrow is a + 1 arrow. Then draw the appropriate return road from 25 to 7. For our three examples, the return roads are shown below.

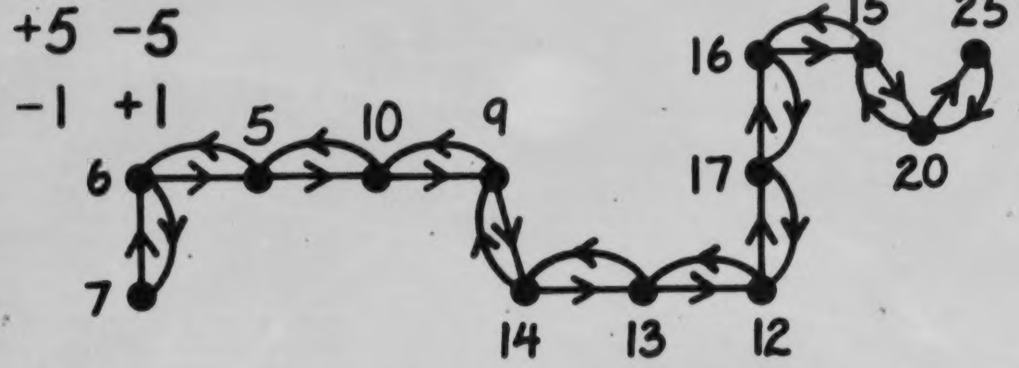
Example 1


+5 -5
-1 +1
7
12
17
22
27
25
26

Example 2


+5 -5
-1 +1
11
10
15
25
20
7
12

Example 3


+5 -5
-1 +1
5
10
9
6
7
16
15
25
20
17
14
13
12

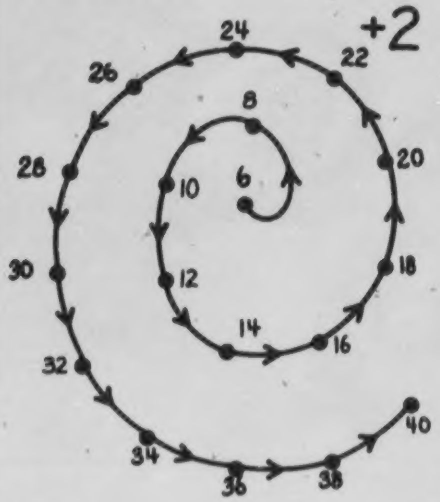
Distribute the students' copies of the Workbook A CARAVAN OF PROBLEMS #1. Ask students to first correct any mistakes from the first week's work on this workbook and then to continue. You may wish to discuss collectively any problem with which many students had difficulty the first week. At the end of the lesson, collect the workbooks for your review.

A CARAVAN OF PROBLEMS

#1

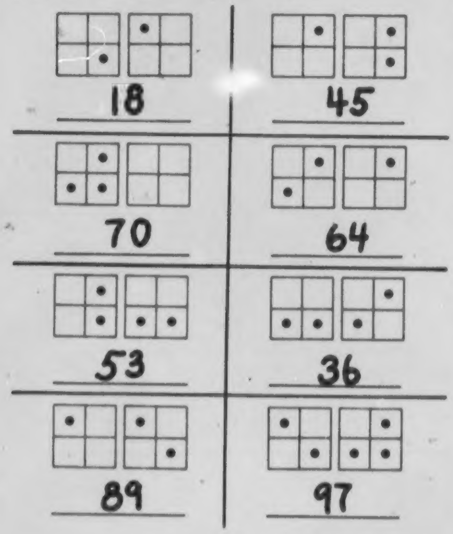
A Caravan of Problems #1

Label the dots.


+2
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40

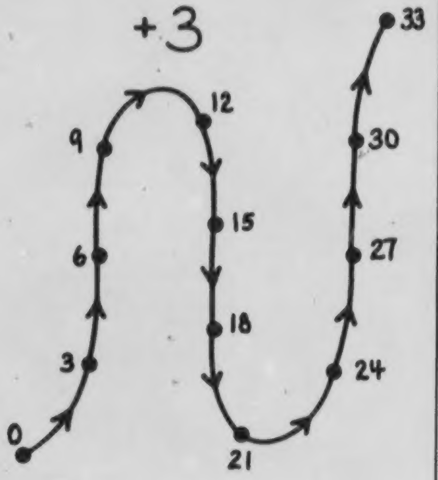
2

What number is on the Minicomputer?


18
45
70
64
53
36
89
97

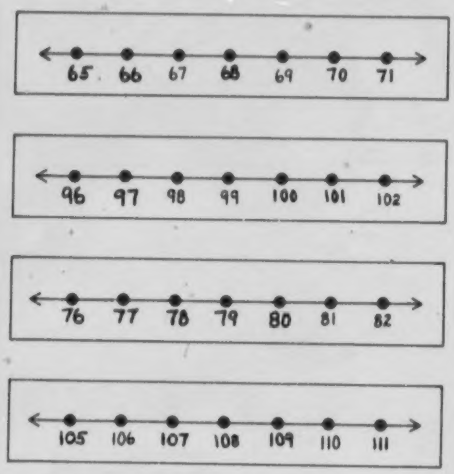
3

Label the dots.


+3
0
3
6
9
12
15
18
21
24
27
30
33

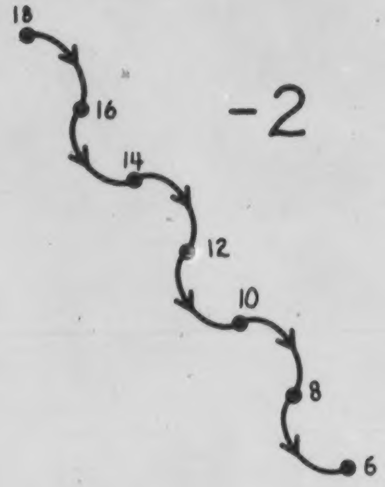
4

Label the dots on these number lines.


65
66
67
68
69
70
71
96
97
98
99
100
101
102
76
77
78
79
80
81
82
105
106
107
108
109
110
111

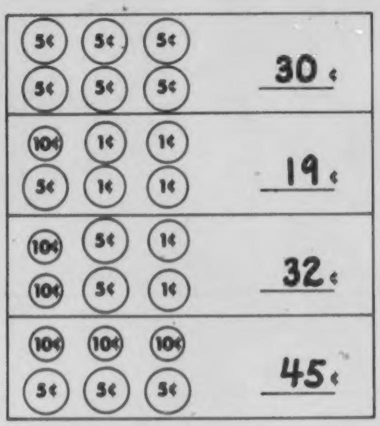
5

Label the dots.



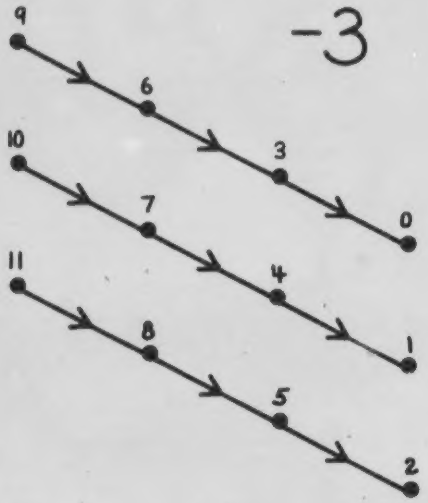
18
16
14
12
10
8
6
-2

How much money?



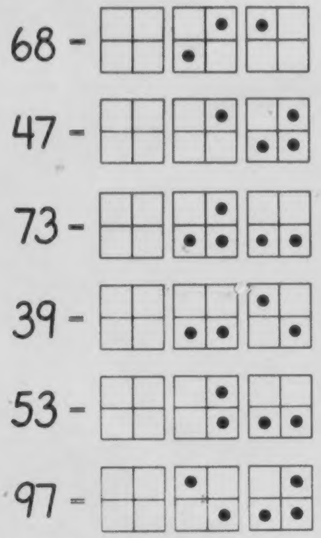
5¢ 5¢ 5¢
5¢ 5¢ 5¢
30¢
10¢ 1¢ 1¢
5¢ 1¢ 1¢
19¢
10¢ 5¢ 1¢
10¢ 5¢ 1¢
32¢
10¢ 10¢ 10¢
5¢ 5¢ 5¢
45¢

Label the dots.



9
6
3
0
10
7
4
1
11
8
5
2
-3

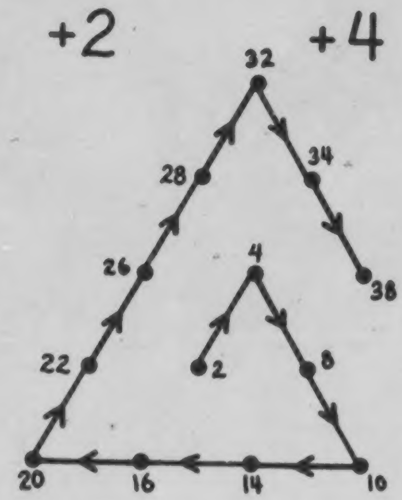
Put these numbers on the Minicomputer.



68 -
47 -
73 -
39 -
53 -
97 =

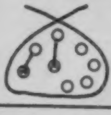
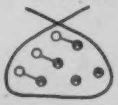
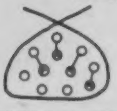
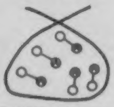
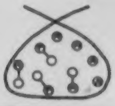
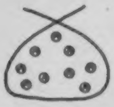
Standard configuration is given here for each number; however, many other solutions are possible.

Label the dots.


+2
+4
32
34
28
26
4
38
22
2
8
20
16
14
10

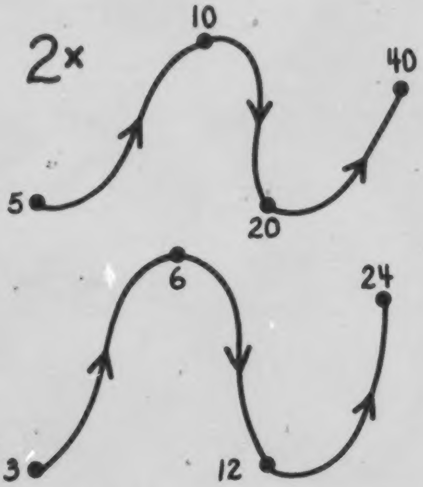
10

Finish these number stories.

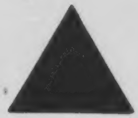
$6+\hat{2}=$ 4 	$3+\hat{5}=$ $\hat{2}$ 
$\hat{4}+7=$ 3 	$\hat{5}+5=$ 0 
$4+\hat{8}=$ $\hat{4}$ 	$\hat{9}+0=$ $\hat{9}$ 

11

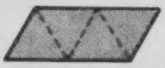
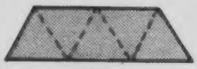
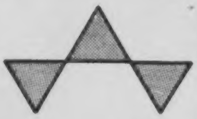
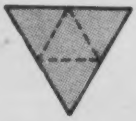
Label the dots.


2×
10
40
5
20
6
24
3
12

12

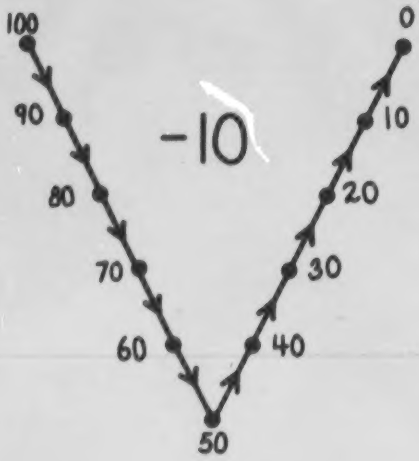


Compare the areas of shapes below with the area of this red triangle.
Circle your answer.

This one is done for you.  Bigger [✓ Same] Smaller	 [✓ Bigger] Same Smaller
 Bigger Same [✓ Smaller]	 Bigger [✓ Same] Smaller

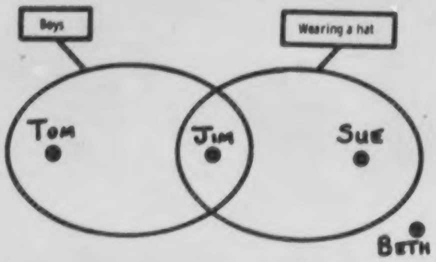
13

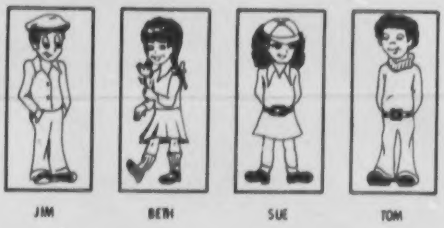
Label the dots.


-10
100
90
80
70
60
50
40
30
20
10
0

14

Label the dots with the names of the children pictured below.


Boys
Wearing a hat
Tom
Jim
Sue
Beth


JIM
BETH
SUE
TOM

15

CODE

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26

Decode:

W	H	A	T
20 + 3	2 x 4	7 - 6	2 x 10

M	O	N	T	H
14 - 1	3 x 5	2 x 7	4 x 5	10 - 2

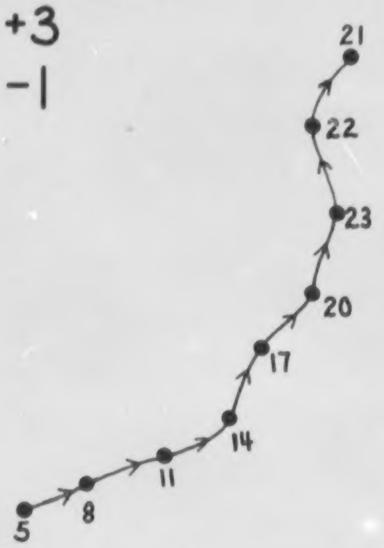
I	S	I	T	?
3 x 3	20 - 1	10 - 1	21 - 1	

Answer: February*

16

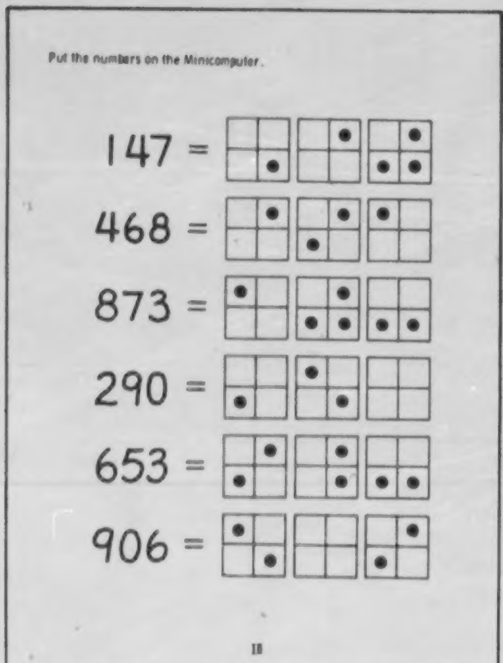
*The answer depends on the month in which the workbook is used.

Build a red-blue road from 5 to 21.


+3
-1
21
22
23
20
17
14
11
8
5

17

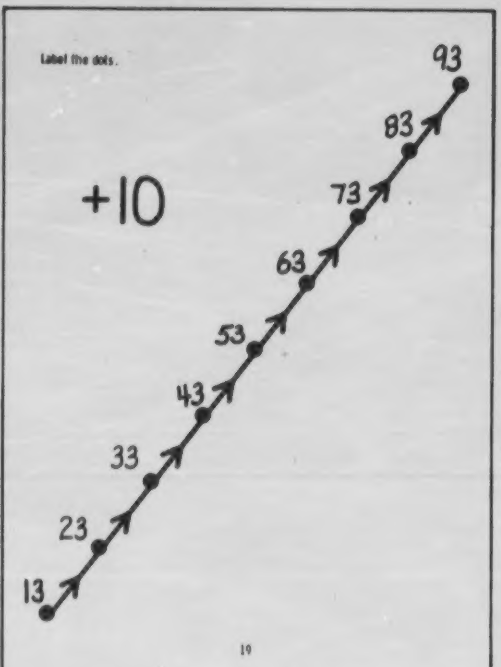
Many solutions are possible.



Put the numbers on the Minicomputer.

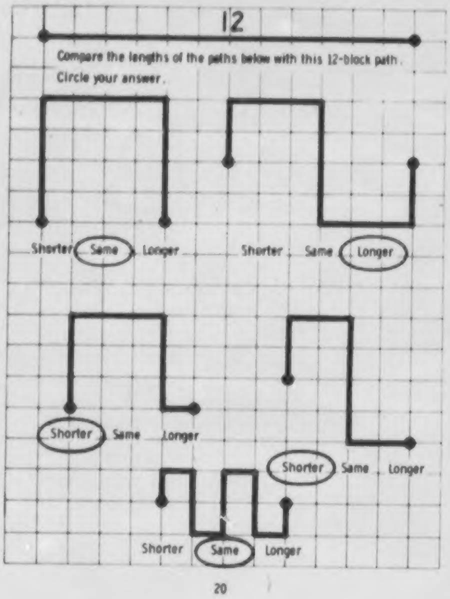
147 =
468 =
873 =
290 =
653 =
906 =
18

Standard configuration is given here for each number; however, many other solutions are possible.



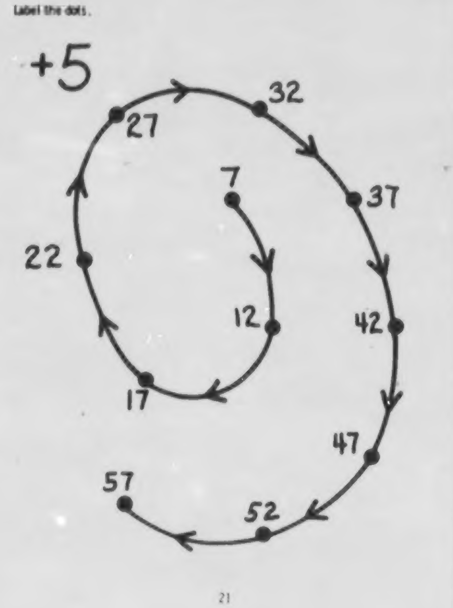
Label the dots.

+10
13
23
33
43
53
63
73
83
93
19



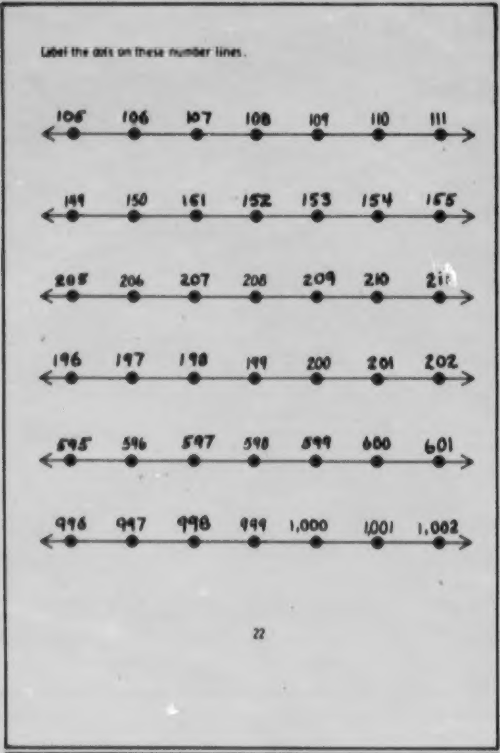
Compare the lengths of the paths below with this 12-block path. Circle your answer.

12
Shorter [✓ Same] Longer
Shorter Same [✓ Longer]
[✓ Shorter] Same Longer
[✓ Shorter] Same Longer
Shorter [✓ Same] Longer
20



Label the dots.

+5
7
12
17
22
27
32
37
42
47
52
57
21



Label the dots on these number lines.

105 106 107 108 109 110 111

149 150 151 152 153 154 155

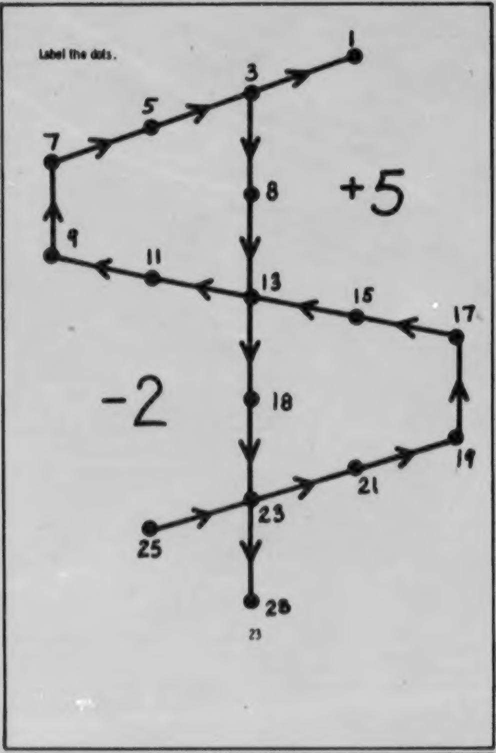
205 206 207 208 209 210 211

196 197 198 199 200 201 202

595 596 597 598 599 600 601

996 997 998 999 1,000 1,001 1,002

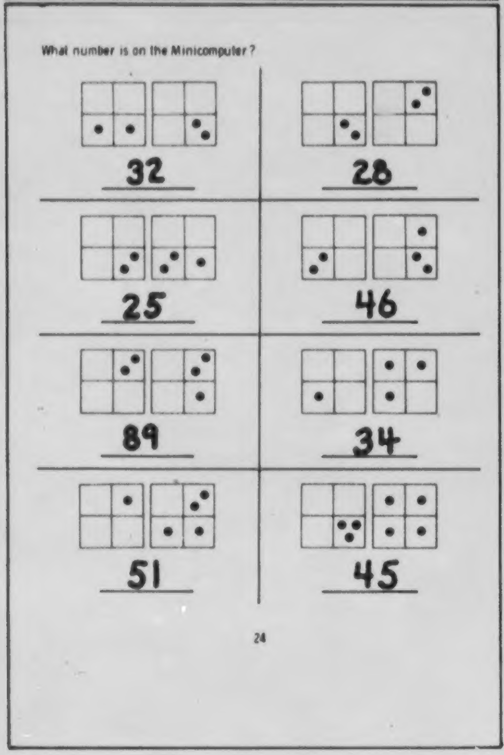
22



Label the dots.

1
3
5
7
9
11
13
15
17
19
21
23
25
8
18
28
+5
-2

23



What number is on the Minicomputer?

32

28

25

46

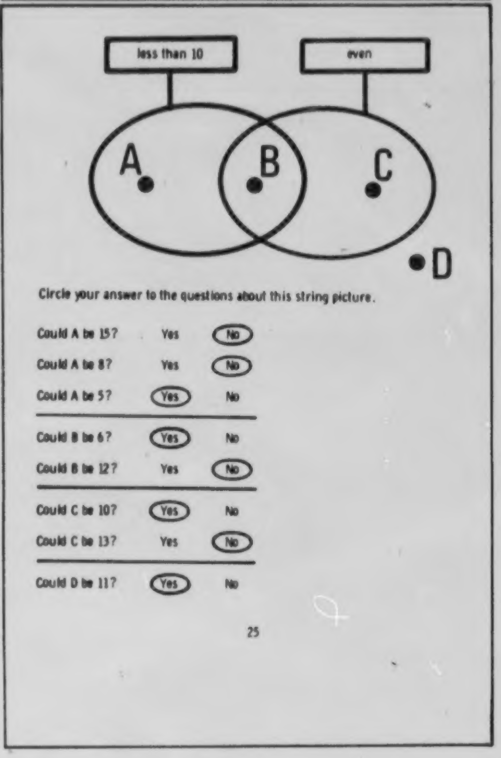
89

34

51

45

24



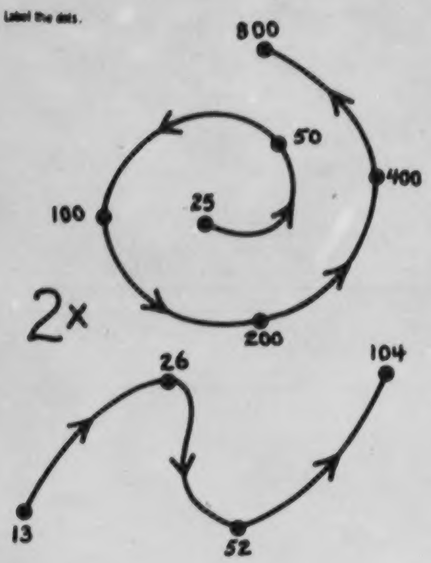
less than 10
even
A
B
C
D

Circle your answer to the questions about this string picture.

Could A be 15?	Yes	[✓ No]
Could A be 8?	Yes	[✓ No]
Could A be 5?	[✓ Yes]	No
Could B be 6?	[✓ Yes]	No
Could B be 12?	Yes	[✓ No]
Could C be 10?	[✓ Yes]	No
Could C be 13?	Yes	[✓ No]
Could D be 11?	[✓ Yes]	No

25

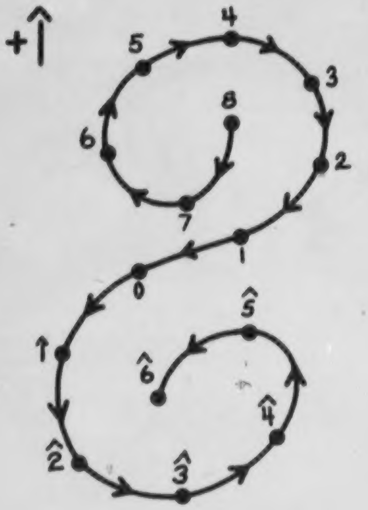
Label the dots.


800
50
400
25
100
2x
200
26
104
13
52

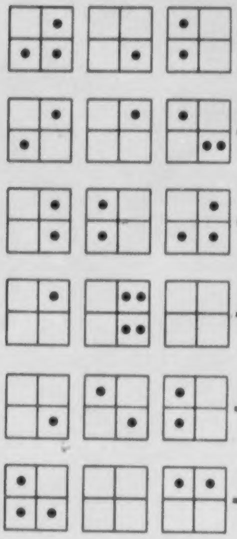
Show some ways to make 50¢.
One is done for you.

Quarters	Dimes	Nickels
25¢	10¢	5¢ 5¢ 5¢
25¢		5¢ 5¢ 5¢ 5¢ 5¢
	10¢ 10¢ 10¢ 10¢ 10¢	
	10¢ 10¢ 10¢ 10¢	5¢ 5¢
	10¢ 10¢ 10¢	5¢ 5¢ 5¢ 5¢
	10¢ 10¢	5¢ 5¢ 5¢ 5¢ 5¢ 5¢
	10¢	5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢
		5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢ 5¢

Label the dots.


+↑̂
4
5
3
8
6
2
7
1
0
5̂
↑
6̂
4̂
2̂
3̂

What number is on the Minicomputer?



– 720

– 650

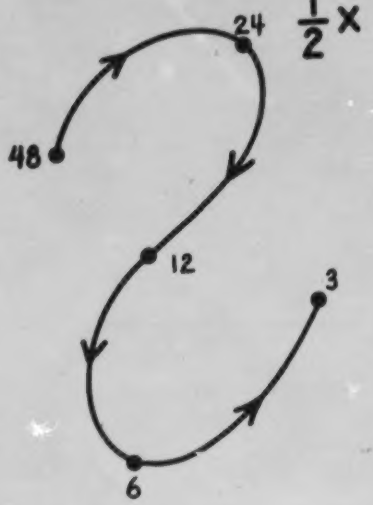
– 607

– 500

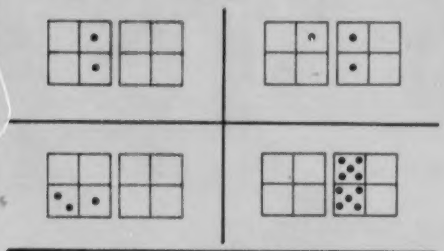
– 200

– 1,112

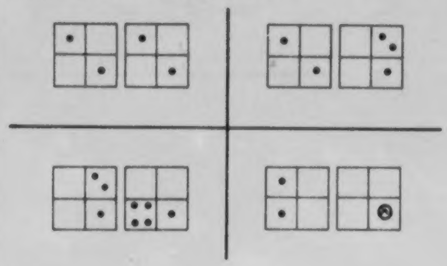
Label the dots.



Find four ways to put 50 on the Minicomputer.



Find four ways to put 99 on the Minicomputer.



Many solutions are possible.

CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

W	H	E	N
25 - 2	12 - 4	½ x 10	2 x 7

I	S
$10 + \hat{1}$	20 - 1

Y	O	U	R
5 x 5	3 x 5	3 x 7	3 x 6

B	I	R	T	H	D	A	Y
½ x 4	3 x 3	2 x 9	4 x 5	½ x 16	10 - 6	$5 + \hat{4}$	30 - 5

?

Answer: Student's birth date

W3 Detective Story #1

CAPSULE LESSON SUMMARY

Solve a detective story in which the clues involve the Minicomputer, an arrow picture, and a string picture.

MATERIALS

Teacher: Minicomputer set; colored chalk
Student: Minicomputer set; paper; pencil

NOTE: Each student should have an individual Minicomputer sheet*, one regular checker, and one negative checker.

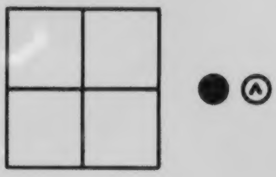
DESCRIPTION OF LESSON

Write a large "$\hat{1}$" on a slip of paper and fold the paper so that the "$\hat{1}$" is hidden.

T: I wrote the name of a secret number on this paper. Listen carefully to the clues and you can discover which number it is.

First clue

Display one demonstration Minicomputer board and draw one regular checker and one negative checker near it.



T: The secret number can be put on the ones' board of the Minicomputer using exactly one regular checker and one negative checker. Which number could be the secret number? Use your Minicomputer to help you decide.

---

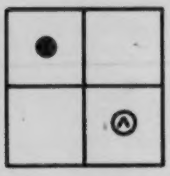
* Each sheet has two boards.

W 3

Check each number suggested by asking a student to put it on the Minicomputer using one regular and one negative checker. Record correct possibilities on the board, spacing the numbers appropriately so they can be recorded in order. After three or four possibilities have been found, ask the following questions.

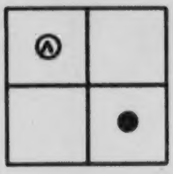
T: What is the largest number that can be put on the ones' board using exactly one regular and one negative checker?

S: 7.



T: What is the smallest?

S: $\hat{7}$ (read: negative seven).



Continue until all the possible numbers have been found. Be prepared to give suggestions if the students have difficulty finding some of the numbers. Refer to Eli the Elephant and magic peanuts whenever it would be helpful. You should have this list on the board.

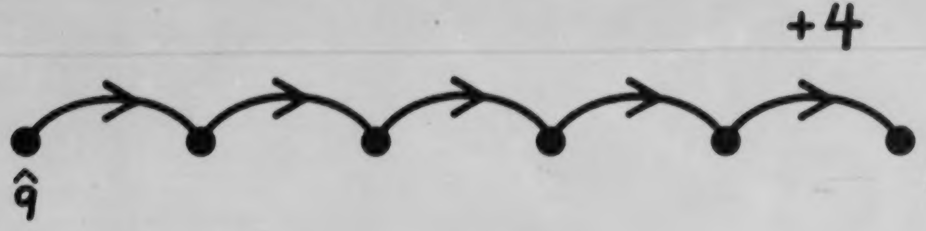
$\hat{7}, \hat{6}, \hat{4}, \hat{3}, \hat{2}, \hat{1}, 0, 1, 2, 3, 4, 6, 7$

Ask the students to try to put 5 and $\hat{5}$ on the Minicomputer using only one regular and one negative checker for each. Conclude that the secret number cannot be 5 nor $\hat{5}$.

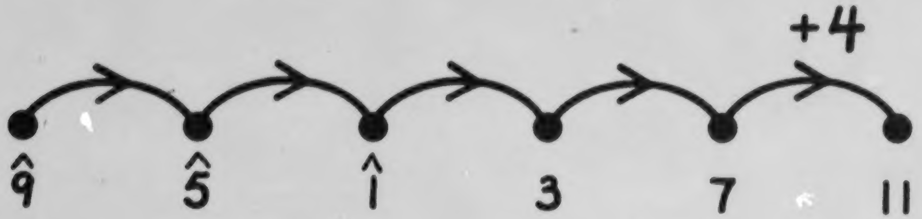
T: Now we know that the secret number is one of these numbers, but we don't know which one. We need another clue.

Second clue

Draw this arrow picture on the board.



Point to the dot for $\hat{9}$ and trace the first arrow as you say, "Negative nine plus four is . . . ?" If your class is not certain that the second dot from the left should be labeled "$\hat{5}$", draw the appropriate picture of magic and regular peanuts and calculate "$\hat{9}$ + 4". Continue until all the dots have been labeled.



T: The secret number is in our list and in this arrow picture. Which numbers could be the secret number?

S: $\hat{1}$ is in the arrow picture and in the list.

S: 3 and 7 are in the arrow picture and in the list.

W 3

T: Are there any other numbers in both?

S: No.

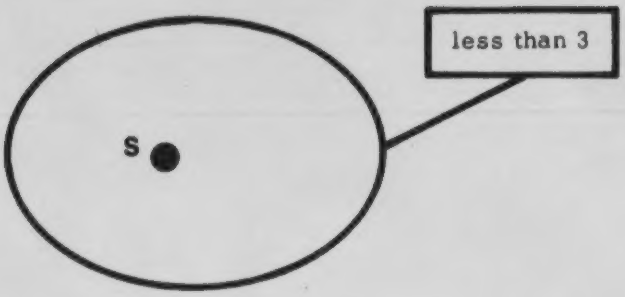
Erase the board except for these three numbers.

$$\hat{1}, 3, 7$$

T: Now we know that the secret number is either $\hat{1}$ or 3 or 7. There is only one more clue.

Third clue

Draw this string picture on the board.



T (pointing to "s"): The secret number is inside this string. When you know the secret number, write it on a piece of paper.

Check the answers of most of the students before asking someone to give the answer aloud.

T: What is the secret number?

S: $\hat{1}$.

Reveal that "$\hat{1}$" is written on your paper.

W4 Length and Area (Lesson 1)

CAPSULE LESSON SUMMARY

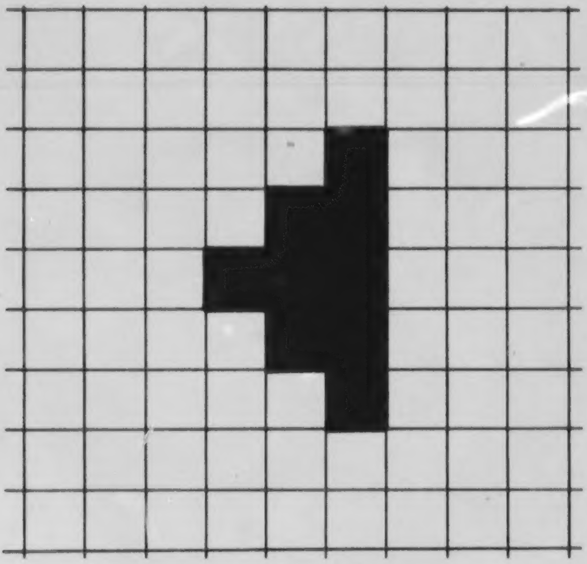
Draw shapes on a grid all with area 9 ■ (grid squares). Compare the lengths of the borders of these shapes and determine which shape has the longest border and which shape has the shortest border. Begin work in the Workbook LENGTH AND AREA.

MATERIALS

Teacher: Grid board[*]; colored chalk
Student: Worksheet W4; colored pencils; Workbook LENGTH AND AREA

DESCRIPTION OF LESSON

Display a grid board and color in this shape on the board. Provide a heavy border on the shape.



T: What is the area of this shape? How many little squares . . . ?

S: 9 little squares.

---

[*] See the NOTE ON GRIDS in the introduction to the Geometry strand.

W 4

Write this information on the board.

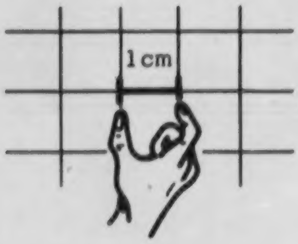
Area = 9 ■

T (tracing the border of the shape): This is the border of my shape. I'd like to find out how long the border of this shape is. How can I do it?

S: Count the blocks.

T: Yes. Let's pretend that the side of one little square (or that one block) is one centimeter.

Indicate the side of one square with your fingers and write "1 cm" on the board.



T: How long is the border of this shape in centimeters?

Call on a student to do the counting.

S: 16 cm.

Write this information on the board.

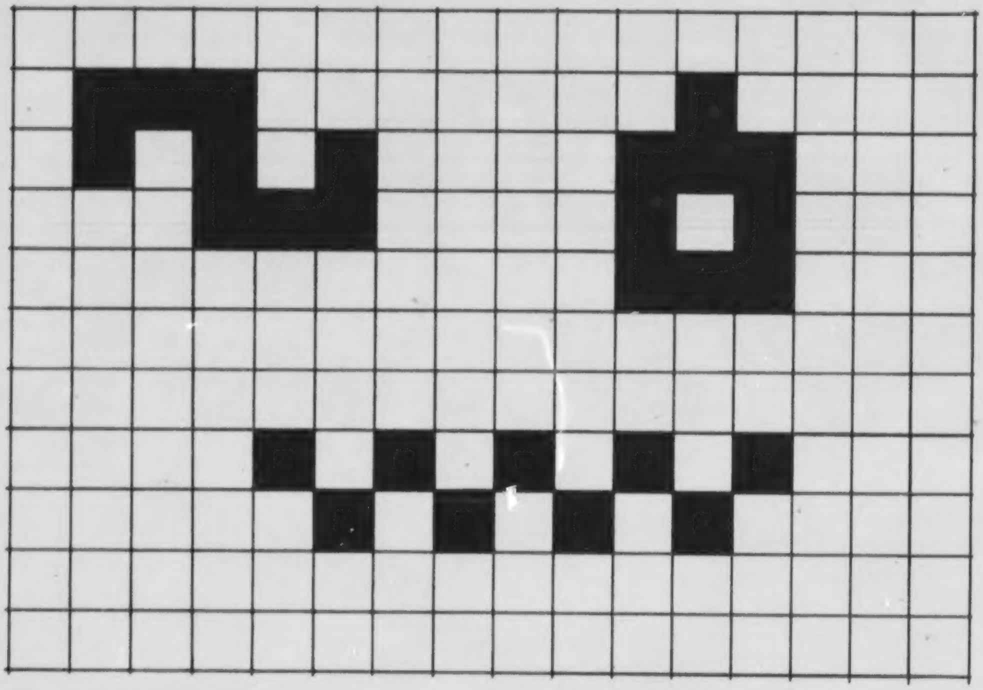
Area = 9 ■

Length of border = 16 cm

Distribute copies of Worksheet W4 and colored pencils.

T: On your worksheet, color in any shape you like that has area 9 ■. Then find the length of the border of your shape.

Let the students work independently for a few minutes. Help those students who have difficulty getting started. If necessary, encourage students not to divide the grid squares in forming their shapes. This might make finding the length of the border difficult. As you observe the work, find three students who have drawn shapes with different length borders and ask them to copy their shapes on the demonstration grid board. For example, these three shapes might be put on the board.



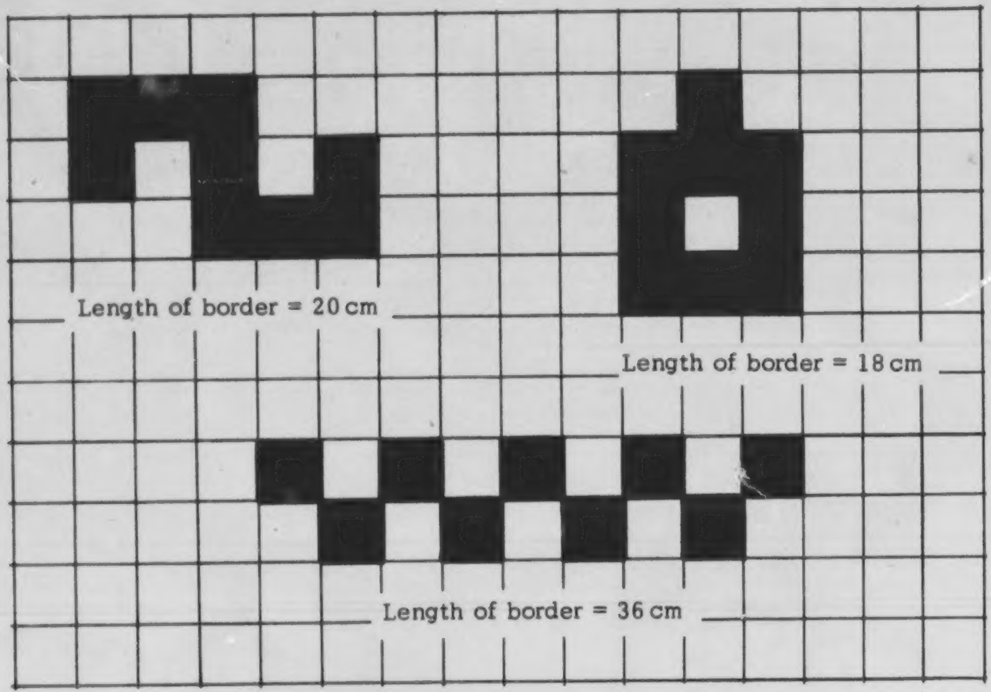
T: All three of these shapes on the board have area 9 little squares.

Provide each shape with a heavy border as you say:

T: Let's look at the border of each shape. Which shape has the longest border? Which shape has the shortest border?

W 4

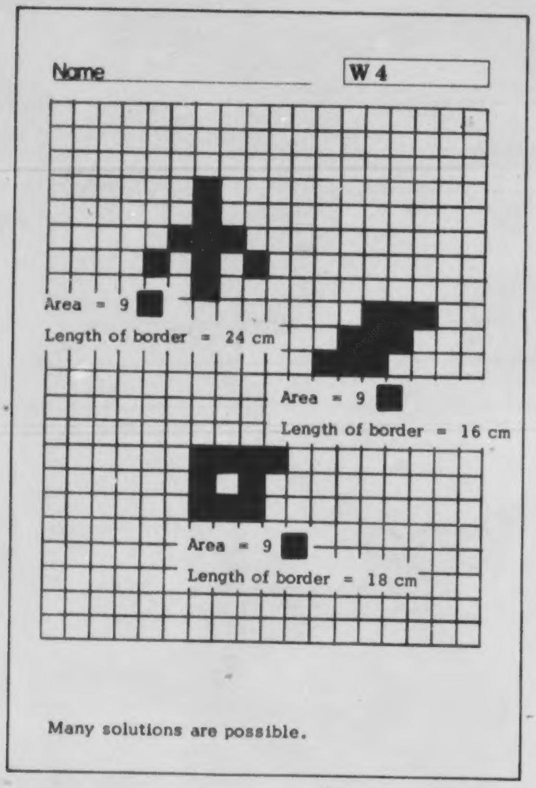
Call on students to find the length of the border of each shape and write the information on the board. Then note the shape with the longest border and the shape with the shortest border.



NOTE: For a shape like the one at the upper right above, be careful to count that part of the border which makes a hole in the shape.

Distribute individual copies of the Workbook LENGTH AND AREA and let the students work independently for the rest of the period. If many students have difficulty with a particular page, you may wish to have a full group discussion of that page.

At the end of the lesson, collect the workbooks. Check them and have them ready for use in the second lesson on this workbook (W5).



Name

W 4

Area = 9 ■

Length of border = 24 cm

Area = 9 ■

Length of border = 16 cm

Area = 9 ■

Length of border = 18 cm

Many solutions are possible.

W 5

W5 Length and Area (Lesson 2)

CAPSULE LESSON SUMMARY

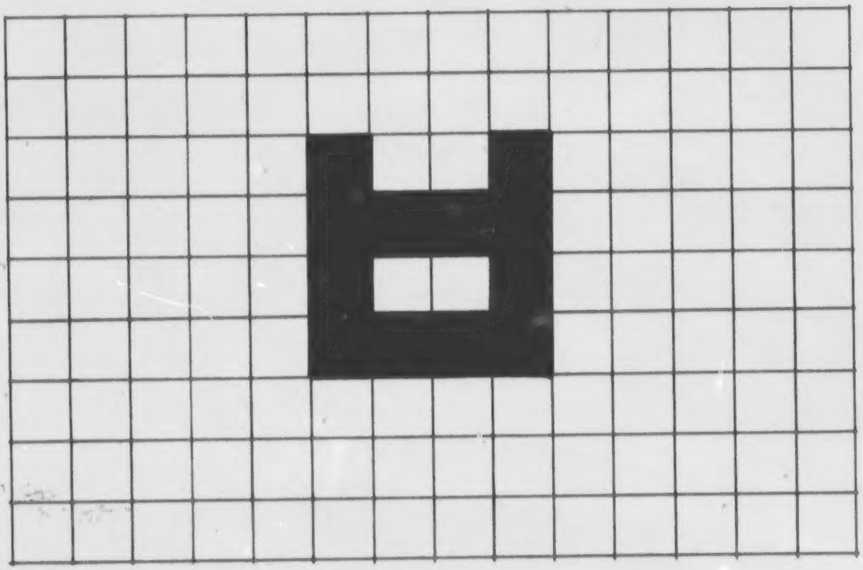
Draw a shape of area 12 ■ (grid squares) on a grid board and find the length of its border. Then draw other shapes (not allowing divided grid squares) of the same area but with longer and shorter borders. Find that the shortest possible border is 14 cm and the longest possible border is 48 cm. Continue work in the Workbook LENGTH AND AREA. (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: Grid board*; colored chalk
Student: Worksheet W5; colored pencils;
Workbook LENGTH AND AREA

DESCRIPTION OF LESSON

Display a grid board and color in this shape on the board. Provide a heavy border on the shape.



T: What is the area of this shape? How many little squares?

S: 12 little squares.

---

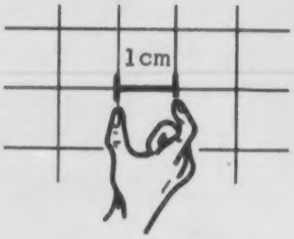
* See the NOTE ON GRIDS in the introduction to the Geometry strand.

Write this information on the board.

Area = 12 ■

T (tracing the border of the shape): I would like to find the length of the border of this shape. Let's pretend that the side of the little square is one centimeter long.

Indicate the side of one square with your fingers and write "1 cm" on the board.



T: How long is the border of this shape in centimeters?

Call on a student to do the counting. Be sure the student counts that part of the border around the hole in the shape.

S: 24 cm.

Write this information on the board.

Area = 12 ■

Length of border = 24 cm

W 5

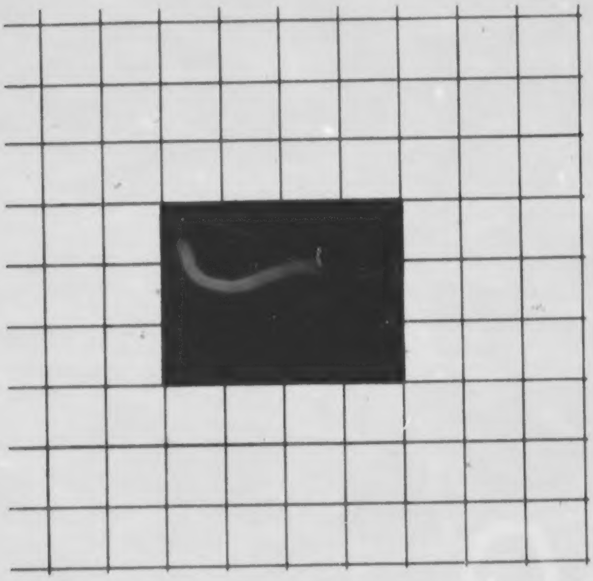
Distribute Worksheet W5 and colored pencils.

T: On your worksheet, color in a shape with area 12 ■ but with a very short border. Try to find a shape that has a border shorter than 24 cm. Then color in another shape with area 12 ■ but with a long border. Try to find a shape that has a border longer than 24 cm.

Let the students work independently for several minutes. Encourage students to experiment. Again, ask students not to divide the grid squares in their shapes. After a short while, call the class' attention back to the grid board.

T: Who has a shape with area 12 ■ and with a border shorter than 24 cm?

Let students announce their results and call on one with the shortest possible border to put his or her shape on the board. A shape (not allowing divided grid squares) with shortest possible border is illustrated below. Record the area and length of border information near the shape.

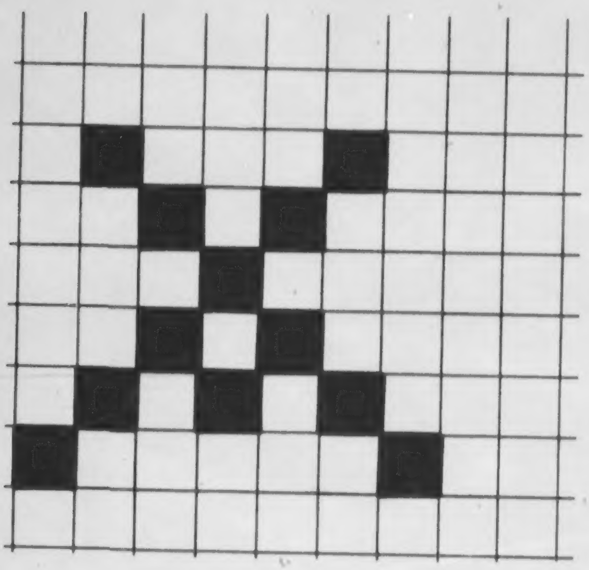


Area = 12 ■

Length of border = 14 cm

T: Now, who has a shape with area 12 ■ and with a border longer than 24 cm?

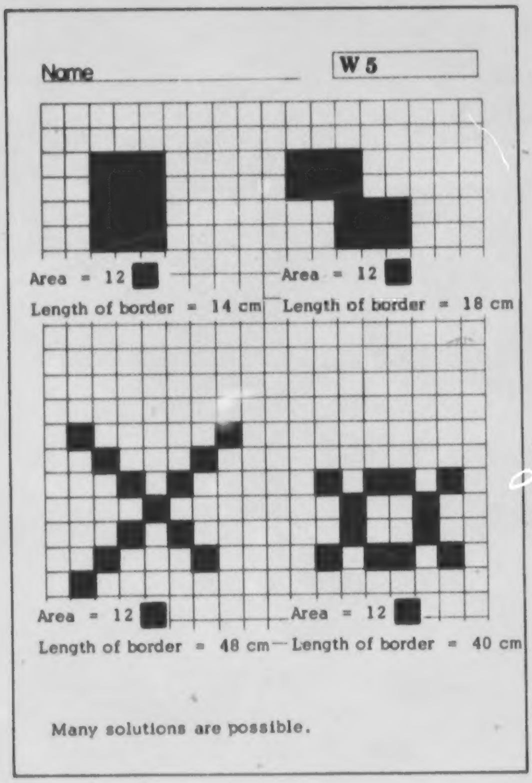
Let students announce their results and call on one with the longest possible border to put his or her shape on the board. A shape (not allowing divided grid squares) with longest possible border is shown below. Record the area and length of border information near the shape.



Area = 12 ■
Length of border = 48 cm

Return the students' copies of the Workbook LENGTH AND AREA and let them work independently for the rest of the period. If many students have difficulty with a particular page, you may wish to have a full group discussion about that page.

At the end of the lesson, collect the workbooks for your review.



Name

W 5

Area = 12 ■

Length of border = 14 cm

Area = 12 ■

Length of border = 18 cm

Area = 12 ■

Length of border = 48 cm

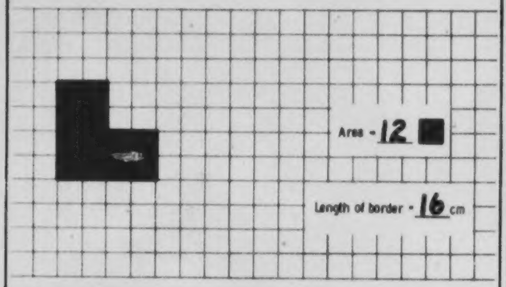
Area = 12 ■

Length of border = 40 cm

Many solutions are possible.

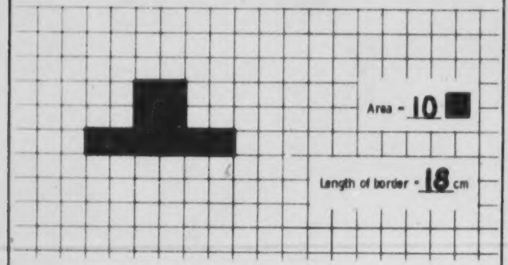
LENGTH
AND
AREA

Length and Area



Area = **12** ■

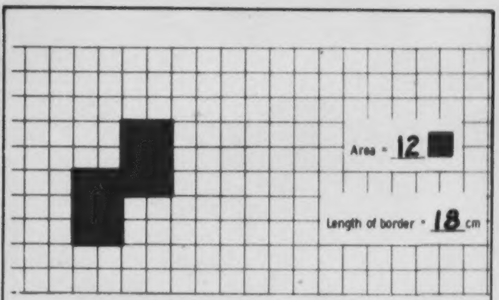
Length of border = **16** cm



Area = **10** ■

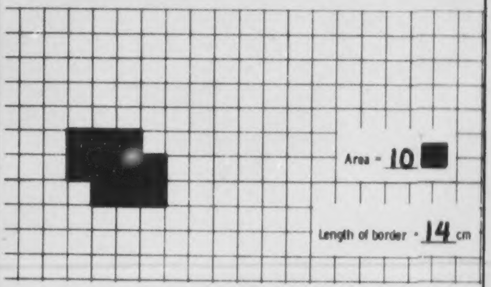
Length of border = **18** cm

2



Area = **12** ■

Length of border = **18** cm

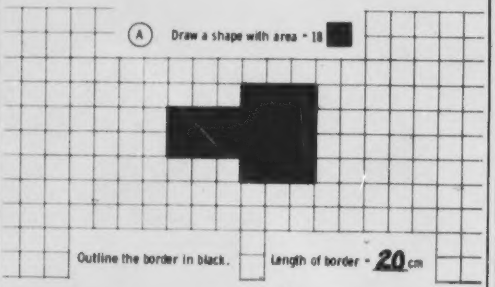


Area = **10** ■

Length of border = **14** cm

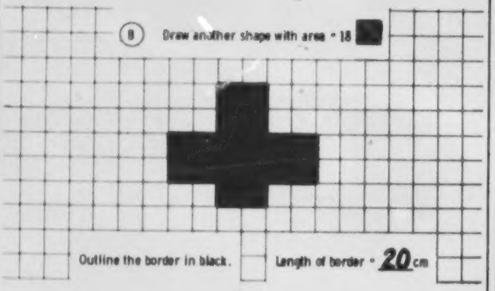
3

Ⓐ Draw a shape with area = 18 ■



Outline the border in black. Length of border = **20** cm

Ⓑ Draw another shape with area = 18 ■

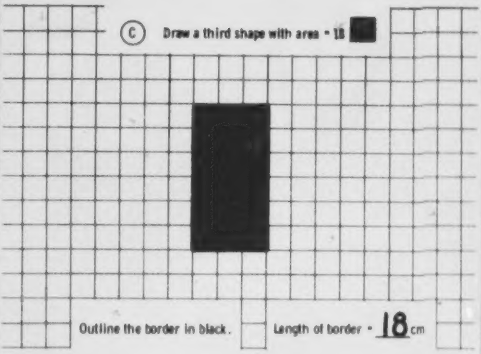


Outline the border in black. Length of border = **20** cm

4

Many solutions are possible.

Ⓒ Draw a third shape with area = 18 ■



Outline the border in black. Length of border = **18** cm

Look at your three shapes.

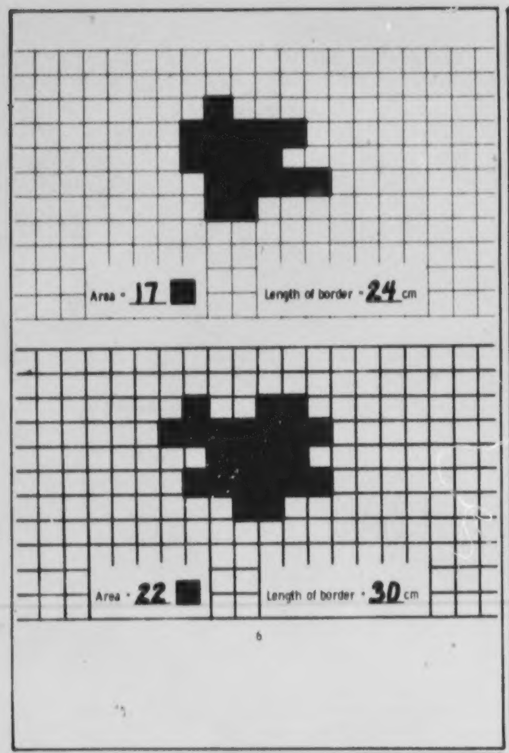
Circle your answers.

Which shape has the longest border? [✓ A] or [✓ B] C

Which shape has the shortest border? A B [✓ C]

5

Answers depend on the shapes drawn by the student.

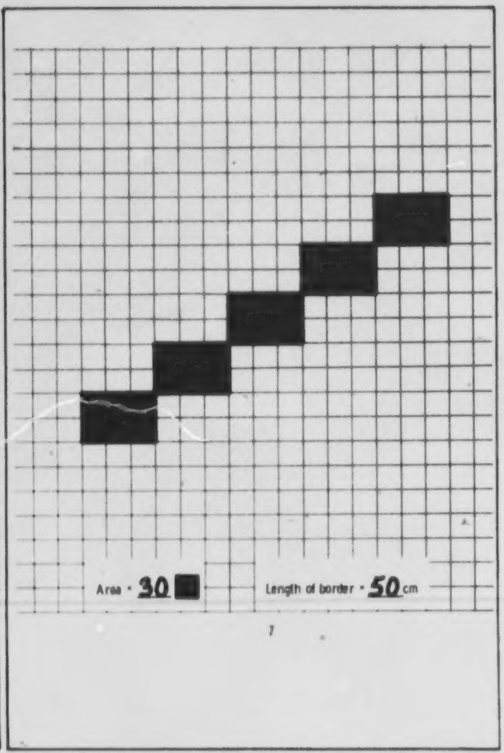

Area = 17 ■

Length of border = 24 cm

Area = 22 ■

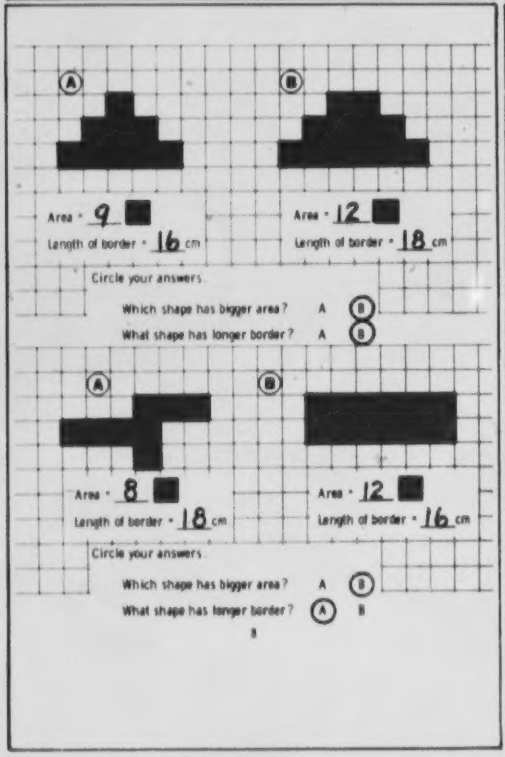
Length of border = 30 cm

6


Area = 30 ■

Length of border = 50 cm

7



Area = 9 ■

Length of border = 16 cm

Area = 12 ■

Length of border = 18 cm

Circle your answers.

Which shape has bigger area? A [✓ B]

What shape has longer border? A [✓ B]

Area = 8 ■

Length of border = 18 cm

Area = 12 ■

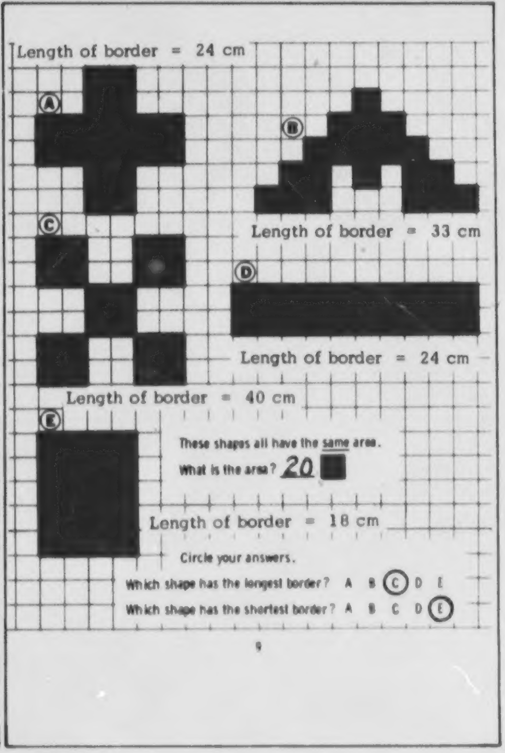
Length of border = 16 cm

Circle your answers.

Which shape has bigger area? A [✓ B]

What shape has longer border? [✓ A] B

8



Length of border = 24 cm

Length of border = 33 cm

Length of border = 24 cm

Length of border = 40 cm

These shapes all have the same area.

What is the area? 20 ■

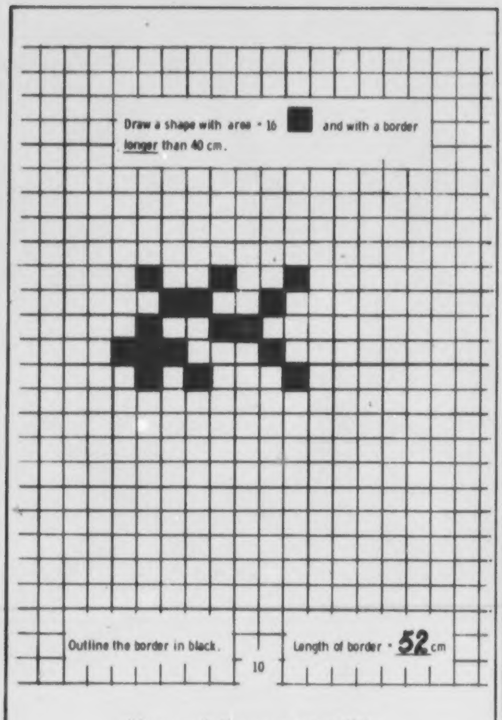
Length of border = 18 cm

Circle your answers.

Which shape has the longest border? A B [✓ C] D E

Which shape has the shortest border? A B C D [✓ E]

9



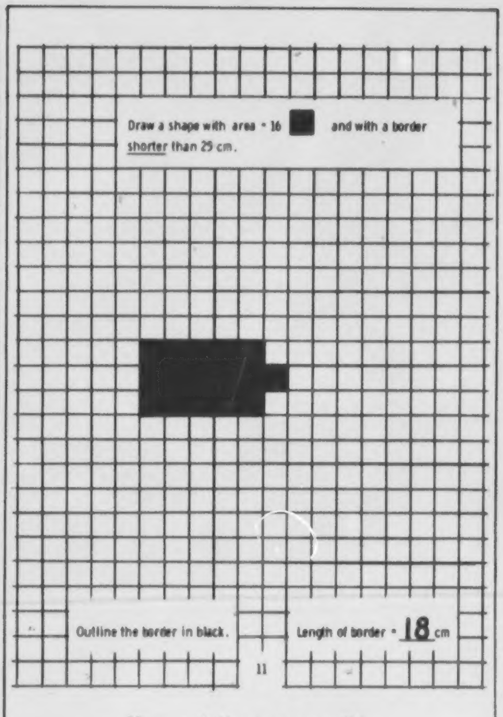
Draw a shape with area = 16 ■ and with a border longer than 40 cm.

Outline the border in black.

Length of border = 52 cm

10

Many solutions are possible.



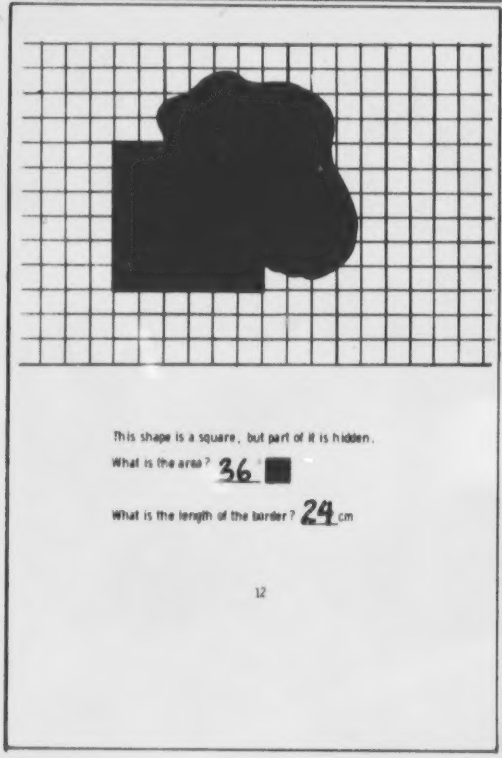
Draw a shape with area = 16 ■ and with a border shorter than 25 cm.

Outline the border in black.

Length of border = 18 cm

11

Many solutions are possible.

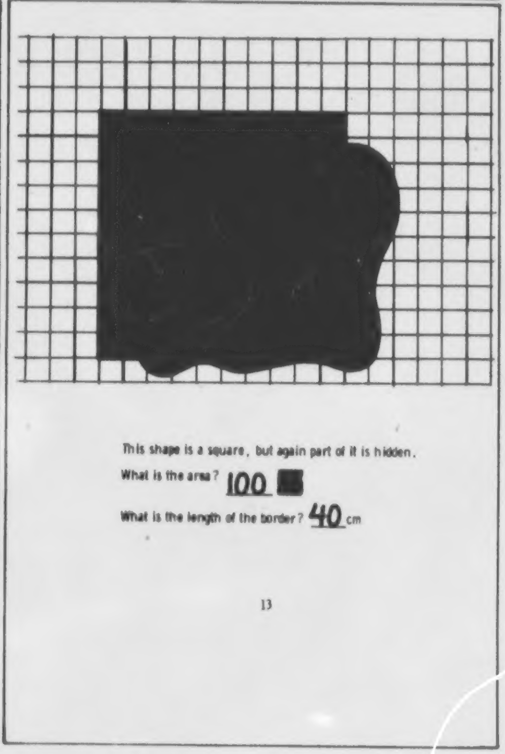


This shape is a square, but part of it is hidden.

What is the area? 36 ■

What is the length of the border? 24 cm

12

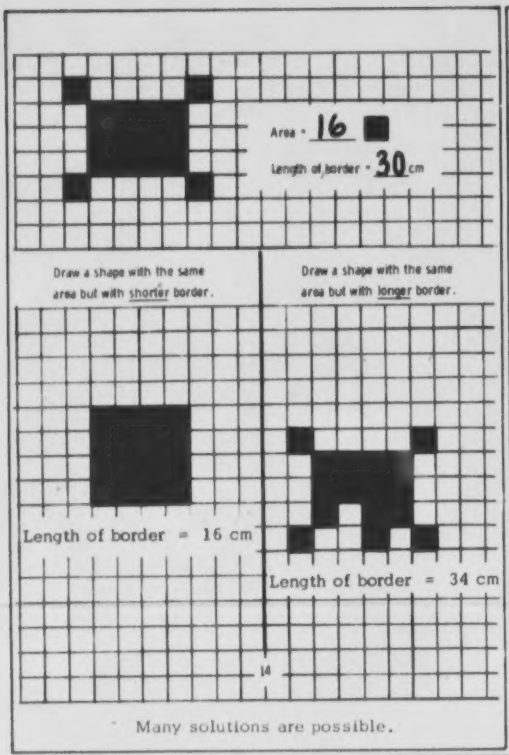


This shape is a square, but again part of it is hidden.

What is the area? 100 ■

What is the length of the border? 40 cm

13



Area = 16 ■

Length of border = 30 cm

Draw a shape with the same area but with shorter border.

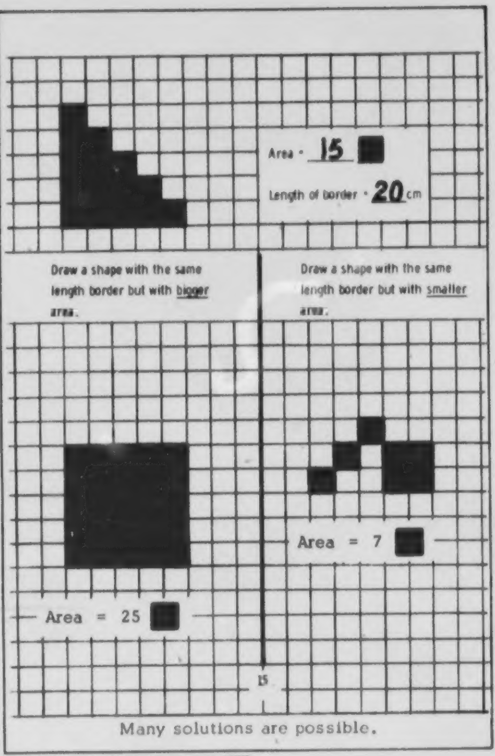
Length of border = 16 cm

Draw a shape with the same area but with longer border.

Length of border = 34 cm

14

Many solutions are possible.



Area = 15 ■

Length of border = 20 cm

Draw a shape with the same length border but with bigger area.

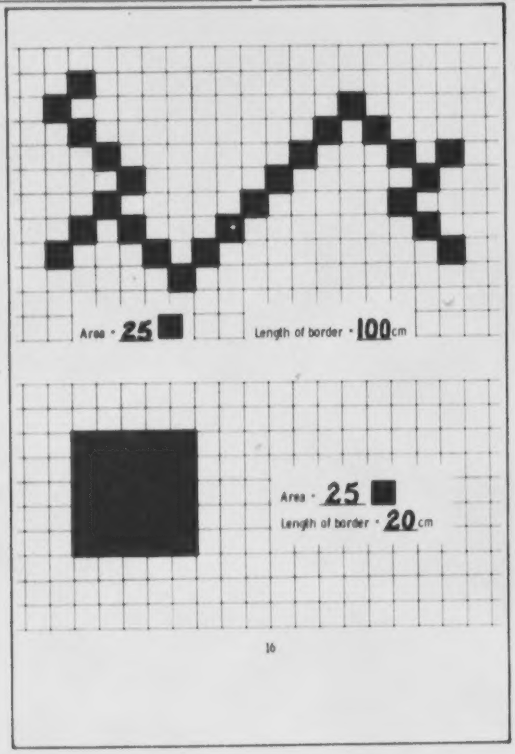
Area = 25 ■

Draw a shape with the same length border but with smaller area.

Area = 7 ■

15

Many solutions are possible.



Area = 25 ■ Length of border = 100 cm

Area = 25 ■

Length of border = 20 cm

16

W6 Detective Story #2

CAPSULE LESSON SUMMARY

Solve a detective story in which the secret number is not an even number and in which the other clues involve the hand-calculator and a string picture.

MATERIALS

Teacher: Two hand-calculators*; colored chalk; Minicomputer set

Student: Paper; pencil; Worksheets W6*, **, ***, and ****

DESCRIPTION OF LESSON

Give two students the hand-calculators. Students who sit close to students with calculators will be able to observe the use of the calculator and to verify what number is on the display. You should describe each step on the calculator for the entire class.

First clue

T: Using several clues we are going to try to discover a secret number. The calculator will help us with the first clue. We will use the calculator to count by elevens (from 0). The first clue is that the secret number is one of the numbers which will appear on the display.

Turn on the calculators. What number is on the display?

S: 0.

T: Listen carefully to my directions. Press $\boxed{+}\ \boxed{1}\ \boxed{1}\ \boxed{=}$ (read: plus eleven equals). What number is on the display?

S: 11.

---

* See ROLE AND USE OF HAND-CALCULATORS on page 14 of the General Introduction.

257

W 6

Record "11" on the board.

T: Press $\boxed{=}$ again. What number is on the display?

S: 22.

Record "22" under "11" on the board.

T: What number will we get if we press $\boxed{=}$ again? (33)
Press $\boxed{=}$.
What number is on the display?

S: 33.

Record "33" in the list forming on the board.

T: What number will we get next? (44)
Press $\boxed{=}$.
What number is on the display?

S: 44.

Record "44" in the list.

11
22
33
44

T: Do you notice a pattern in this list of numbers?

S: Here (pointing to the tens' column) the numbers go 1, 2, 3, 4, and here (pointing to the ones' column) they also go 1, 2, 3, 4.

Ask the students to continue predicting and verifying on the calculator multiples of 11 (in order) until the list includes 99.

Allow several students to predict what the next multiple of 11 is before asking the students with calculators to press $\boxed{=}$ again. Begin a second column of the list with "110". Continue in the same manner at least until your list includes 143. If some students can predict accurately the larger numbers, ask them to explain how they find them. Put three dots under the last number in your list.

11
22
33
44
55
66
77
88
99

110
121
132
143
.
.
.

T: Why did I put three dots?

S: To show we could go on and on with more numbers.

Put the calculators away or ask the students to turn them off.

Second clue

T: Our second clue is that the secret number is not an even number. Using this information, tell me which of the numbers in our list cannot be the secret number.

W 6

NOTE: It is possible that a student may comment that all numbers which are not even are odd. This is a natural conclusion since the students' experience is almost entirely in the realm of the integers and since they are not as well-acquainted with non-integer rational numbers (such as 0.5, $\frac{1}{3}$, 6.32, and so on) in the second grade. In this case, you should rephrase the student's statement: "All the numbers in this list which are not even are odd."

As students indicate which of the numbers listed are even, cross them out.

11
~~22~~
33
~~44~~
55
~~66~~
77
~~88~~
99

~~110~~
121
~~132~~
143
.
.
.

T: Does anyone see a pattern?

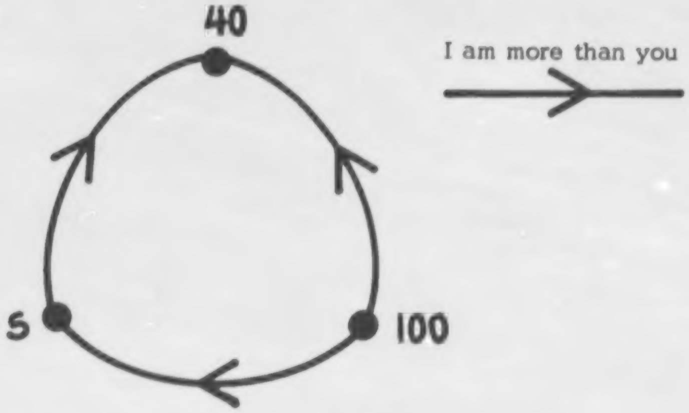
It may be difficult for students to verbalize that every other number in the list is an even number and they may need prompting.

T: Should we cross off the three dots?

S: No, because there are more odd numbers (which are multiples of 11).

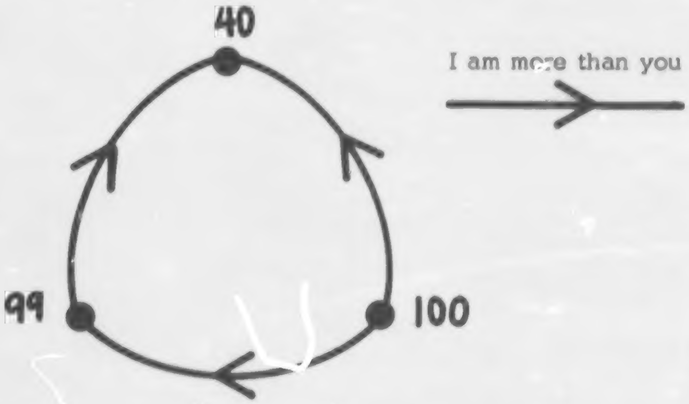
Third clue

Draw this arrow picture on the board.



T: Where is the secret number in this picture? (At s)
What are the red arrows for? ("I am more than you")
Is 100 more than 40 (trace the appropriate arrow)? (Yes)
Which of these numbers (point to the list) could be the secret number?

For each number your students suggest, label s with that number, then check if the number stories told by the arrows and the numbers in the picture are correct. Suppose a student suggests that 99 could be the secret number.



Trace the arrow from 99 to 40.

W 6

S: 99 is more than 40.

T: Is that true? (Yes)

Trace the arrow from 100 to 99.

S: 100 is more than 99.

T: Is that true? (Yes)
99 could be the secret number.

Circle "99" in the list.

Suppose a student incorrectly suggests that 33 could be the secret number.

S: 33 is more than 40.

T: Is that true? (No)
Could the secret number be 33? (No)

Cross off "33" in the list.

Continue until all the numbers in your list are either crossed out or circled.

~~11~~, ~~22~~, ~~33~~, ~~44~~, (55), ~~66~~, (77), ~~88~~, (99), ~~110~~, ~~121~~, ~~132~~, ~~143~~...

T: Can we cross off the three dots?

S: Yes, because if we go on, the numbers we get are all more than 100.

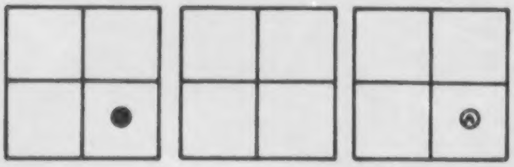
Cross off the three dots. Erase everything which has been crossed off.

Fourth clue

Display three Minicomputer boards. Hold up one regular checker and one negative checker.

T: We know that the secret number is 55 or 77 or 99. There is one more clue. The secret number can be put on the Minicomputer with one regular and one negative checker.

Ask the students to write the secret number on a piece of paper for you to check. Then ask someone to give the answer aloud and to put it on the Minicomputer using one regular and one negative checker.



$= 100 + \hat{1} = 99$

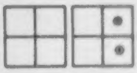
Decide with the class that neither 55 nor 77 can be put on the Minicomputer using one regular and one negative checker.

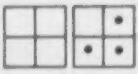
Worksheets W6*, **, ***, **** are available for individual work.

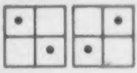
Name

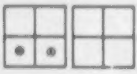
W 6 *

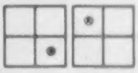
What number is on the Minicomputer?

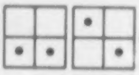
 - $\hat{5}$

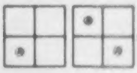
 - 7

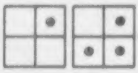
 - 96

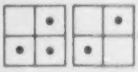
 - $\widehat{30}$

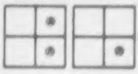
 - $\widehat{18}$

 - 39

 - $\widehat{29}$

 - $\widehat{47}$

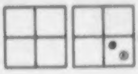
 - 76

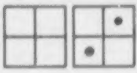
 - $\widehat{51}$

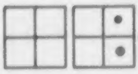
Name

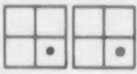
W 6 **

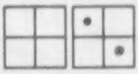
What number is on the Minicomputer?

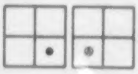
 - 0

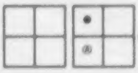
 - 2

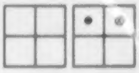
 - 3

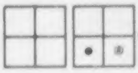
 - 9

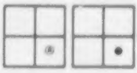
 - 7

 - 8

 - 6

 - 4

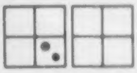
 - 1

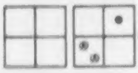
 - $\hat{9}$

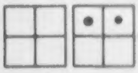
Name

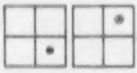
W 6 ***

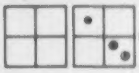
What number is on the Minicomputer?

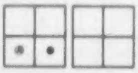
 - 0

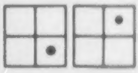
 - 0

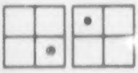
 - $\hat{4}$

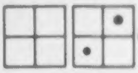
 - 6

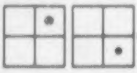
 - 6

 - $\widehat{10}$

 - $\hat{6}$

 - $\hat{2}$

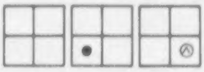
 - $\hat{2}$

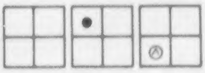
 - $\widehat{39}$

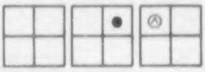
Name

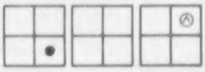
W 6 ****

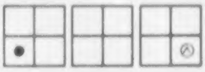
Put these numbers on the Minicomputer.
Use at least one negative checker for each number.

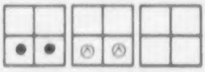
19 = 

78 = 

32 = 

96 = 

199 = 

270 = 

Many solutions are possible.

W7 A Caravan of Problems #2 (Lesson 1)

CAPSULE LESSON SUMMARY

Explore the effect of moving, removing, or adding some checkers to a configuration on the Minicomputer — in each case, does the numerical value increase, decrease, or stay the same? Estimate a number on the Minicomputer and then make plays, stopping periodically to get better estimates, until its standard configuration is obtained. Begin the Workbook A CARAVAN OF PROBLEMS #2.

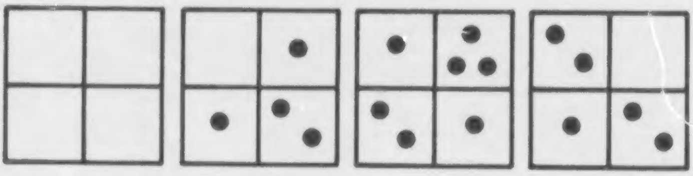
MATERIALS

Teacher: Minicomputer set

Student: Workbook A CARAVAN OF PROBLEMS #2; colored pencils; pencil

DESCRIPTION OF LESSON

Put this configuration on the Minicomputer.



T: Is the number on the Minicomputer more than 500? How do you know?

A possible dialogue is given here.

S: Yes, there is a checker on the 400-square and a checker on the 200-square. 400 + 200 = 600 and 600 is more than 500.

T: Is this number more than 800? How do you know?

S: Yes, there is 800 on the hundreds' board and there are more checkers on the other boards.

T: Is this number more than 900? How do you know?

S: The number on the hundreds' board is 800 and there is more than 100 on the tens' board (80 + 20 = 100), so this number is more than 900.

T: Is this number more than 1,000? How do you know?

It is possible that a student will be able to explain and to convince the class that this number is more than 1,000. In any case, your class should estimate the number to be more than 900 and possibly more than 1,000.

T: We know that this number is more than 900 (1,000). We do not need to know exactly which number is on the Minicomputer to compare it to other numbers.

Write these words on the board near the Minicomputer.

More

Same

Less

T: I am going to move, remove, or add some checkers. Each time tell me if the number on the Minicomputer is more than, the same as, or less than before.

Move a checker from the 1-square to the 10-square. Point to each of the three words in turn as you say:

T: If you think the number is more than before, raise your hand.
If you think the number is still the same, raise your hand.
If you think the number is less than before, raise your hand.

Repeat the move very obviously if many students do not know that the number on the Minicomputer is more than the previous number.

T: How much more is this number than the number we had before? How do you know?

S: It is 9 more, because you moved the checker from the 1-square to the 10-square.

Continue this activity with the following or similar moves.

- Make a "100 + 100 = 200" play. (Same)
- Move a checker from the 8-square to the 4-square. (4 less)
- Move a checker from the 2-square to the 8-square. (6 more)
- Move a checker from the 20-square to the 80-square. (60 more)
- Move a checker from the 200-square to the 800-square. (600 more)
- Remove two checkers from the 8-square and put one checker on the 4-square. (12 less)

After you make several moves yourself, invite students to transform the number on the Minicomputer. When appropriate, ask the students exactly how much more or less a number is than the previous one.

At the end of this activity you will have some number displayed on the Minicomputer. Allow students to guess what the number is and record their guesses on the board. Guide the class in estimating this new number in the same way as you did the beginning number. When the class has a fairly good estimate or when there is disagreement about an estimate, invite students to make some plays. After several plays are made, guide the class to make a closer estimate.

Continue making plays until the standard configuration is obtained. Invite a student to write the number below the Minicomputer, and decide which guess is the closest.

Distribute copies of the Workbook A CARAVAN OF PROBLEMS #2 and let the students work independently. At the end of the lesson, collect the workbooks for your review.

W 8

W8 A Caravan of Problems #2 (Lesson 2)

CAPSULE LESSON SUMMARY

Collectively solve the addition problem "437 + 283 = ?". Continue working in the Workbook A CARAVAN OF PROBLEMS #2. (This is the second of two lessons using this workbook.)

MATERIALS

Teacher: None

Student: Workbook A CARAVAN OF PROBLEMS #2; colored pencils

DESCRIPTION OF LESSON

Write this problem on the board.

$$437 + 283 =$$

T: How could we calculate "437 + 283"?

Perhaps someone will suggest writing the problem vertically.

$$\begin{array}{r} 437 \\ +\,283 \\ \hline \end{array}$$

T: What do you see in the ones' column?

S: "7" and "3".

T: 7 + 3 = . . . ?

S: 10.

T: Where do we show the 10?

S: In the tens' column by writing "1" above "3".

T: What should I write below the ones' column?

S: "0", because there are not any extra ones.

$$\begin{array}{r} 4\overset{1}{3}7 \\ +283 \\ \hline 0 \end{array}$$

T: What do you see in the tens' column?

S: "1" and "3" and "8".

T: 1 ten + 3 tens + 8 tens = . . . ?

S: 12 tens.

T: Where do we show the 12 tens?

S: We show 10 tens by writing a "1" in the hundreds' column, because 10 tens is 1 hundred.

T: What should I write below the tens' column?

S: "2", because there are two extra tens.

$$\begin{array}{r} \overset{1}{4}\overset{1}{3}7 \\ +283 \\ \hline 20 \end{array}$$

W 8

T: What do you see in the hundreds' column?

S: "1" and "4" and "2".

T: 1 hundred + 4 hundreds + 2 hundreds = . . . ?

S: 7 hundreds.

T: What should I write below the hundreds' column?

S: "7".

$$\begin{array}{r} \overset{1}{4}\overset{1}{3}7 \\ +283 \\ \hline 720 \end{array}$$

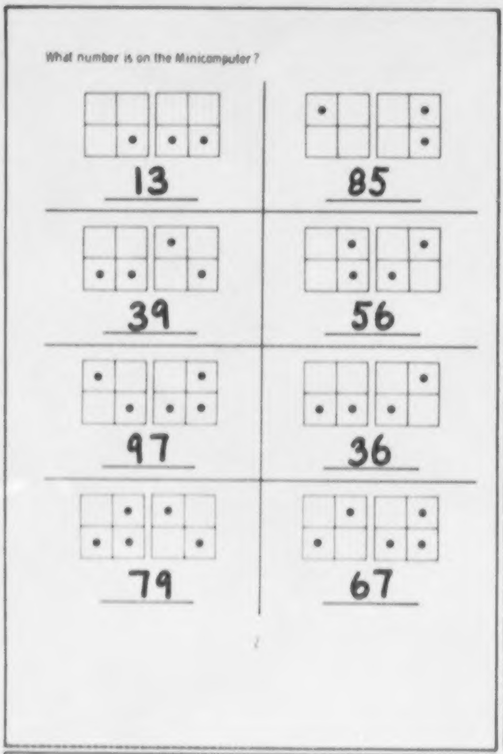
Conclude that 437 + 283 = 720.

Distribute the students' copies of the Workbook A CARAVAN OF PROBLEMS #2. Allow about 30 minutes for independent work and then collect the workbooks for your review.

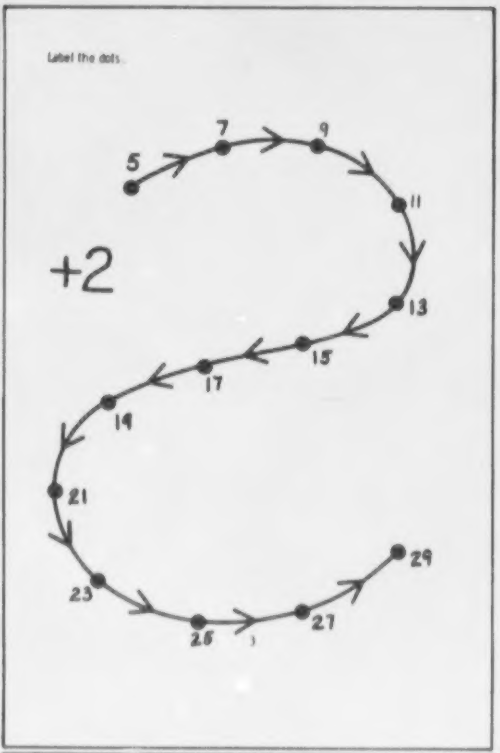
A
CARAVAN OF
PROBLEMS
2

A Caravan of Problems #2

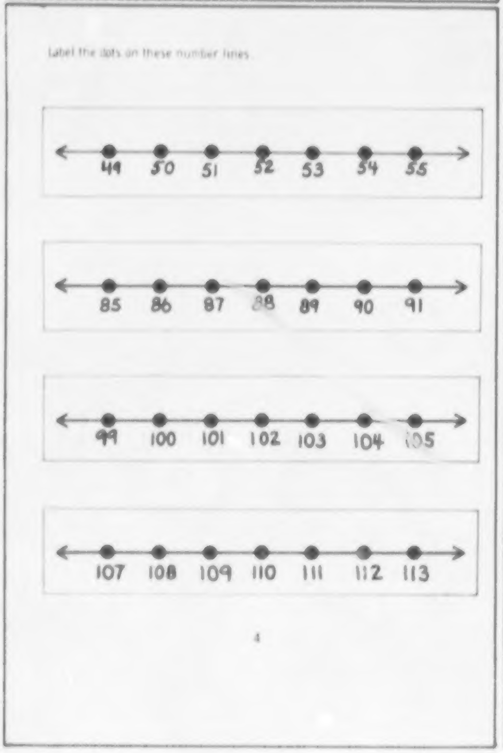
What number is on the Minicomputer?


13
85
39
56
97
36
79
67
2

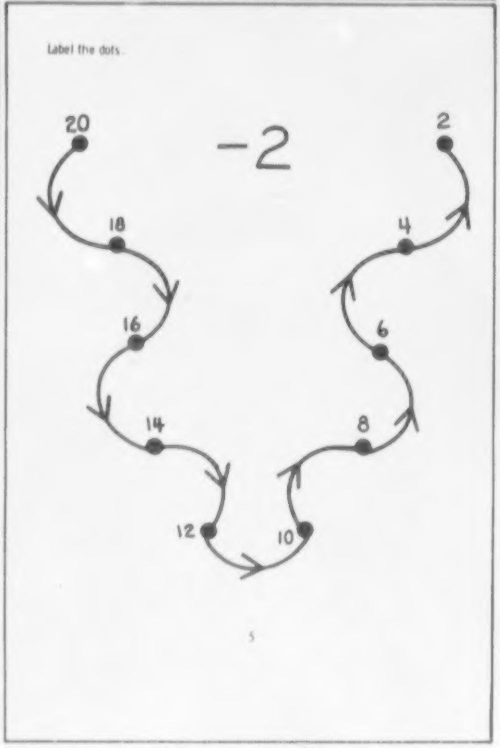
Label the dots.


+2
5
7
9
11
13
15
17
19
21
23
25
27
29
3

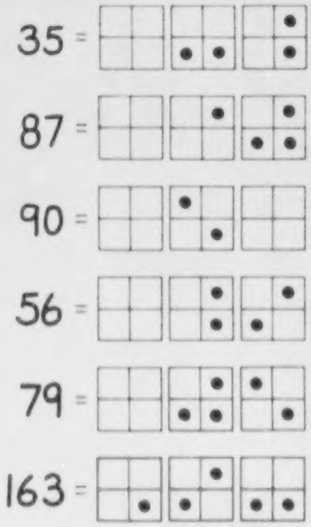
Label the dots on these number lines.


49 50 51 52 53 54 55
85 86 87 88 89 90 91
99 100 101 102 103 104 105
107 108 109 110 111 112 113
4

Label the dots.

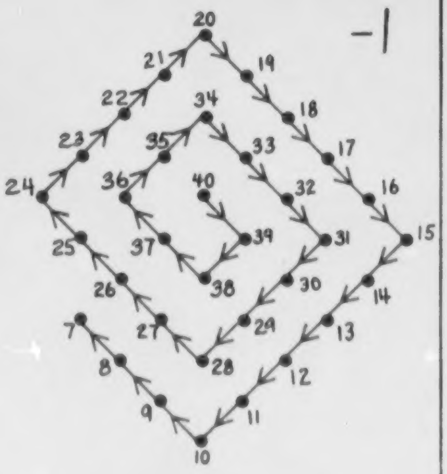

−2
20
18
16
14
12
10
8
6
4
2
5

Put the numbers on the Minicomputer.


35 =
87 =
90 =
56 =
79 =
163 =

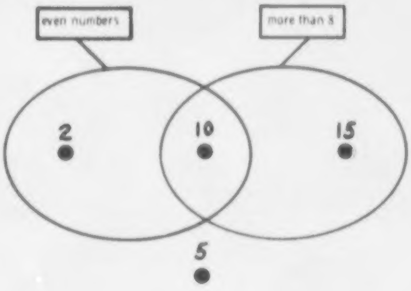
Standard configurations are given here; however, many other solutions are possible.

Label the dots.

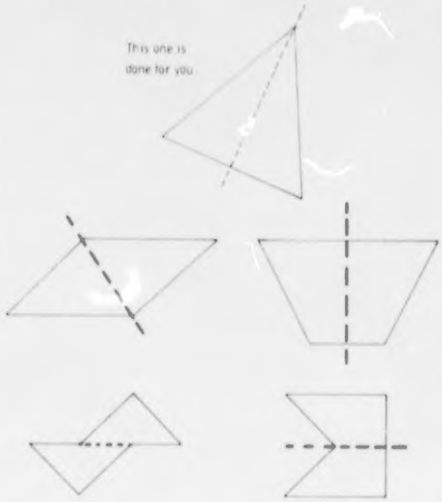

-1
20
21
19
22
34
18
23
35
33
17
24
36
40
32
16
15
25
37
39
31
38
30
14
26
29
13
7
27
28
8
12
9
11
10

Label the dots in this picture with these numbers:

2 5 10 15

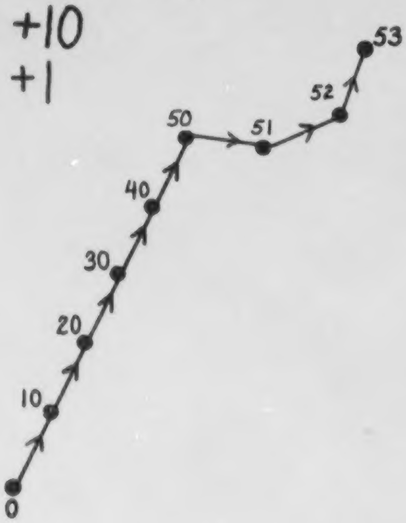

even numbers
more than 8
2
10
15
5

Draw a line which cuts each shape in half.


This one is done for you.

Other solutions are possible.

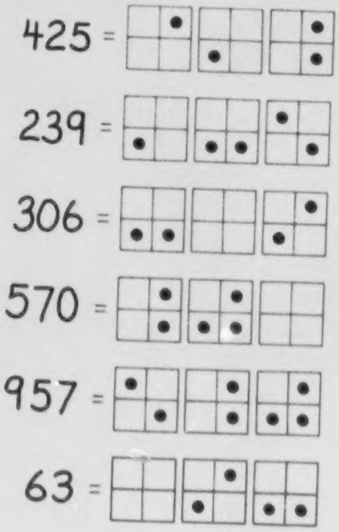
Build a red-blue road from 0 to 53.


+10
+1
0
10
20
30
40
50
51
52
53

10

Many solutions are possible.

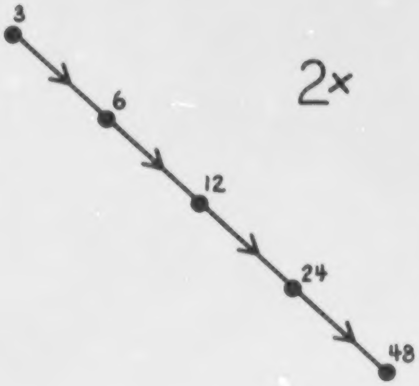
Put the numbers on the Minicomputer.


425 =
239 =
306 =
570 =
957 =
63 =

11

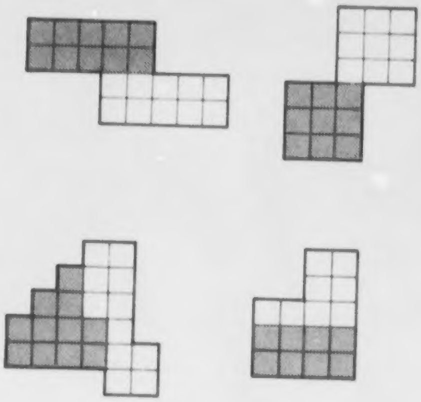
Standard configurations are given here; however, many other solutions are possible.

Label the dots.


3
6
12
24
48
2x

12

Shade one-half of each shape.

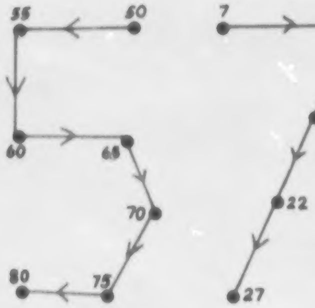


13

Many solutions are possible.

Draw all the missing red arrows.

+5



Calculate:

$\begin{array}{r}25\\+15\\\hline 40\end{array}$ $\begin{array}{r}52\\+64\\\hline 116\end{array}$

$\begin{array}{r}350\\+240\\\hline 590\end{array}$ $\begin{array}{r}416\\+322\\\hline 738\end{array}$

$\begin{array}{r}512\\+329\\\hline 841\end{array}$ $\begin{array}{r}321\\+284\\\hline 605\end{array}$

CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

$\underset{10-2}{\underline{\text{H}}}$ $\underset{3\times5}{\underline{\text{O}}}$ $\underset{24-1}{\underline{\text{W}}}$

$\underset{4\times5}{\underline{\text{T}}}$ $\underset{5-4}{\underline{\text{A}}}$ $\underset{2\times6}{\underline{\text{L}}}$ $\underset{14-2}{\underline{\text{L}}}$

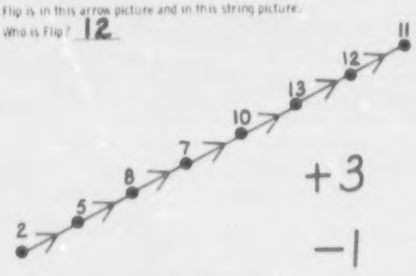
$\underset{10-9}{\underline{\text{A}}}$ $\underset{20-2}{\underline{\text{R}}}$ $\underset{10-5}{\underline{\text{E}}}$

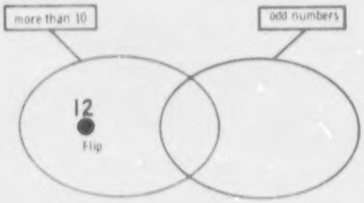
$\underset{5\times5}{\underline{\text{Y}}}$ $\underset{16-1}{\underline{\text{O}}}$ $\underset{22-1}{\underline{\text{U}}}$?

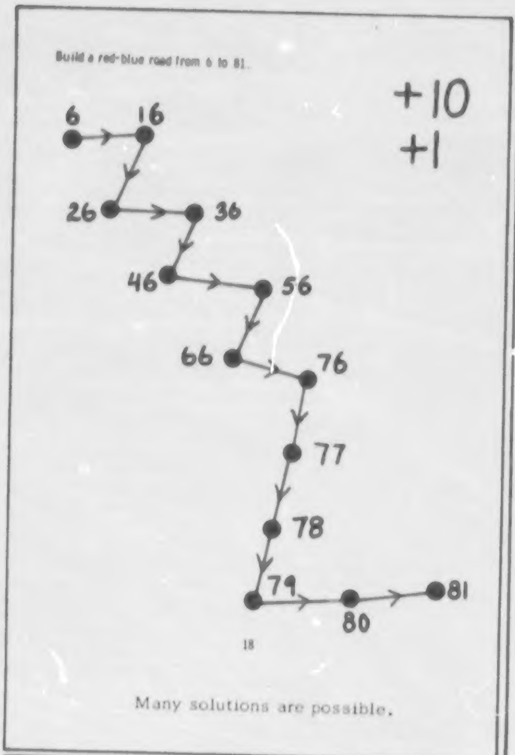
Answer: ____* cm

*Be prepared to measure students' heights in centimeters.

Flip is a secret number.
Flip is in this arrow picture and in this string picture.
Who is Flip? 12





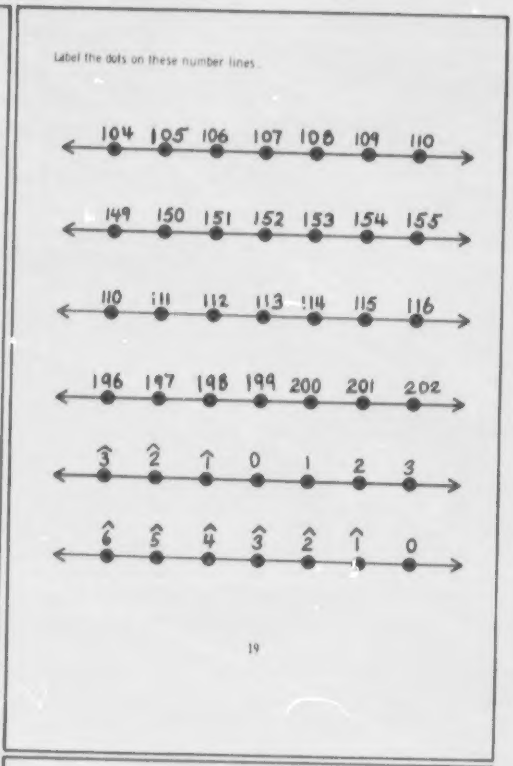


Build a red-blue road from 6 to 81.

+10
+1
6
16
26
36
46
56
66
76
77
78
79
80
81

18

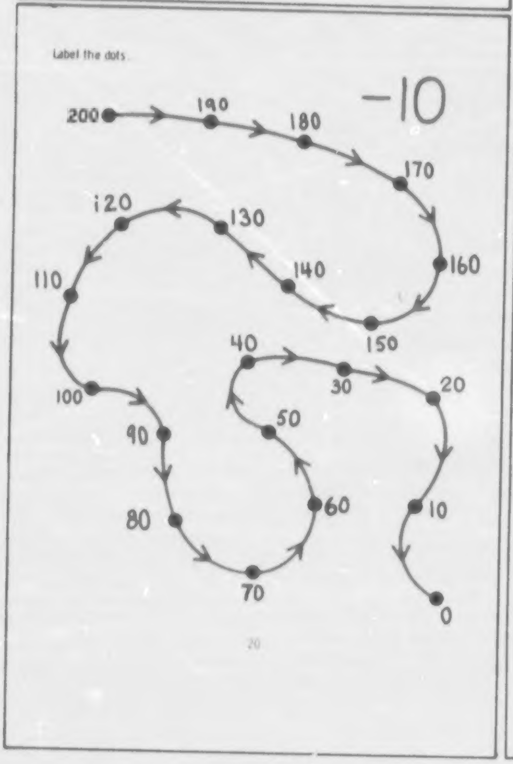
Many solutions are possible.



Label the dots on these number lines.

104 105 106 107 108 109 110
149 150 151 152 153 154 155
110 111 112 113 114 115 116
196 197 198 199 200 201 202
$\hat{3}$ $\hat{2}$ $\hat{1}$ 0 1 2 3
$\hat{6}$ $\hat{5}$ $\hat{4}$ $\hat{3}$ $\hat{2}$ $\hat{1}$ 0

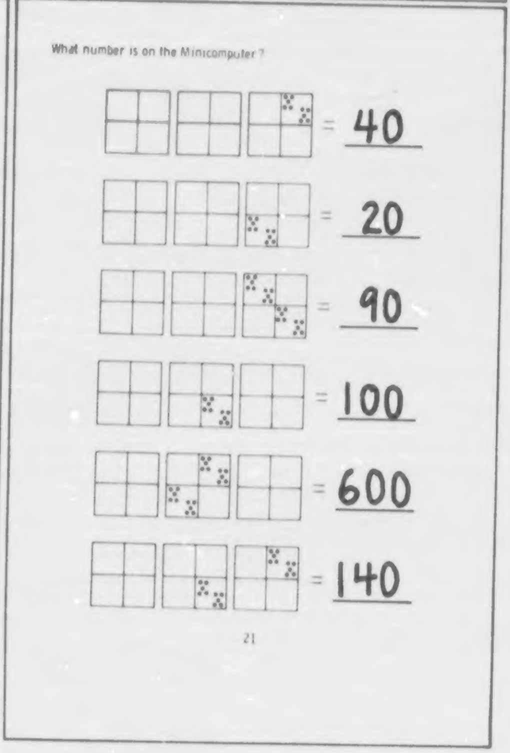
19



Label the dots.

-10
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0

20

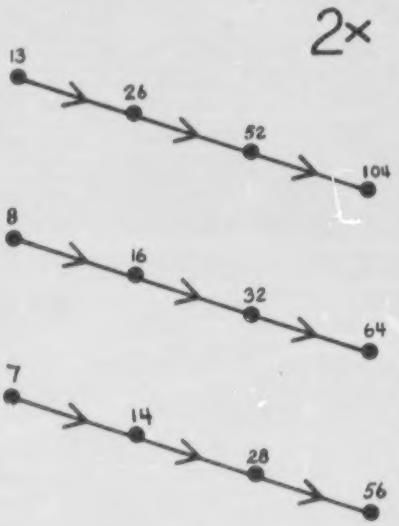


What number is on the Minicomputer?

= 40
= 20
= 90
= 100
= 600
= 140

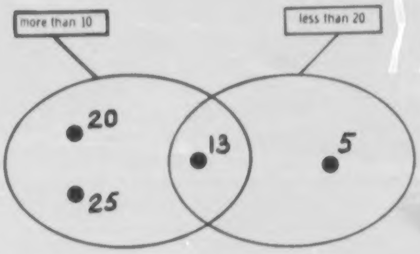
21

Label the dots.

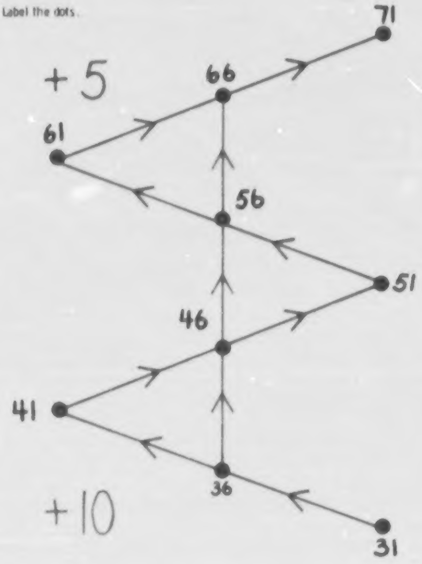

2×
13
26
52
104
8
16
32
64
7
14
28
56

Label the dots in this picture with these numbers:

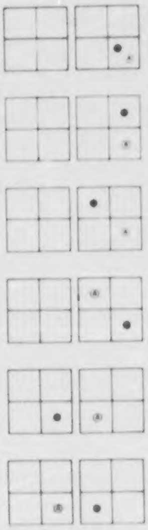
5 13 20 25


more than 10
less than 20
20
25
13
5

Label the dots.


+5
+10
71
66
61
56
51
46
41
36
31

What number is on the Minicomputer?



= $1+\hat{1}=0$

= $4+\hat{1}=3$

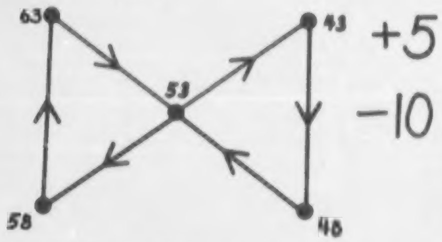
= $8+\hat{1}=7$

= $\hat{8}+1=\hat{7}$

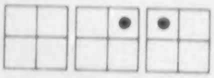
= $10+\hat{2}=8$

= $\widehat{10}+2=\hat{8}$

Muf is a secret number.
Muf is in this arrow picture.

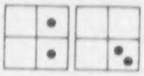


Muf can be put on the Minicomputer with exactly two checkers (positive or negative).
Put Muf on this Minicomputer.

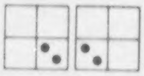


Who is Muf? 48

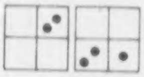
What number is on the Minicomputer?



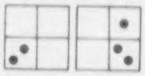
52



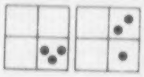
24



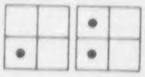
85



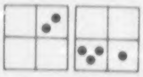
46



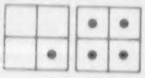
39



30

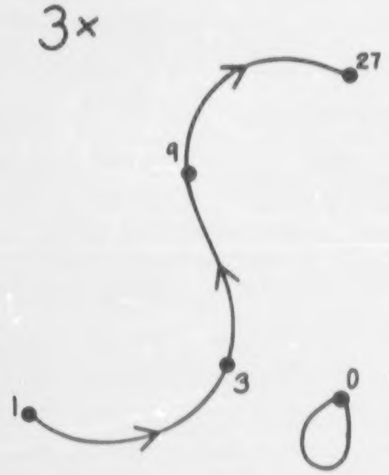


87

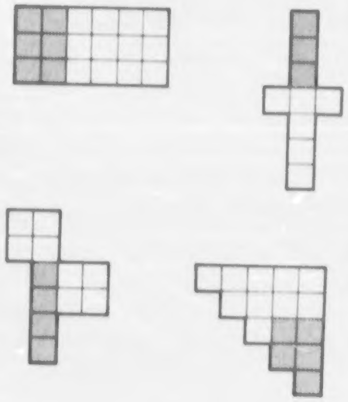


25

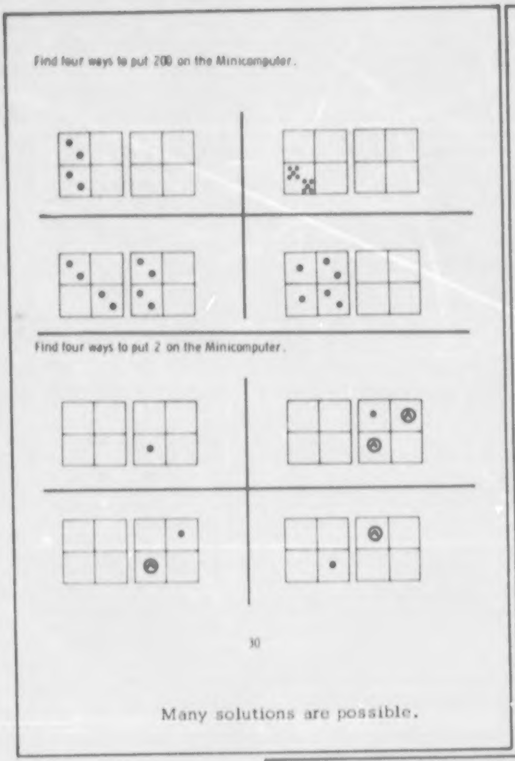
Label the dots.



Shade one-third of each shape.



Many solutions are possible.

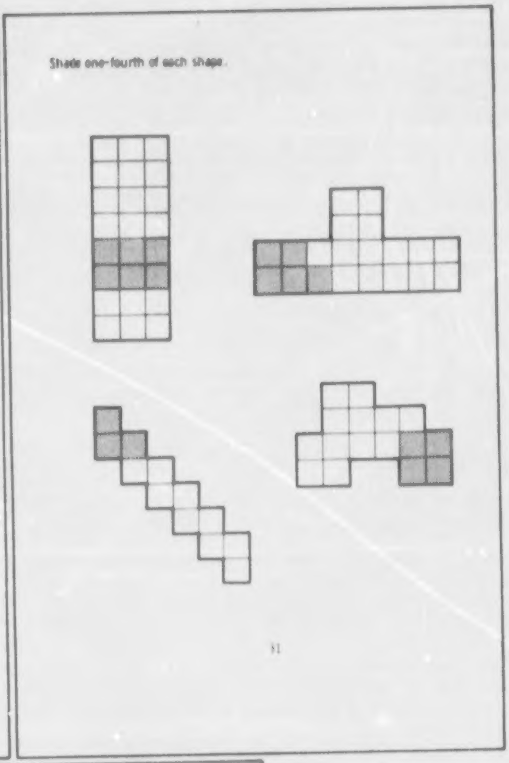


Find four ways to put 200 on the Minicomputer.

Find four ways to put 2 on the Minicomputer.

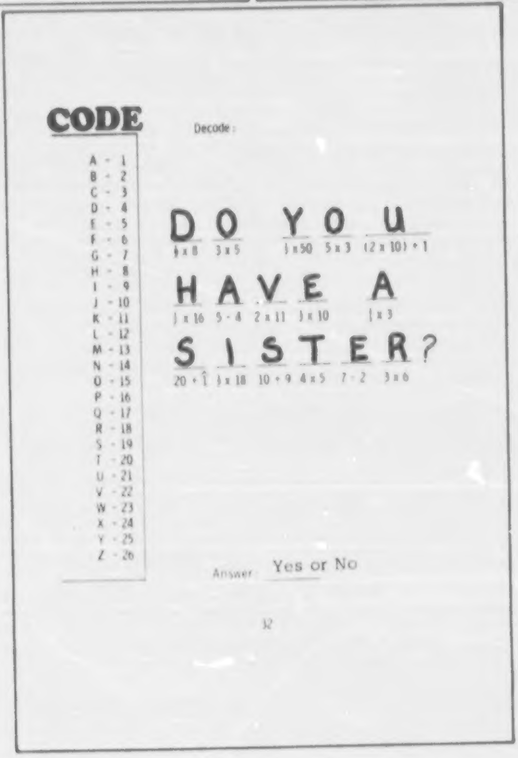
30

Many solutions are possible.



Shade one-fourth of each shape.

31



CODE

A - 1
B - 2
C - 3
D - 4
E - 5
F - 6
G - 7
H - 8
I - 9
J - 10
K - 11
L - 12
M - 13
N - 14
O - 15
P - 16
Q - 17
R - 18
S - 19
T - 20
U - 21
V - 22
W - 23
X - 24
Y - 25
Z - 26

Decode:

D O Y O U
½ x 8 3 x 5 ½ x 50 5 x 3 (2 x 10) + 1

H A V E A
½ x 16 5 - 4 2 x 11 ½ x 10 ⅓ x 3

S I S T E R?
20 - 1 ½ x 18 10 + 9 4 x 5 7 - 2 3 x 6

Answer: Yes or No

32

W9 Detective Story #3

CAPSULE LESSON SUMMARY

Solve a detective story in which the secret number is less than 100 and in which the other clues involve the hand-calculator, the Minicomputer, and a string picture.

MATERIALS

Teacher: Two hand-calculators[#]; Minicomputer set; colored chalk

Student: None

DESCRIPTION OF LESSON

Give two students the hand-calculators. Students who sit close to students with calculators will be able to observe the use of the calculator and to verify what number is on the display. You should describe each step on the calculator for the entire class.

First clue

T: We are going to try to discover a secret number from several clues. The calculator will help us with the first clue. We will use the calculator to count by nines (from 0). The first clue is that the secret number is one of the numbers which will appear on the display.

Turn on the calculators. Which number is on the display?

S: 0.

T: Listen carefully and follow my directions. Press $\boxed{+}$ $\boxed{9}$ $\boxed{=}$. What number is on the display?

S: 9.

[#] See ROLE AND USE OF HAND-CALCULATORS on page 14 of the General Introduction.

W 9

Record "9" on the board.

T: Press $\boxed{=}$ again. What number is on the display?

S: 18.

Record "18" under "9". Line up the ones' digits.

$$\begin{array}{r} 9 \\ 18 \end{array}$$

T: What number will we get if we press $\boxed{=}$ again?

Accept three or four predictions of what the next number will be.

T: Press $\boxed{=}$. What number is on the display?

S: 27.

Write "27" in the list forming on the board and accept three or four predictions of what the next number will be.

T: Press $\boxed{=}$. What number is on the display now?

S: 36.

Record "36" in the list.

$$\begin{array}{r} 9 \\ 18 \\ 27 \\ 36 \end{array}$$

T: Does anyone see a pattern?

S: Here (pointing to the tens' column) the numbers go forward: 1, 2, 3,.. and here (pointing to the ones' column) they go backward: 9, 8, 7....

If no one sees a pattern yet, ask students with calculators to press $\boxed{=}$ again. Record that the next number is 45 and again ask if anyone sees a pattern.

Continue this exercise as long as your students show interest. With each number, first ask for predictions and then ask the students to press $\boxed{=}$. When you get to the bottom of the board begin a new column of numbers. Draw three dots at the end of your list.

9
18
27
36
45
54
63
72
81
90
99
108
117
126
135
144
153
162
·
·
·

T: What are these three dots for?

S: To show that the numbers in the list go on and on.

Second clue

T: The second clue is that the secret number is less than 100. Using this information, tell me which of the numbers in our list cannot be the secret number.

W 9

As students indicate which of the numbers listed are more than 100[#], cross them out.

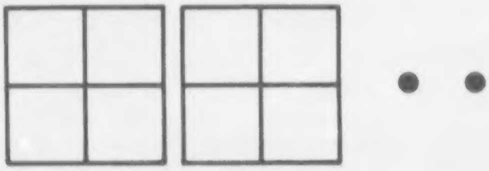
T (pointing to the three dots): Could the secret number be any of the numbers in our list which we didn't write on the board?

S: No, because those numbers are larger than 100.

Cross off the three dots.

Third clue

Display two Minicomputer boards and two regular checkers.



T: The secret number can be put on the Minicomputer with exactly two regular checkers. Which of the numbers in the list could be the secret number?

When a student suggests a number, ask the student to put the number on the Minicomputer with exactly two regular checkers. Circle correct possibilities for the secret number and cross out any number that the class decides cannot be shown with two regular checkers. Continue until all the remaining possibilities in the list have been considered. Erase everything which has been crossed off.

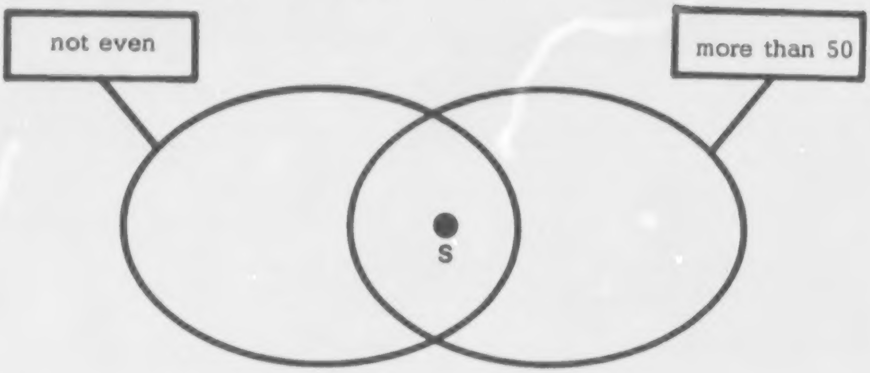
9 81

18 90

[#] 100 is not on the list.

Fourth clue

Draw this string picture on the board.



T: We know that the secret number is 9 or 18 or 81 or 90. The fourth clue is that the secret number (point to "s") is in this string picture. When you know which number is the secret number, whisper it to me.

When most students have had an opportunity to whisper an answer to you, ask:

T: Can you tell me some numbers which are not even?

Let students make suggestions. Indicate if a number is not even by saying either, "Yes, that number is not an even number," or "No, that is an even number."

T: The secret number is in the blue string so it is not even. Which of the numbers in our list are not even? (9 and 81)
Where would 9 be in this string picture? (Inside the blue string and outside the red string)

Ask a student to point to the correct region.

T: Where would 81 be in this string picture? (In the middle)
Is 81 not even? (Yes) Is 81 more than 50? (Yes)

Conclude that 81 is the secret number.

W 10

W10 Nora's Neighborhood

CAPSULE LESSON SUMMARY

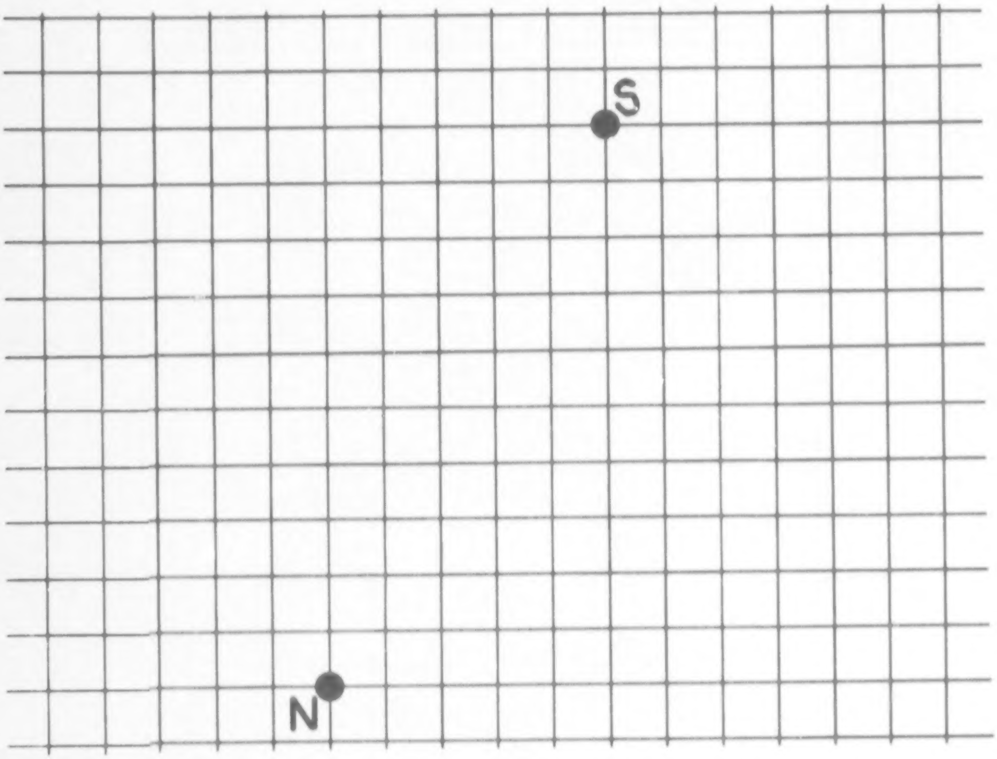
Review the idea of taxi-geometry paths between two points on a grid. Find short paths and long paths and compare the lengths (numbers of blocks) of several paths. Discuss the notion of a round trip starting at one point, going to another, and then returning to the first point. Use the workbook to review several ideas from previous taxi-geometry lessons.

MATERIALS

Teacher: Grid board[#]; colored chalk
Student: Workbook NORA'S NEIGHBORHOOD; colored pencils

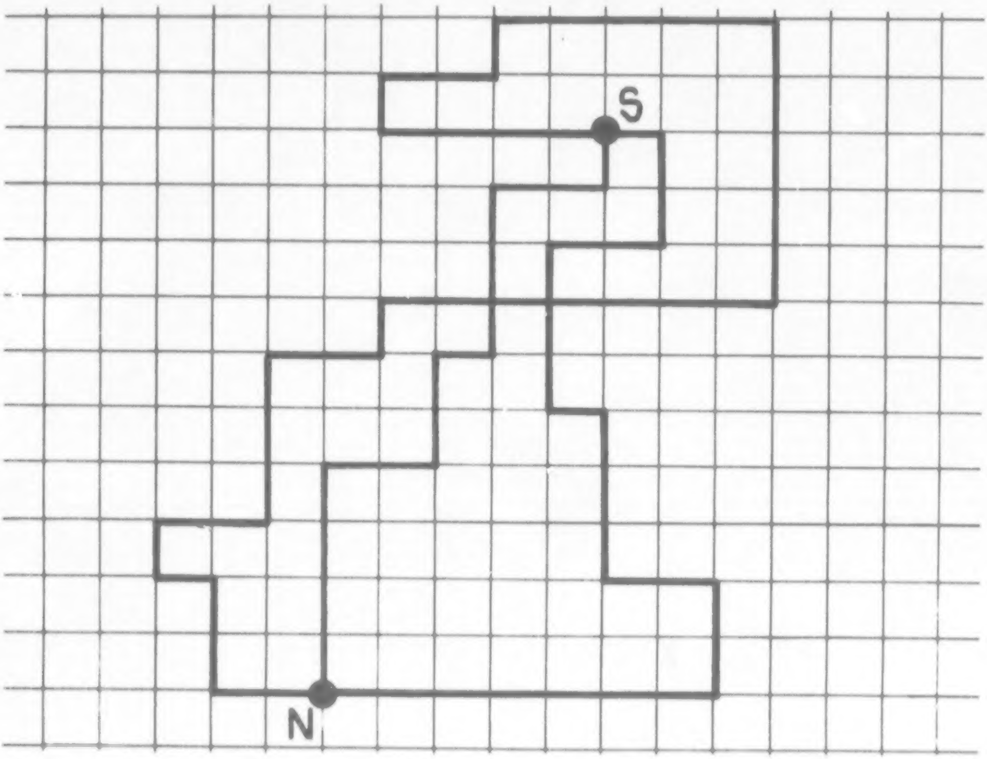
DESCRIPTION OF LESSON

Display your grid board with a dot labeled "N" and a dot labeled "S" as illustrated below.



[#] See the NOTE ON GRIDS in the introduction to the Geometry strand.

Briefly recall that this is a map of a city where Nora lives, that Nora's house is at N and the school is at S. Let the class recall that paths from N to S must follow the lines of the grid. Then ask several students to trace paths from N to S. Using three different colors, draw three of the suggested paths; choose one rather long path, one shortest path, and one in between. The following illustration provides an example,



T: Which of these paths is the longest? Which is the shortest?

S: The red path is the longest.

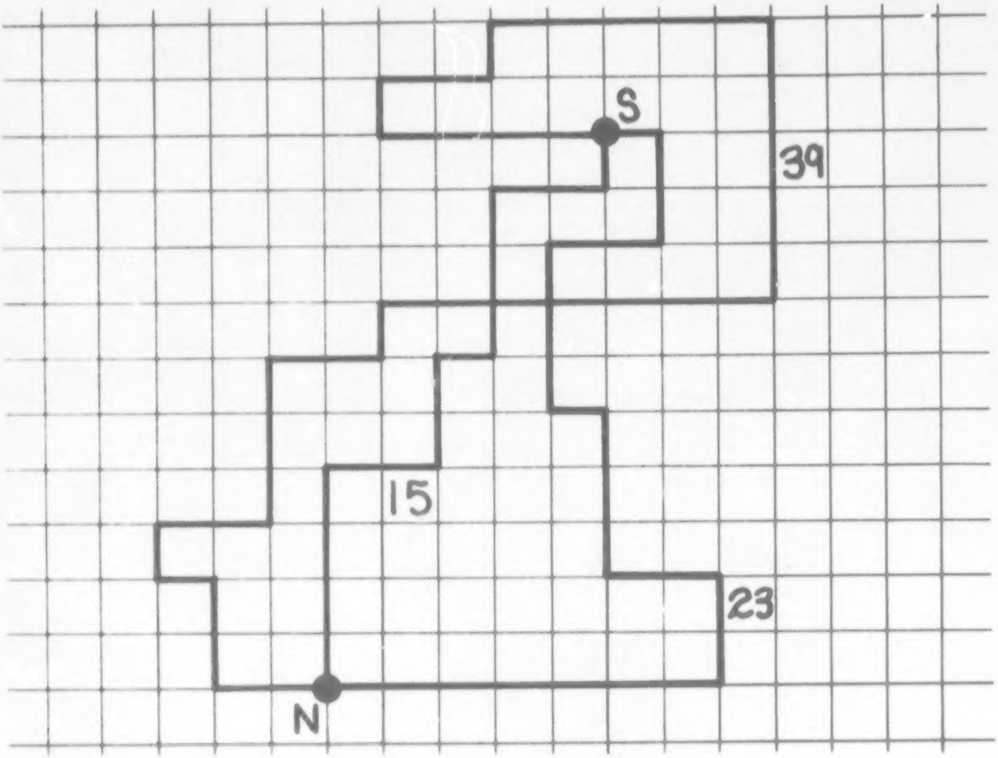
S: The blue path is the shortest.

T: How could we check to be sure?

W 10

S: Count the number of blocks in each path.

Ask students to make a count for each path and to record the length on the board in the appropriate color near the path.



T: Each of these paths is also a path from S to N.

Trace each path starting at S and ending at N to illustrate this point.

T: If Nora is at school and takes the green path home, how long is her walk?

S: 23 blocks.

T: Suppose Nora takes the red path going to school (from N to S) and then returns home along the blue path (from S to N). How long is her round trip?

S: 54 blocks, because 39 + 15 = 54.

T: Very good. How long would a trip be if Nora took the green path from N to S and returned along the green path from S to N?

S: 46 blocks, because 23 + 23 = 46 (or 2 x 23 = 46).

Distribute copies of the Workbook NORA'S NEIGHBORHOOD and let the students work independently for the rest of the period. If many students have difficulty with a particular page, you may wish to have a short full group discussion of that page.

NORA'S NEIGHBORHOOD

Nora's Neighborhood

Here are some paths from Nora's house (N) to Grandmother's house (G).

How long are these paths?

RED PATH: 14 blocks

BLUE PATH: 12 blocks

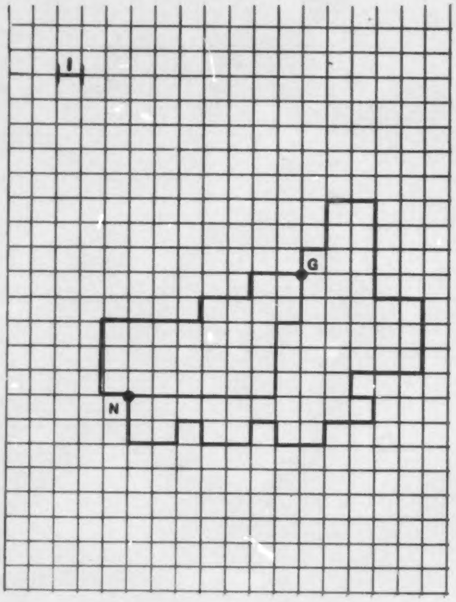
BLACK PATH: 38 blocks

1 Which path is the shortest? (Circle your answer.)

Red [✓ Blue] Black

2 Which path is the longest? (Circle your answer.)

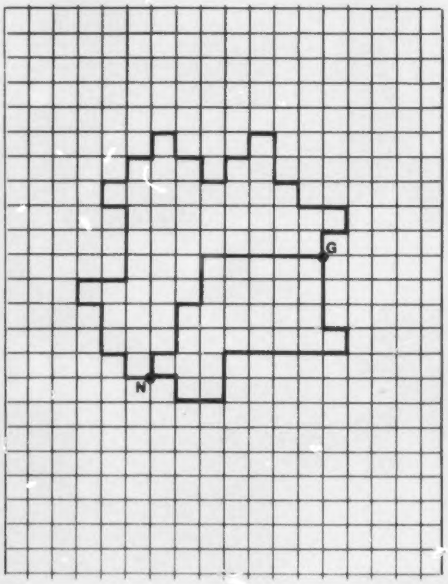
Red Blue [✓ Black]



Today it is sunny. Nora wants to walk to Grandmother's house.

N = Nora's house
G = Grandmother's house

Draw 3 paths from N to G:
- a long RED path
- a short BLUE path
- a medium GREEN path.



Many solutions are possible.

It is raining. Nora is
in a hurry.

N • Nora's house
S • School

Draw two very short paths from N to S.
Color one RED and one BLUE.

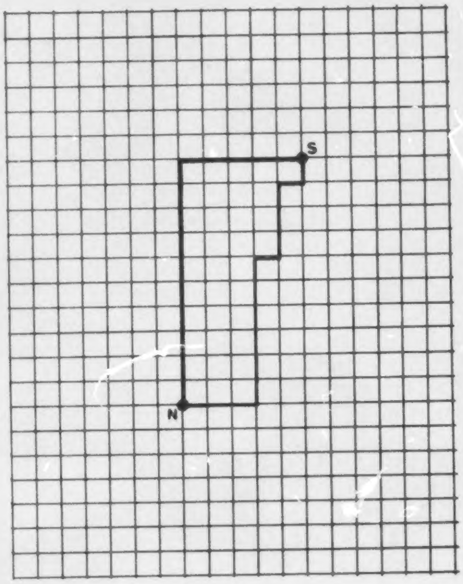
How long are your paths ?

RED path : 15 blocks
BLUE path : 15 blocks

It is still raining when Nora goes home.
She can return on the RED path or the BLUE path.

How long are these paths from S to N ?

RED path : 15 blocks
BLUE path : 15 blocks

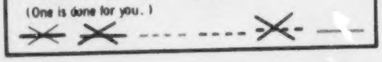


Other solutions are possible.

It is raining.

Nora takes a shortest path
from N to S.

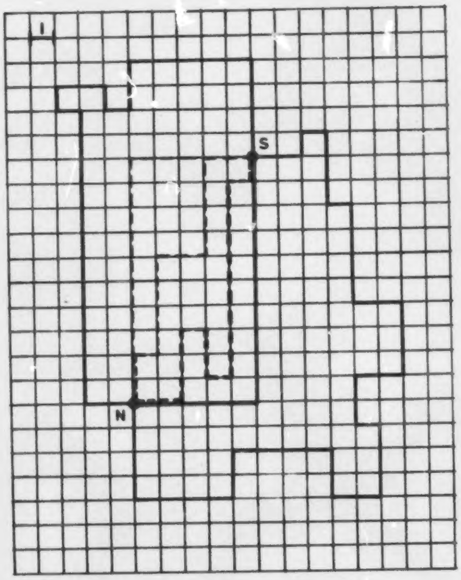
Cross out the paths she would not take.
(One is done for you.)



If Nora starts at school, these are also paths from
S to N.

Nora takes the ______ path from N to S.
She returns on the - - - - path from S to N.
How long is the round trip? 50 blocks

Nora takes the ______ path from N to S.
She returns on the ______ path from S to N.
How long is the round trip? 30 blocks



N = Nora's house
L = Library

Draw these paths from N to L.

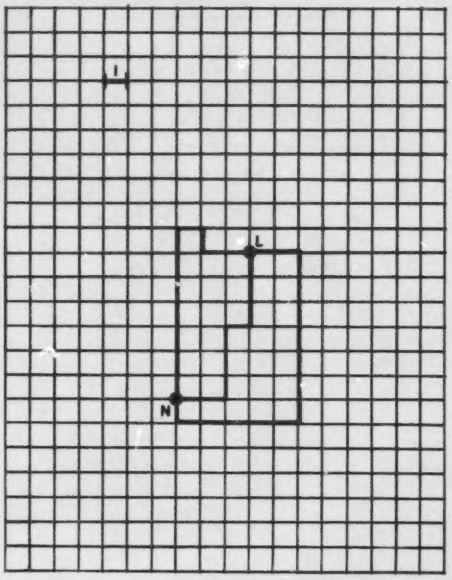
A RED path : 9 blocks
A BLUE path : 11 blocks
A GREEN path : 15 blocks

Can you find a 10 block path from N to L? No

Your paths are also paths from L to N.

Nora takes the RED path from N to L.
She returns on the BLUE path from L to N.
How long is the round trip? 20 blocks

Nora takes the GREEN path from N to L.
She returns on the GREEN path from L to N.
How long is the round trip? 30 blocks



Other solutions are possible.

Angela, Brad, and Charles are Nora's friends.

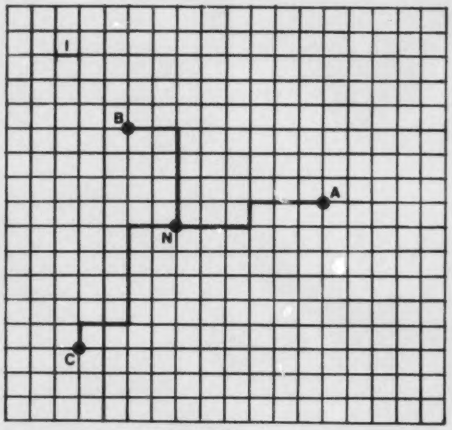
A = Angela's house
B = Brad's house
C = Charles' house

Draw a shortest RED path from N to A.
Draw a shortest BLUE path from N to B.
Draw a shortest GREEN path from N to C.

How long are your paths?

RED path : 7 blocks
BLUE path : 6 blocks
GREEN path : 9 blocks

Which friend lives closest to Nora? Brad



Other solutions are possible.

How long is the shortest round trip Nora can make from

N to A and A to N? 14 blocks
N to B and B to N? 12 blocks
N to C and C to N? 18 blocks

A shortest path from Nora's house to Kristy's house is four blocks. Color RED all the places where Kristy could live.

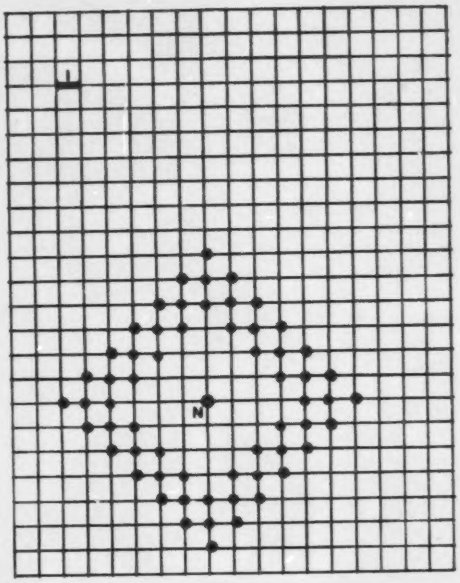
How many red dots? 16

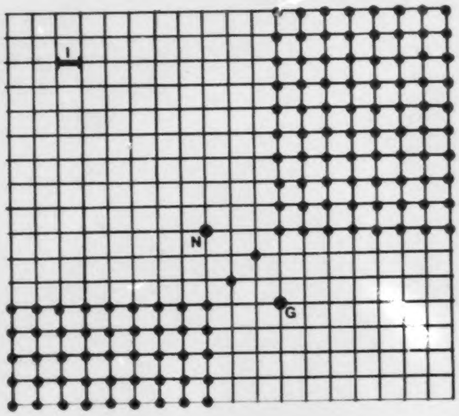
A shortest path from Nora's house to Daniel's house is five blocks. Color BLUE all the places where Daniel could live.

How many blue dots? 20

A shortest path from Nora's house to Gary's house is six blocks. Color GREEN all the places where Gary could live.

How many green dots? 24





Nora and Gary want to meet at a place where they each must walk the same number of blocks. Color RED the places where they can meet.

W11 Detective Story #4

CAPSULE LESSON SUMMARY

Solve a detective story in which the clues involve a string picture and two arrow pictures.

MATERIALS

Teacher: Colored chalk

Student: None

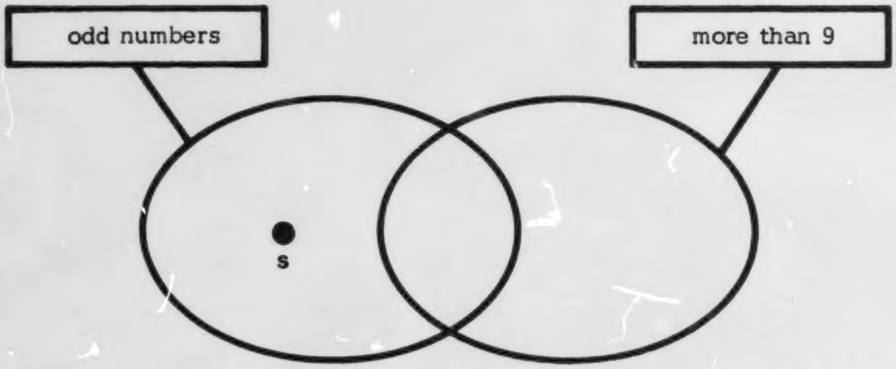
DESCRIPTION OF LESSON

Write a large "7" on a slip of paper and fold the paper so that the "7" is hidden.

T: I wrote the name of a secret number on this paper. You are going to be detectives. From my clues, you will try to discover the secret number.

First clue

Draw this string picture on the board.



T: Your first clue is on the board. What does this string picture tell you about the secret number (point to "s")?

The students might first tell you that the secret number is odd. Ask them to name some odd numbers including some which are relatively large and some which are negative.

T: What else does this string picture tell you about the secret number?

S: It's not more than 9.

If someone says that the secret number is less than 9, you should express some doubt. Commend a student that says the secret number could be 9 or less than 9. It is not necessary for you to suggest that the secret number could be 9 at this time in the lesson if no student suggests it.

T: What are some odd numbers which are not more than 9?

If necessary, ask specifically about negative numbers.

T: Let's make a list of numbers which could be the secret number. Who can give us a number which could be the secret number?

S: 3.

Trace the appropriate strings as you ask the questions.

T: Is 3 an odd number? (Yes)
Is 3 more than 9? (No)
Could the secret number (point to "s") be 3? (Yes)

Record "3" on the board but not in the string picture. Perhaps a student will suggest a number which cannot be the secret number; for example,

S: 15.

T: Is 15 an odd number? (Yes)
Is 15 more than 9? (Yes)

Could the secret number (point to "s") be 15? (No)
Who can show us where 15 belongs in this string picture?

A student should indicate that 15 is in the middle region — the intersection of the two strings.

As students suggest correct possibilities for the secret number, put them in the list forming on the board, spacing them appropriately so that they will be listed in order. After three or four possibilities have been given ask:

T: What is the largest number the secret number could be? (9)

If 9 is not suggested immediately, accept another number that could be the secret number and ask if the secret number could be larger than that number. Continue until someone suggests 9.

At this point you might return to the question of whether the secret number is less than 9. Ask the class if the secret number must be less than 9. Conclude with your class that the secret number could be 9, because 9 is not more than 9.

T: What are some other numbers the secret number could be?

Continue recording correct possibilities until your list includes these numbers.

$$9,\ 7,\ 5,\ 3,\ 1,\ \hat{1},\ \hat{3},\ \hat{5},\ \hat{7},\ \hat{9},\ \widehat{11},\ \widehat{13}$$

NOTE: Your list should contain numbers less than $\hat{9}$, such as $\widehat{11}$, to emphasize that $\widehat{11} < 9$.

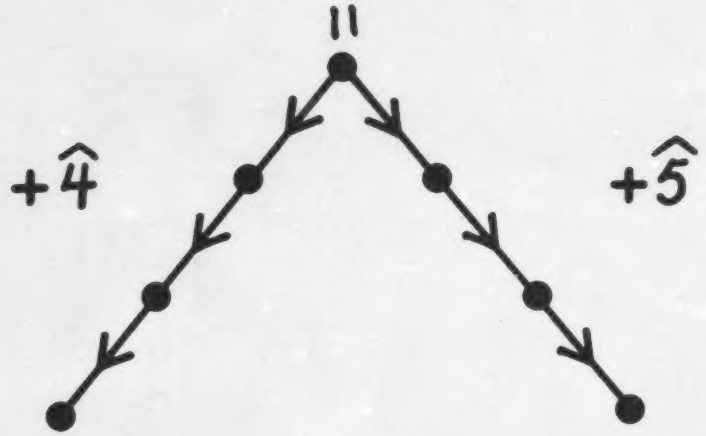
Decide with the class that the list goes on and on, and draw three dots at the end of the list.

$$9,\ 7,\ 5,\ 3,\ 1,\ \hat{1},\ \hat{3},\ \hat{5},\ \hat{7},\ \hat{9},\ \widehat{11},\ \widehat{13},\ \cdots$$

W 11

Second clue

Draw this arrow picture on the board.



T: Your second clue is that the secret number is in this arrow picture. What are the blue arrows for?

Accept any of the following answers:

- plus negative four
- plus four magic (peanuts)
- take away four

T: The blue arrows are for $+\hat{4}$ (read: plus negative four). What are the red arrows for? ($+\hat{5}$)
If we start at 11 and follow the red arrows, will the numbers we meet be getting larger or smaller? (Smaller)
Who can point to where the largest number is in this arrow picture?

A student should point to the dot labeled "11".

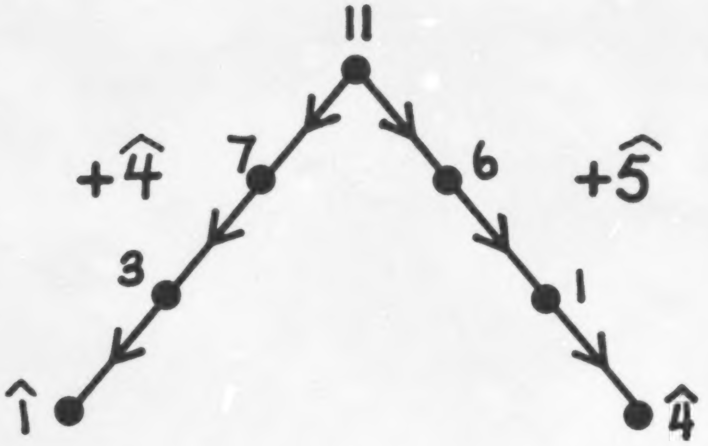
T: How do you know?

S: All the other numbers are smaller than 11.

T: Who can point to where the smallest number is in this arrow picture? (At the dot in the lower right)

This is a difficult question; if a student answers correctly, you may wish to ask why the smallest number is there, but do not expect a well-formed answer. If your class is uncertain where the smallest number is in the arrow picture, wait until all the dots have been labeled and ask the question again.

Invite students to label dots. Refer to a number line or to magic peanuts if a student has difficulty labeling a dot. When all the dots have been labeled, ask the class to notice that the smallest number is at the lower right.



T: The secret number is in this arrow picture and in the list we made. Which numbers could be the secret number?

When a number is suggested, check that the number is included in the list and in the arrow picture. If the number is included in both places, circle it in the list. If the number is not in the arrow picture, cross it off the list. Continue until all the numbers on the board have been considered.

$$\cancel{9}, \textcircled{7}, \cancel{5}, \textcircled{3}, \textcircled{1}, \textcircled{\hat{1}}, \cancel{\hat{3}}, \cancel{\hat{5}}, \cancel{\hat{7}}, \cancel{\hat{9}}, \cancel{\hat{11}}, \cancel{\hat{13}}, \ldots$$

W 11

T (pointing to the three dots): Could the secret number be any of the numbers which belong in this list which we didn't write on the board?

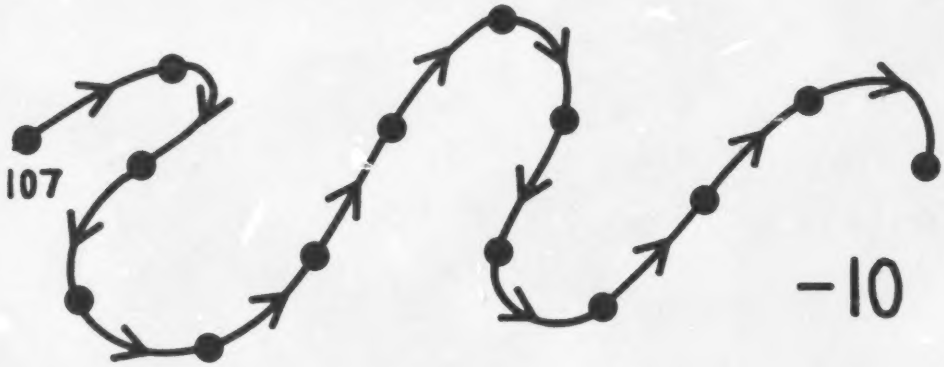
S: No, because the smallest number in the arrow picture is $\hat{4}$ and the other numbers in the list are smaller than $\hat{4}$.

Cross off the three dots.

T: Now we know that the secret number is 7 or 3 or 1 or $\hat{1}$.

Third clue

Draw this arrow picture on the board. You should first erase the previous arrow picture and the parts of the list which have been crossed off.

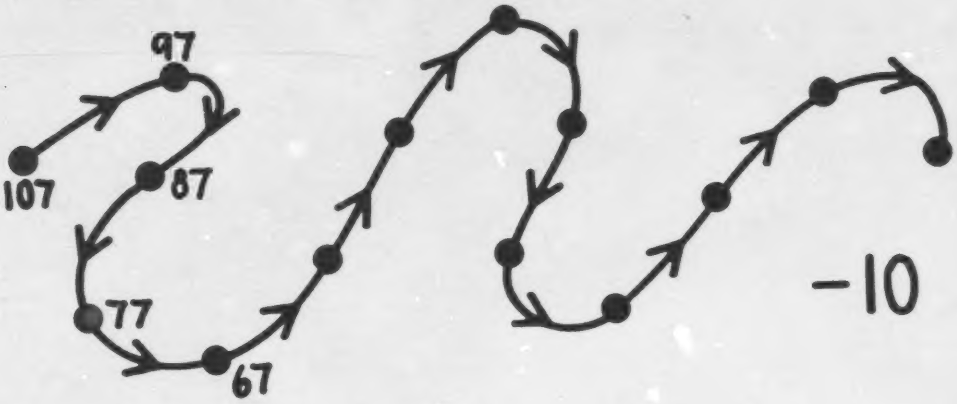


T: The third clue is that the secret number is in this arrow picture.

If some students think they know the secret number, ask them to whisper it to you or to write it on a piece of paper.

T: Let's label some of these dots. What are the red arrows for? (-10)
If we start at 107 and follow the red arrows, will the numbers we meet be getting smaller or larger? (Smaller)
What number is 107 - 10? (97)

If necessary, count backward with the class to find the answer. Continue labeling the dots until the dot for 67 has been labeled.



T: Do you see a pattern?

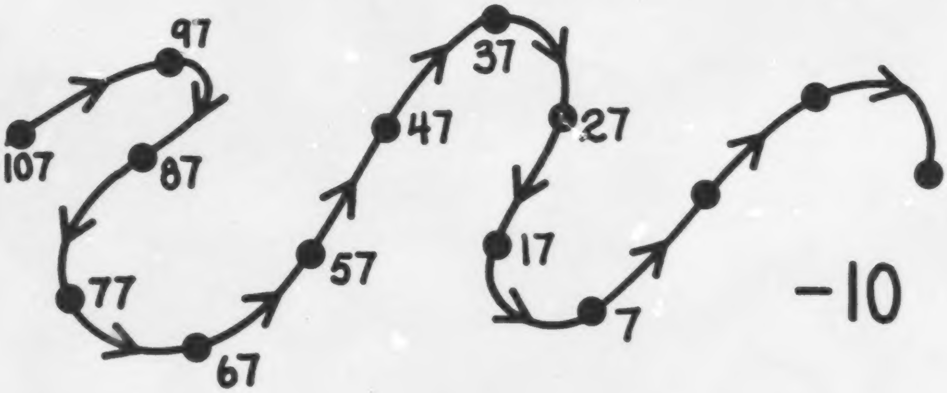
S: All the numbers so far end in 7.

T: Does this tell you anything about the secret number?

S: The secret number must be 7.

T: Who can label the dot for 7 in this arrow picture?

If the students have difficulty finding the correct dot, progressively label the dots from 67 to 7.



W 11

T: Could one of these dots be for 3? (No)
Why not?

Accept any reasonable explanation, but express some doubt if a student says that all the numbers in the arrow picture have 7 as their ones' digit.

Point to "7" and trace the arrow which starts there.

T: What number is 7 - 10? ($\hat{3}$)

If necessary, refer to a number line or count backward with your class.

T: Is one of these dots for the number 1? (No)
Why not? (We skipped over it)
Will we ever meet $\hat{1}$? (No)
Why not? (We skipped over it)

Reveal that "7" is written on the slip of paper.

W12 Fishing for Numbers (Lesson 1)

CAPSULE LESSON SUMMARY

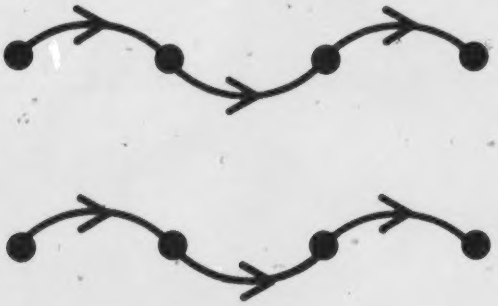
Put the numbers from a string picture in their proper places in an arrow picture which has 2 x and 3 x arrows. Begin the Workbook FISHING FOR NUMBERS.

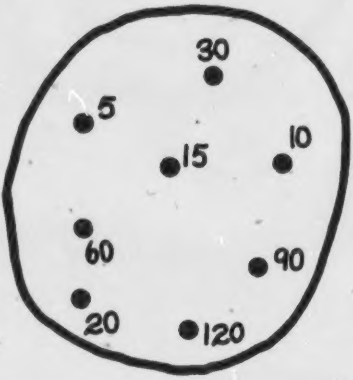
MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS; pencil

DESCRIPTION OF LESSON

Draw these pictures on the board.





T: How many dots are in the arrow picture? (Eight)
How many numbers are in the string picture? (Eight)
Each of the dots in this arrow picture is for one of the numbers in this string picture. Each of the numbers has just one dot in this arrow picture. How can we find out where these numbers go in this arrow picture?

S: We need to know what the arrows are for.

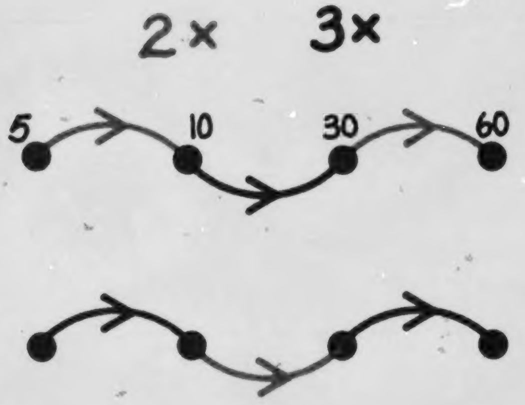
T: The blue arrow if for 2 x and the red arrow is for 3 x.

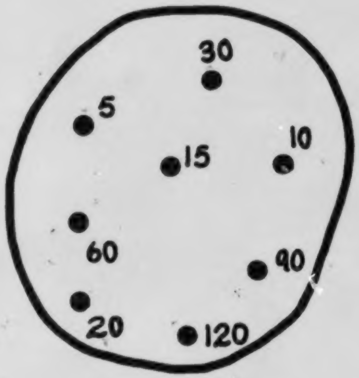
W 12

Write the key to the blue and the red arrows near the arrow picture.

T: Who would like to label a dot in one of these arrow roads?

Do not be concerned if the first number chosen is placed incorrectly in the picture, because the subsequent discussion will be of benefit to the class. For example, suppose a student labels the starting dot of the first arrow road "5". Let students label the other dots in that road. When a dot is labeled, ask the class to check if that number is in the string picture before labeling the next dot in the road.





T: That worked out fine. Now, the other four numbers must go in the bottom arrow road.

Ask for a volunteer to label one of the dots in the second arrow road. Suppose the student labels the starting dot of the second road "10". Point to the dot in the first arrow road labeled "10" as you say:

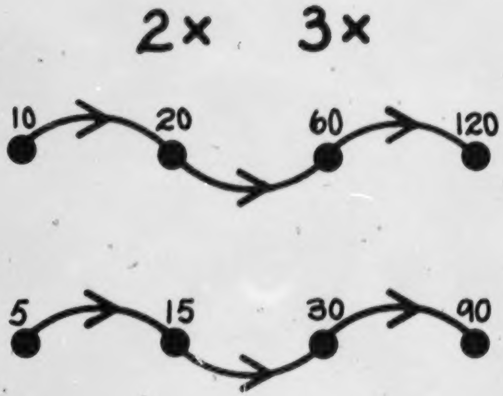
T: We cannot have two different dots for the number 10.

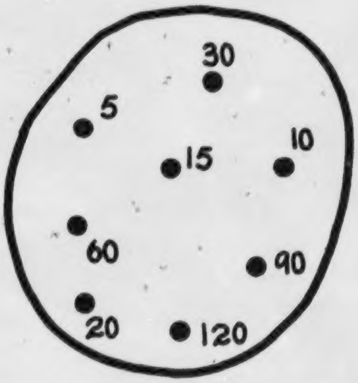
Erase "10" from the second arrow road. Perhaps the next volunteer labels the starting dot "15".

T: What number is 3 x 15? (45) Is 45 in the string picture? (No) Then this road cannot start at "15" (erase "15" from the arrow road).

Continue following the suggestions of your class until someone suggests erasing all the labels (including those from the first road) for the dots and starting over again.

It is impossible to predict how your class will begin labeling the dots in the arrow picture. Pursue each suggestion and see where it leads. If it leads to an incorrect situation, let the students discover this and discuss what should be done. Perhaps someone will suggest that the smallest number is the starting number of one of the roads; if so, ask that student to convince the rest of the class. (Do not suggest this yourself.) Continue until all the dots are labeled with the numbers from the string picture.





Distribute copies of the Workbook FISHING FOR NUMBERS. Remind the students of the similar workbook from Part I (also called FISHING FOR NUMBERS.) Explain that the string picture and the arrow picture are sometimes on opposite pages and sometimes on the same page. Emphasize that very likely they will need to use their erasers, and that they always should try more than one starting number before they ask for help.

At the end of the lesson collect the workbooks for your review and save them for use in W13.

W 13

W13 Fishing for Numbers (Lesson 2)

CAPSULE LESSON SUMMARY

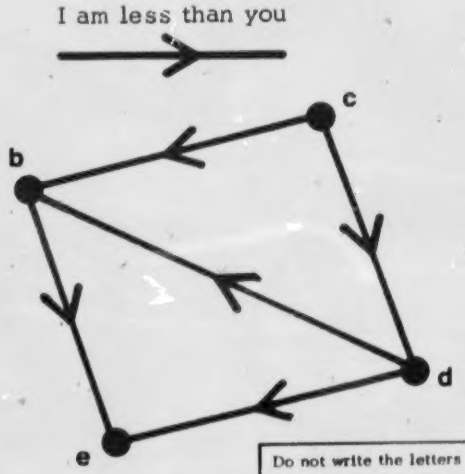
Put the numbers from a string picture in their proper places in an arrow picture which has "I am less than you" arrows. Continue working in the Workbook FISHING FOR NUMBERS. (This is the second of two lessons using this workbook.)

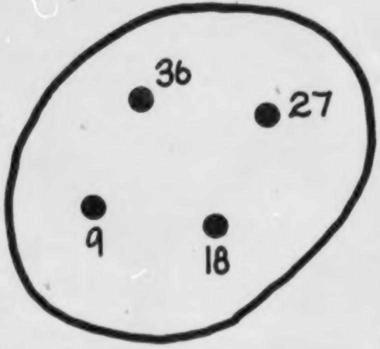
MATERIALS

Teacher: Colored chalk
Student: Workbook FISHING FOR NUMBERS; pencil

DESCRIPTION OF LESSON

Draw these pictures side by side on the board.



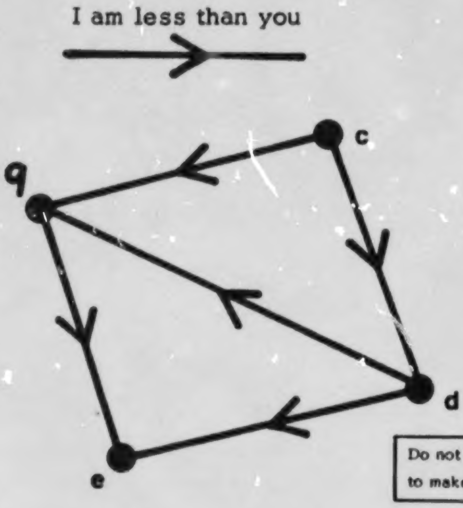


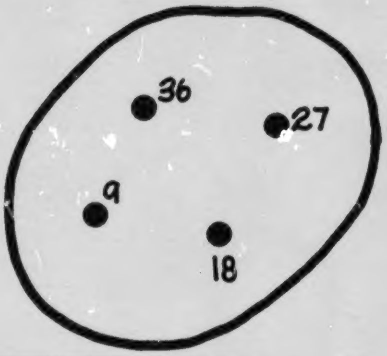
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: How many dots are in the arrow picture? (Four)
How many numbers are in the string picture? (Four)
Each of the dots in this arrow picture is for one of the numbers in this string. Each of the numbers has just one dot in this arrow picture. Who would like to label one of the dots in this arrow picture?

Do not be concerned if the first number chosen is placed incorrectly in the picture, because the subsequent discussion will be of benefit to the class.

For example, suppose a student labels b "9".



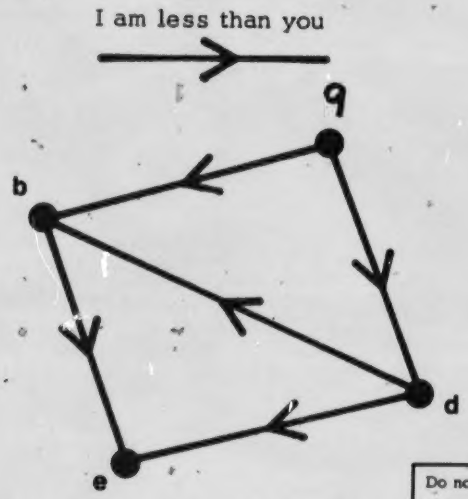


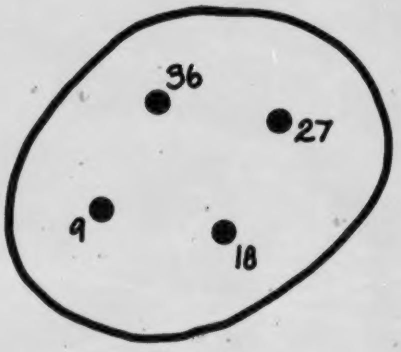
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

T: Could this dot (b) be for "9"? (No)
How do you know?

S: If this number were 9, then this number (at c) would be less than 9, but 9 is the smallest number in this string picture.

Erase "9" from the arrow picture and ask for another volunteer to label one of the dots in the arrow picture. Perhaps a student will label c "9".





Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

W 13

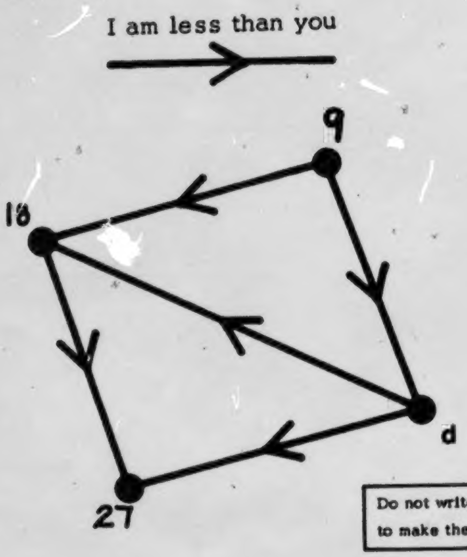
T: Could this dot (c) be for 9?

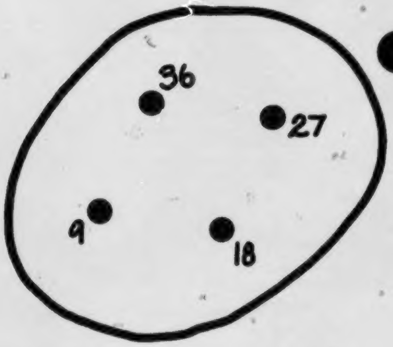
S: We can't tell yet.

T: How can we find out?

S: We need to label the other dots.

Ask someone to label another dot. Continue this activity until all the dots are labeled or until the class discovers that a labeling is incorrect. Suppose b, c, and e have been labeled as in the next illustration. A sample dialogue follows.



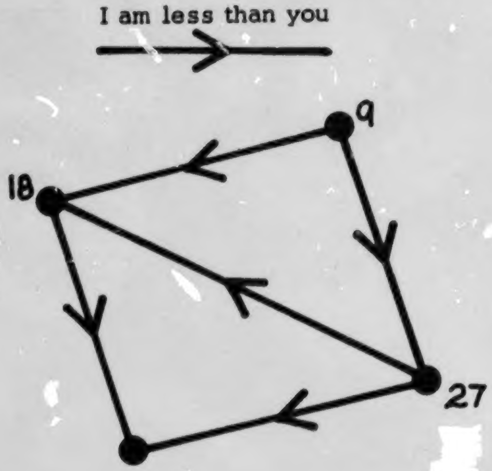


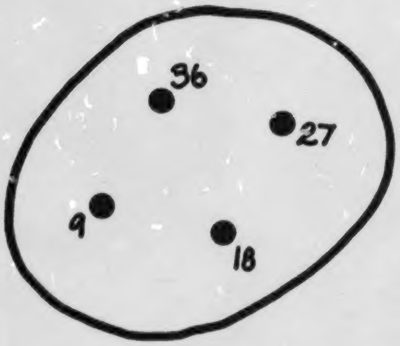
Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.

S: This (point to the dot labeled "27") can't be 27 because this number (at d) is less than 27 and we've already used all the numbers less than 27.

S: I think this number (at d) is 27.

Erase "27" from the arrow picture and label d "27".


I am less than you
9
18
27



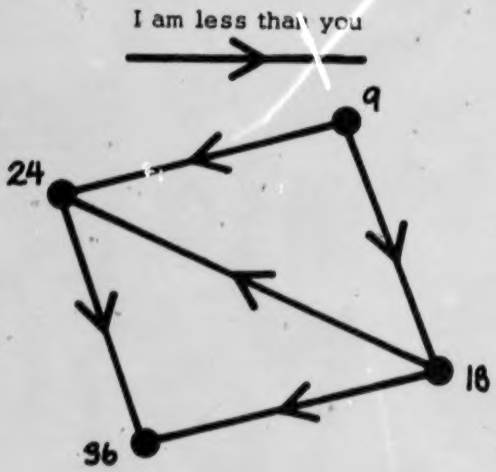
T: Could this dot be for 27?

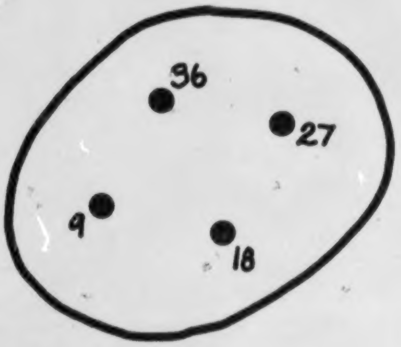
S: No, 27 isn't less than 18.

Erase "27" from the arrow picture.

T: What should we do now?

Follow the students' suggestions and continue this activity until all the dots are correctly labeled.


I am less than you
9
24
18
36



W 13

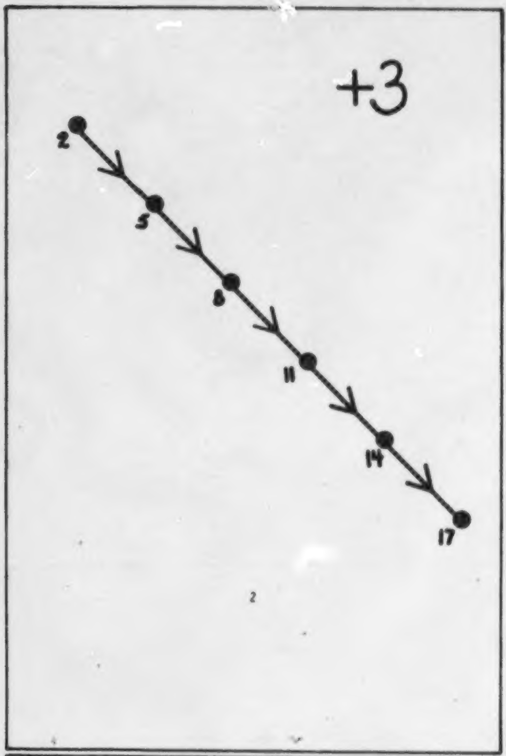
Distribute the students' copies of FISHING FOR NUMBERS. Explain to the students that the string picture and the arrow picture are sometimes on opposite pages and sometimes on the same page. Emphasize that very likely they will need to use their erasers, and that they should always try more than one starting number before they ask for help.

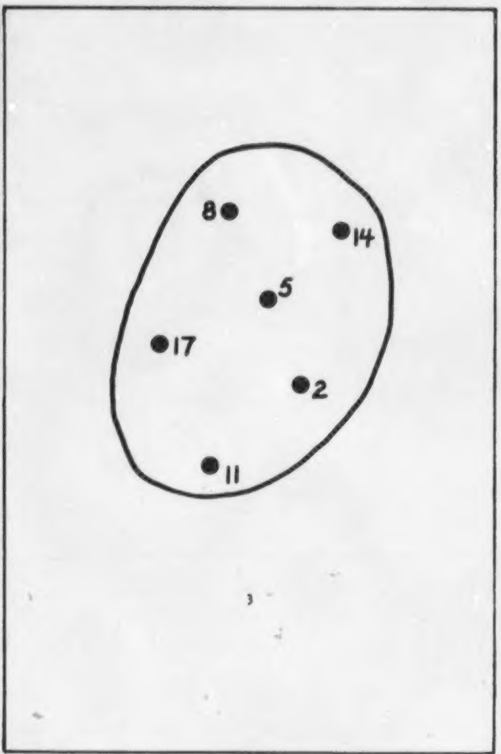
At the end of the lesson, collect the workbooks for your review.

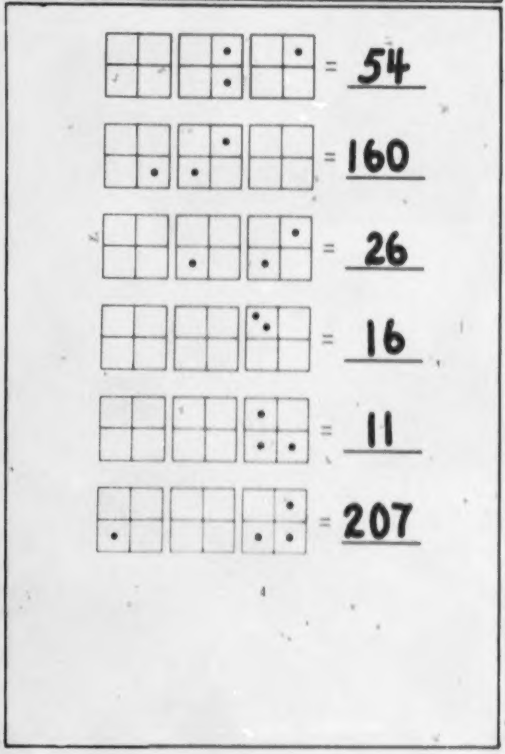
FISHING FOR NUMBERS

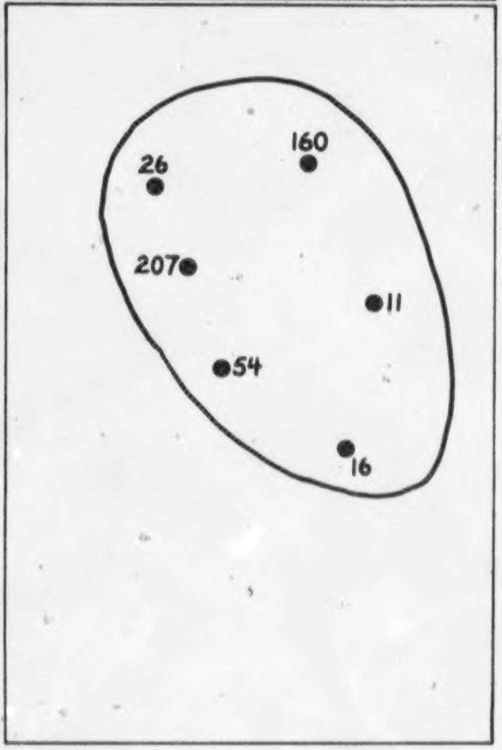
(Part II)

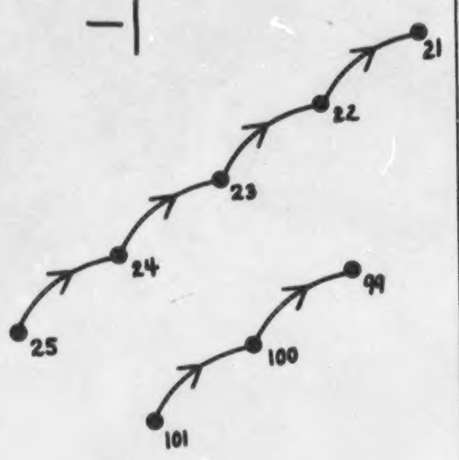
Fishing for Numbers (Part II)


+3
2
5
8
11
14
17
2

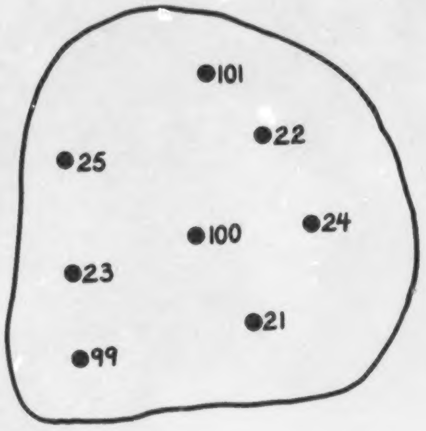

8
14
5
17
2
11
3


= 54
= 160
= 26
= 16
= 11
= 207
4

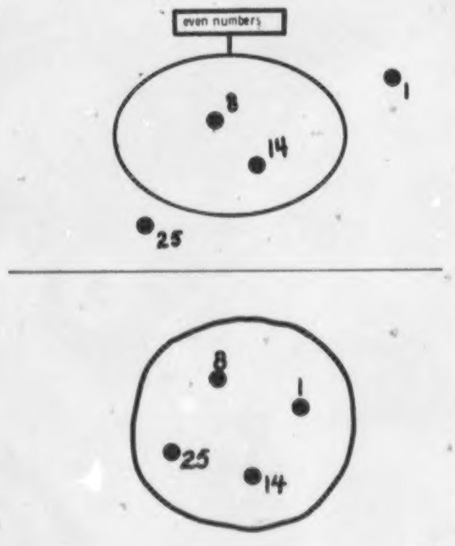

26
160
207
11
54
16
5


-1

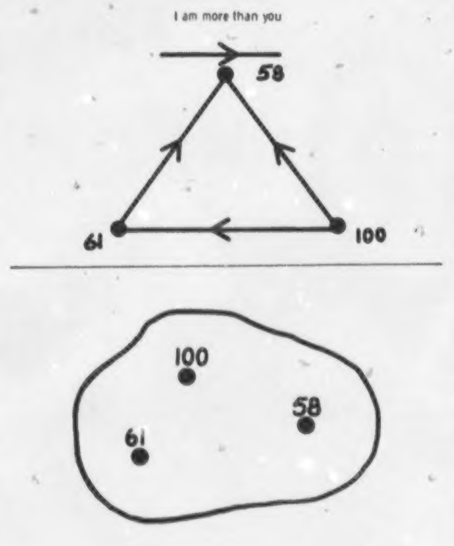
6



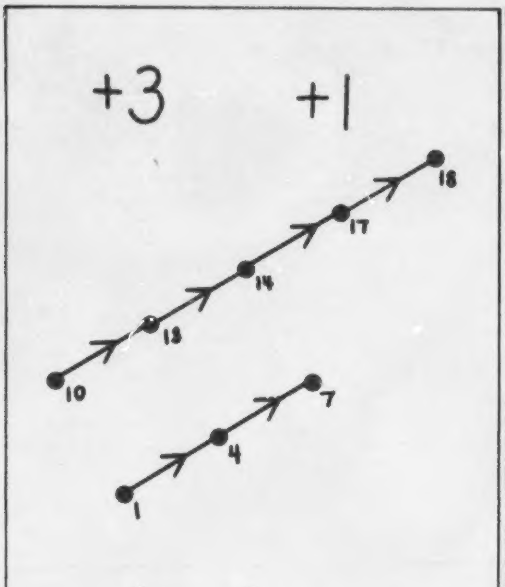
7


even numbers

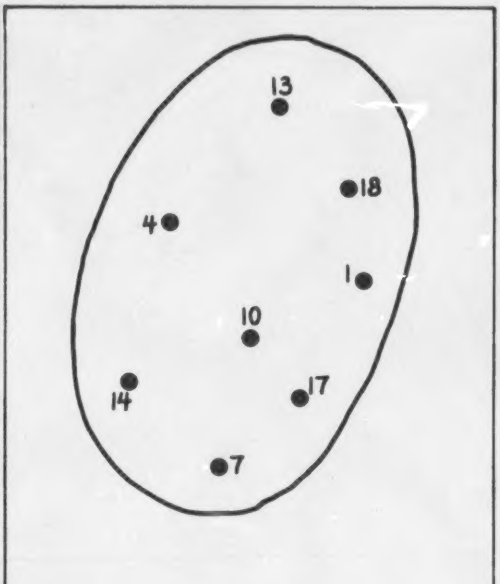
8


I am more than you

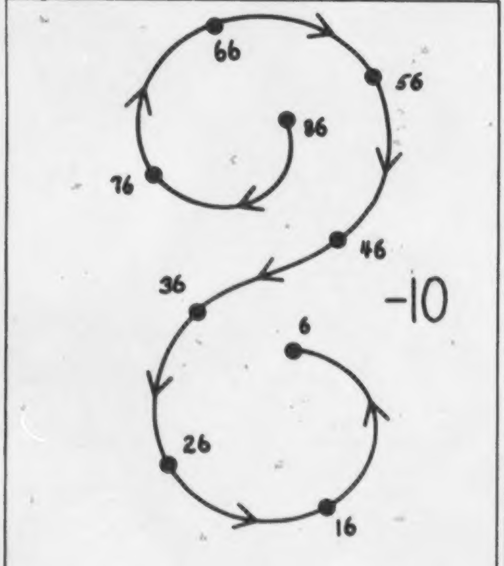
9


+3
+1

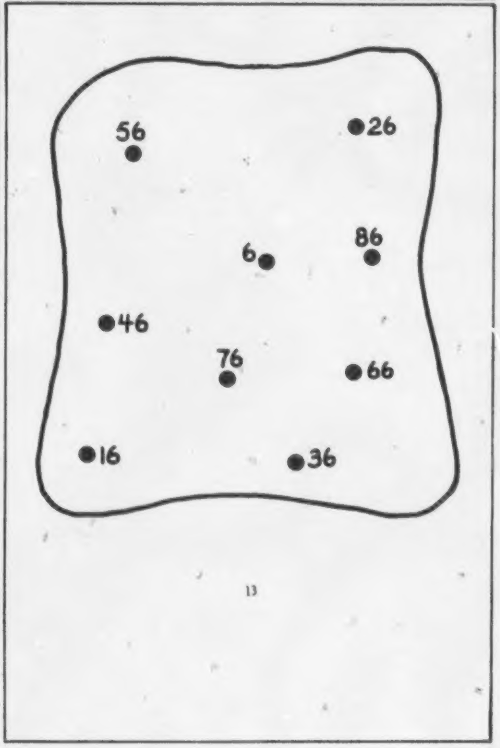
10



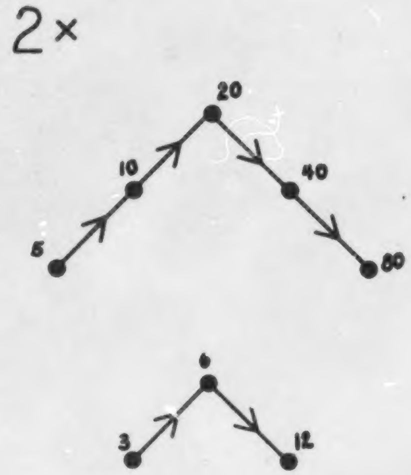
11


-10

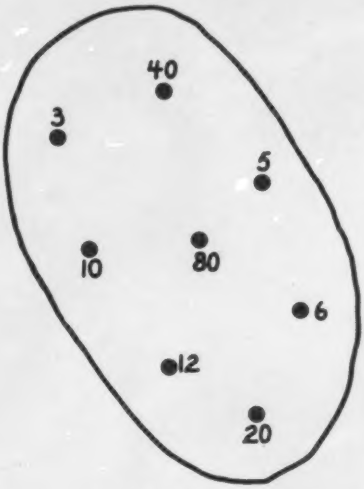
12



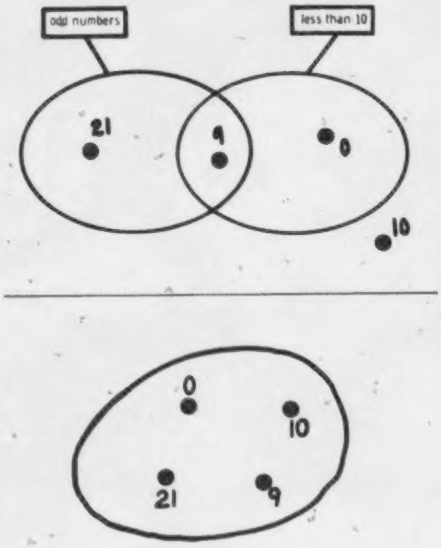
13


2×

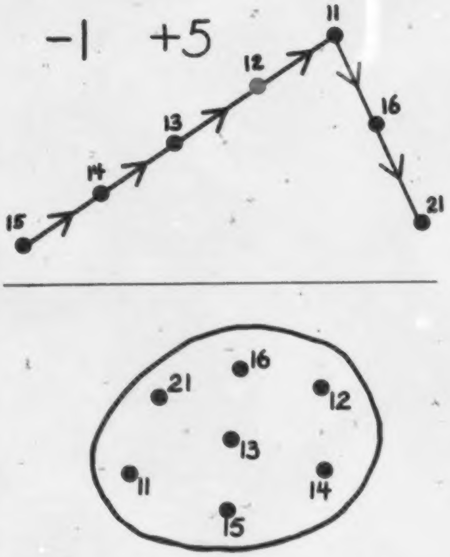
14



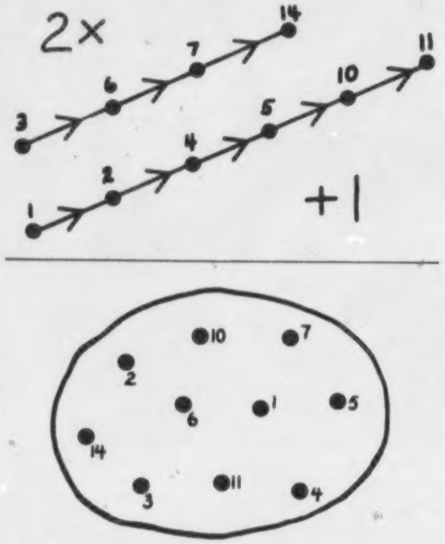
15


odd numbers
less than 10

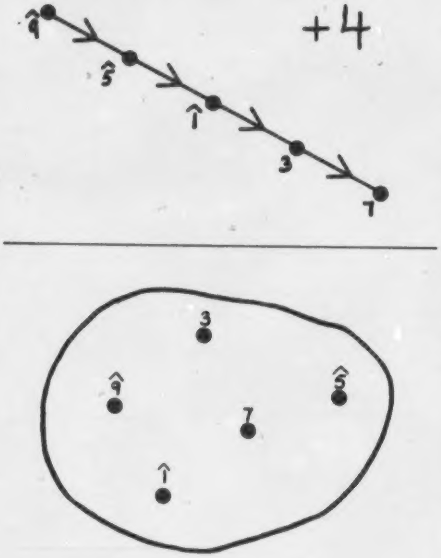
16


−1 +5

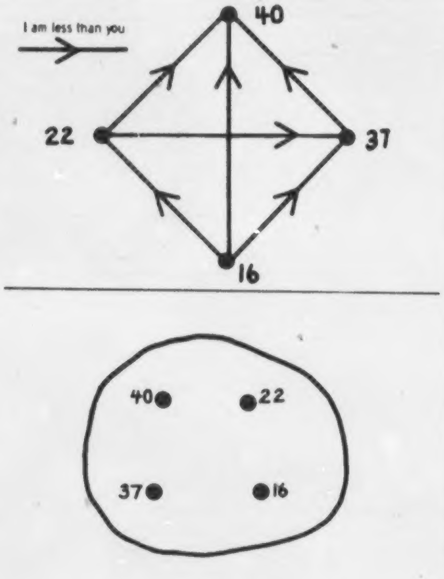
17


2×
+1

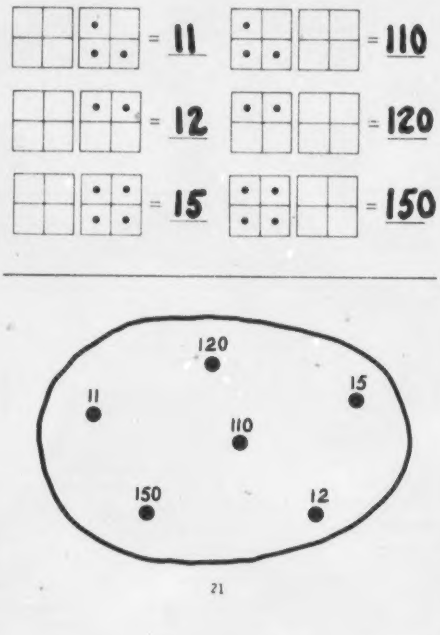
18


9̂
+4
5̂
1̂
3
7
3
9̂
5̂
7
1̂

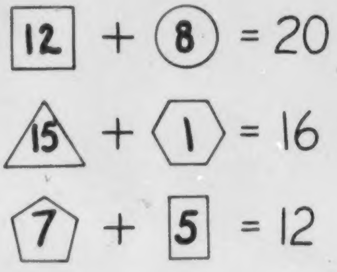
19

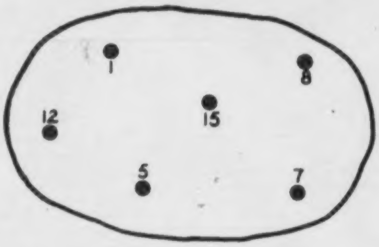

I am less than you

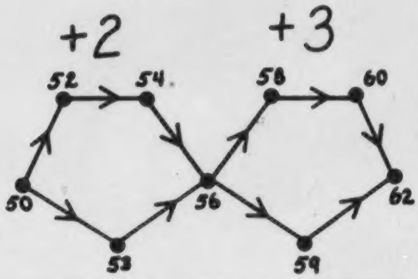
20

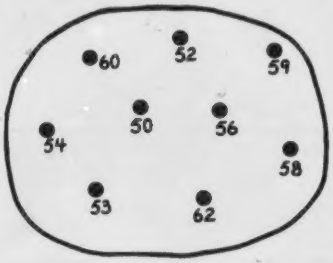

= 11
= 110
= 12
= 120
= 15
= 150
120
11
15
110
150
12

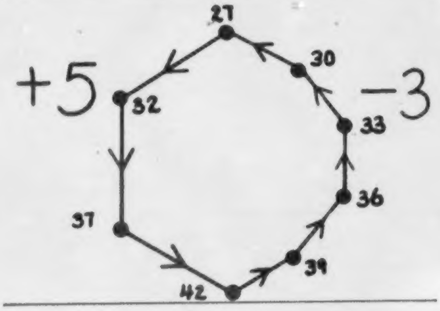
21

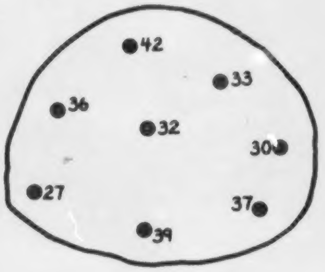

12 + 8 = 20
15 + 1 = 16
7 + 5 = 12

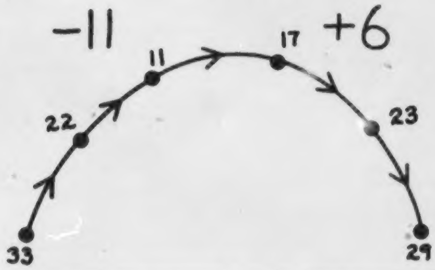


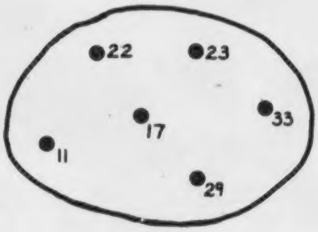

+2
+3

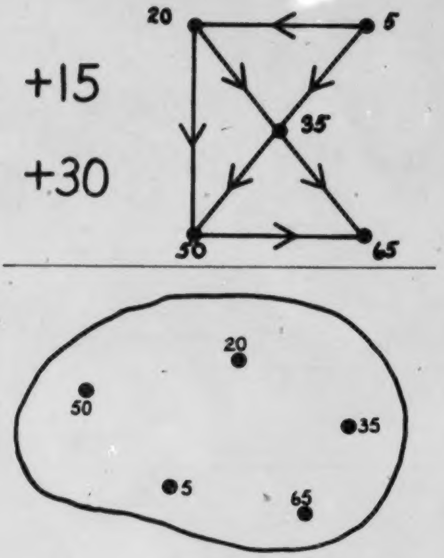



+5
−3

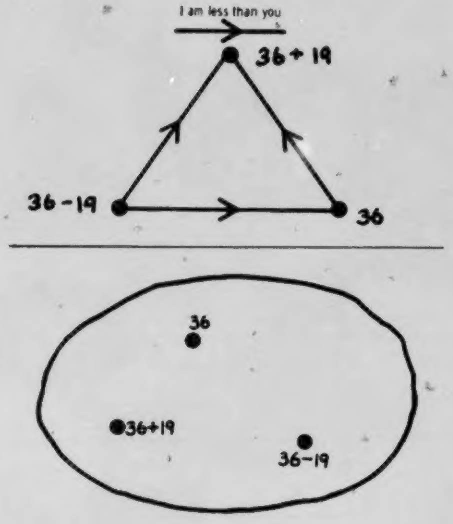



−11
+6

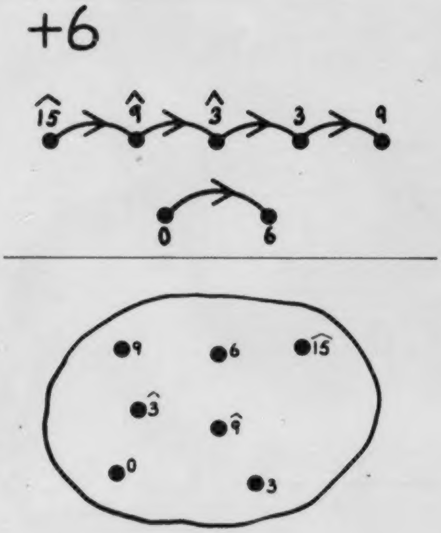



+15
+30

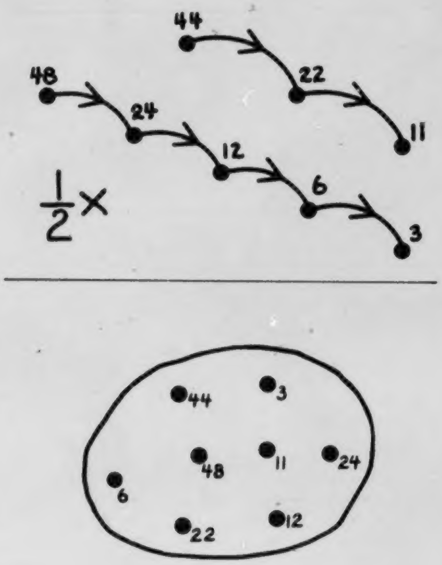
26


I am less than you
36+19
36-19
36
36
36+19
36-19

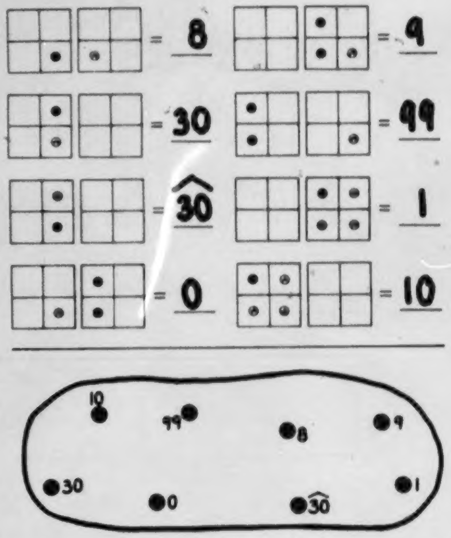
27


+6
$\widehat{15}$
$\hat{9}$
$\hat{3}$
3
9
0
6
9
6
$\widehat{15}$
$\hat{3}$
$\hat{9}$
0
3

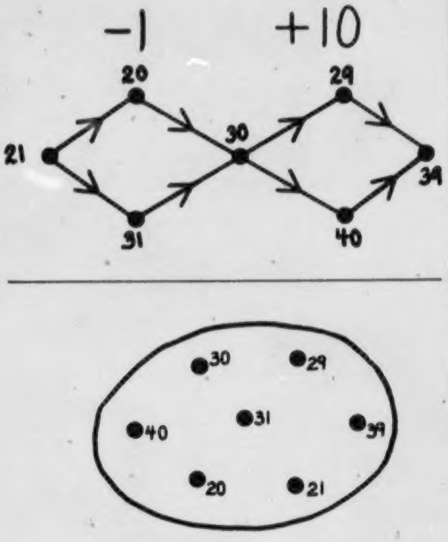
28


$\frac{1}{2}\times$

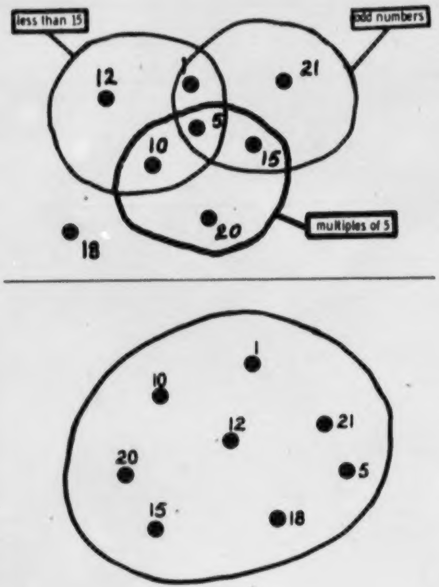
29


= 8
= 9
= 30
= 99
= $\widehat{30}$
= 1
= 0
= 10
10
99
8
9
30
0
$\widehat{30}$
1

30


-1
+10

31


less than 15
odd numbers
multiples of 5

32

W14 Detective Story #5

CAPSULE LESSON SUMMARY

Label floors in a partial picture of the Empire State Building. Find how many floors a person travels on the elevator to get from one floor to another. Discover on which floor of the Empire State Building an executive's office is located through several clues involving order and multiples.

MATERIALS

Teacher: 0-109 Numeral Chart; marking pens or crayons; Minicomputer set; colored chalk

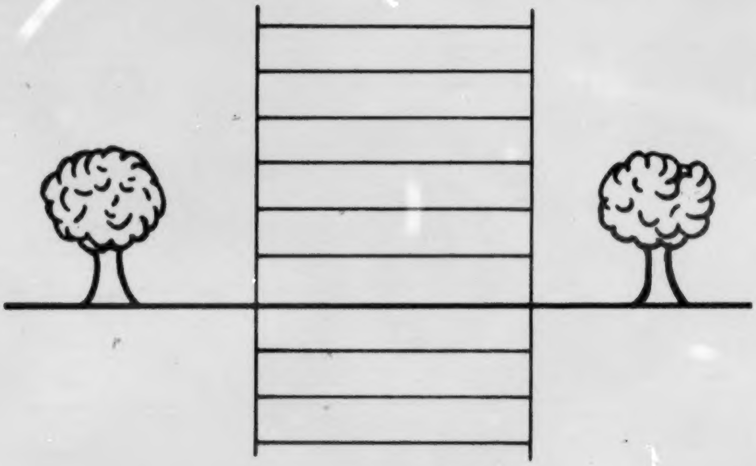
Student: None

DESCRIPTION OF LESSON

Exercise 1

Ask the students if they know about any tall buildings. Encourage them to tell you the names of some tall buildings and where they are located. Some students may have visited some tall buildings and can tell the class about their experiences. If no one suggests the Empire State Building in New York City, suggest it yourself and tell the class that the Empire State Building was the tallest building in the world for many years.

Draw this picture on the board.

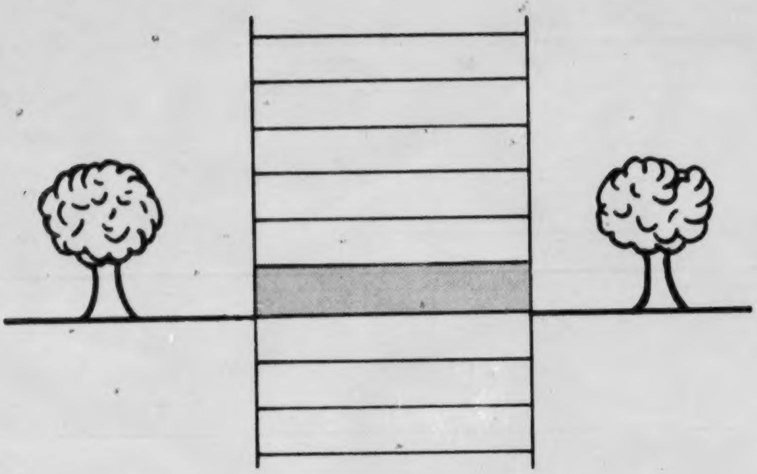


W 14

Explain that the picture on the board shows only the floors of the Empire State Building which are near ground level because it has too many floors to picture all of them.

T: Where is the ground floor in the picture?

The student who volunteers should point to the rectangle which is shaded in this illustration.



NOTE: Some students may be somewhat confused by the use of the word "floor" in this lesson. In this case, "floor" refers to a story or level of the building rather than just the supporting surface at each level.

Discuss briefly the need to have a name for each floor. Indicate that the ground floor is mostly open area and has no offices. Often the ground floor has only a lobby, elevators, and stairways.

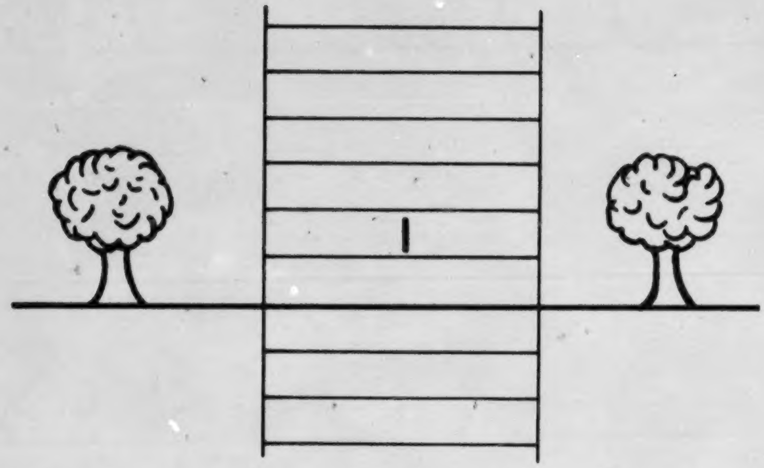
T: Who can show us where the first floor is?

A student should point to the rectangle directly above the rectangle for the ground floor.

T: Where is the second floor? . . . the fifth floor?
Let's assign a number to each of these floors.
What number should we give the first floor?

S: 1.

Write "1" inside the rectangle for the first floor.



With students assisting, label all the floors above the first floor.

T: In many buildings, the ground floor has a letter name instead of a number name. Sometimes the ground floor is called "G" and sometimes it is called "L". Why?

S: "G" indicates ground level and "L" indicates lobby.

T: Let's give our ground floor a number name instead of a letter name. What number should we assign to the ground floor?

S: 0.

Label the ground floor "0" in your picture on the board.

W 14

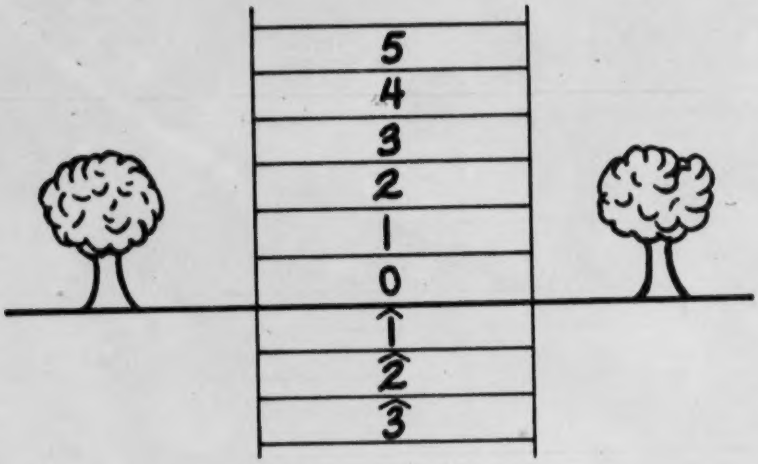
T: In many buildings the floors below the ground are called "B1", "B2", "B3", "B4", and so on. Why?

S: "B" is for basement.

T: What numbers should we assign to the floors below ground level?

S: Negative numbers.

Ask the class to assist you in labeling all the floors shown in the picture which are below ground level.



T: If an office worker walks back to the building after having lunch and rides the elevator to the fifth floor, how many floors would this person go up?

S: Five.

Demonstrate this by counting the floors in the drawing of the building.

T: After work the office worker rides the elevator to the second basement floor (point to "$\hat{2}$") where the cars are parked. What is the number of the floor where the cars are parked? ($\hat{2}$) Should the person go up or down on the elevator? (Down) How many floors does this person go down from floor 5 to floor $\hat{2}$? (Seven)

Demonstrate this by counting the floors in the drawing of the building. Repeat this activity with the following stories or similar ones which involve trips to floors which are included in your drawing.

T: The janitor has an office on floor $\hat{3}$. He is in his office when he receives an emergency call from floor 1. Should he take the elevator up or down? (Up) How many floors should the janitor go up?

S: Four.

T: After he takes care of the emergency on floor 1, he wants to go to floor 4 to check on the heating system. How many floors should he go up?

S: Three.

T: If the janitor now wants to return to his office from floor 4, should he go up or down? (Down)
How many floors down?

S: Seven.

Return to your discussion about the Empire State Building.

T: How many floors above ground do you think the Empire State Building has?

The Empire State Building has 102 floors above ground level.

W 14

Exercise 2: Detective Story

T: Virginia Lockwood is an executive who has an office in the Empire State Building. She likes to write detective stories about secret numbers for her friends. In this story, the secret number is the number of the floor where her office is located. Let's see if you can figure out Ms. Lockwood's secret number.

First clue

T: Her first clue is: The number of the floor where my office is located is one of these numbers.

Tape a 0-109 numeral chart to the board or any other convenient surface.

T: Could Ms. Lockwood's office be in the basement? (No, there are no negative numbers on the chart.)
Could Ms. Lockwood's office be on floor 109? (No)
Why not?

S: There are only 102 floors in the Empire State Building so the secret number can't be 103, 104, 105, 106, 107, 108, or 109.

Indicate that the whole numbers from 103 to 109 cannot be the secret number by crossing them out on the 0-109 Numeral Chart.

T: Could the secret number be 0?

S: No, because there aren't any offices on the ground floor.

Cross out "0" on the 0-109 Numeral Chart.

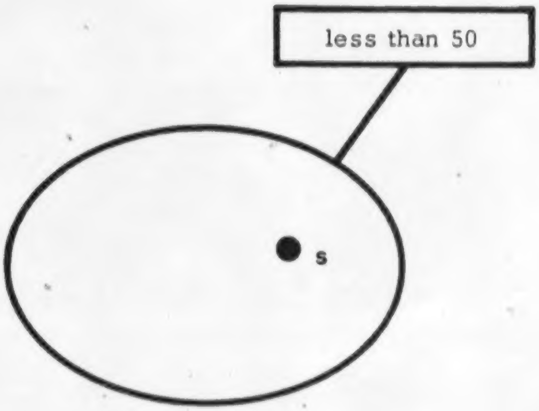
T: Are there any other numbers we can cross out now? (No)

~~0~~	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

Second clue

T: Her second clue is a string picture.

Begin a string picture on the board.



W 14

T: What does this string picture tell us about the secret number (point to "s")?

S: The secret number is less than 50 so Ms. Lockwood's office is on a floor between 0 and 50.

T: Which of these numbers (point to the 0-109 Numeral Chart) cannot be the secret number?

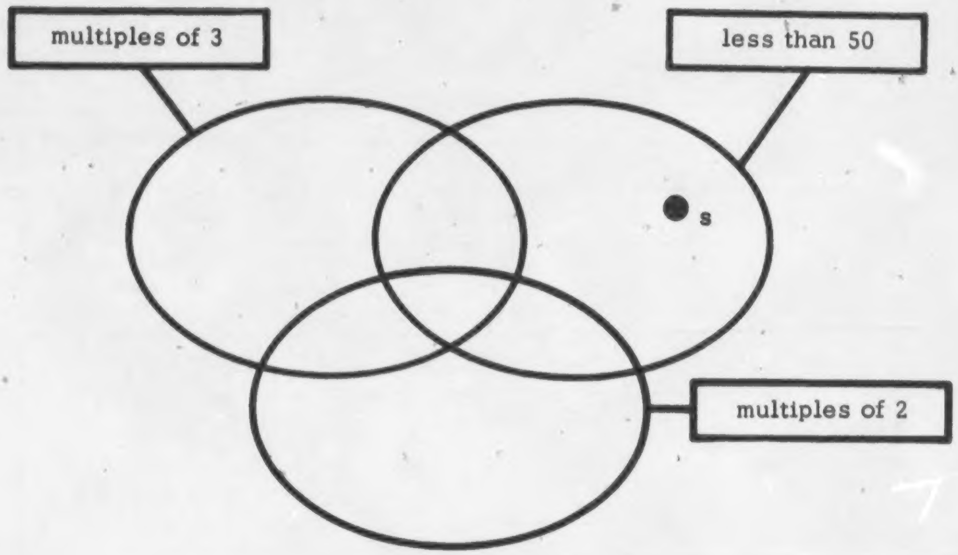
S: 50 and all the numbers larger than 50.

Indicate that the whole numbers from 50 to 102 cannot be the secret number by crossing them out on the 0-109 Numeral Chart.

~~0~~	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
~~50~~	~~51~~	~~52~~	~~53~~	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	~~59~~
~~60~~	~~61~~	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	~~67~~	~~68~~	~~69~~
~~70~~	~~71~~	~~72~~	~~73~~	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	~~79~~
~~80~~	~~81~~	~~82~~	~~83~~	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	~~89~~
~~90~~	~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	~~97~~	~~98~~	~~99~~
~~100~~	~~101~~	~~102~~	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

T: By adding two more strings to the picture, Ms. Lockwood tells us more about the secret number.

Extend your string picture as in the next illustration.



T: What does this green string tell us about the secret number?

S: The secret number is not a multiple of 2.

T: What kind of numbers are not multiples of 2?

S: Odd numbers.

T: The secret number is outside of the green string, so we know the secret number is an odd number. Which of these numbers (point to the 0-109 Numeral Chart) cannot be the secret number?

S: The even numbers.

T: Name them and I'll cross them out.

At first the students may suggest even numbers in no particular order. Emphasize that all the numbers which end in 0, 2, 4, 6, or 8 are even numbers and lead the students to see that entire columns of numbers can be ruled out at a time.

Continue until your chart looks like this one.

~~0~~	1	~~2~~	3	~~4~~	5	~~6~~	7	~~8~~	9
~~10~~	11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19
~~20~~	21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29
~~30~~	31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39
~~40~~	41	~~42~~	43	~~44~~	45	~~46~~	47	~~48~~	49
~~50~~	~~51~~	~~52~~	~~53~~	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	~~59~~
~~60~~	~~61~~	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	~~67~~	~~68~~	~~69~~
~~70~~	~~71~~	~~72~~	~~73~~	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	~~79~~
~~80~~	~~81~~	~~82~~	~~83~~	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	~~89~~
~~90~~	~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	~~97~~	~~98~~	~~99~~
~~100~~	~~101~~	~~102~~	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

T: What does the red string tell us about the secret number?

S: The secret number is not a multiple of 3.

T: Which numbers are the multiples of 3?

S: The numbers we will meet if we start at 0 and count by threes (forward and backward).

T: We know that the secret number is not a multiple of 3. We have already crossed out "0". What is the next number in this chart which is a multiple of 3?

S: 3.

Cross out "3". Continue asking for the next larger multiple of 3 until the students realize that every third number is a multiple of 3. Cross them out on the chart as they are suggested. Then invite several students each to indicate several multiples of 3 and to cross them out on the 0-109 Numeral Chart. Commend any student who notices that every other multiple of 3 is an even (odd) number.

When all of the multiples of 3 between 1 and 50 have been crossed out, your chart should look like this one.

~~0~~	1	~~2~~	~~3~~	~~4~~	5	~~6~~	7	~~8~~	~~9~~
~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19
~~20~~	~~21~~	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29
~~30~~	31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~
~~40~~	41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49
~~50~~	~~51~~	~~52~~	~~53~~	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	~~59~~
~~60~~	~~61~~	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	~~67~~	~~68~~	~~69~~
~~70~~	~~71~~	~~72~~	~~73~~	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	~~79~~
~~80~~	~~81~~	~~82~~	~~83~~	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	~~89~~
~~90~~	~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	~~97~~	~~98~~	~~99~~
~~100~~	~~101~~	~~102~~	~~103~~	~~104~~	~~105~~	~~106~~	~~107~~	~~108~~	~~109~~

Third clue

T: Ms. Lockwood's third clue is: "The secret number is 1 less than a multiple of 5". Which of these numbers are 1 less than a multiple of 5?

If your students have difficulty with this clue, first ask for a multiple of 5 and then ask which number is 1 less than that multiple of 5. A student may observe that all the numbers which end in 4 or 9 are 1 less than a multiple of 5.

W 14

Continue this activity until your class concludes that the secret number is 19 or 29 or 49. Write these numbers on the board.

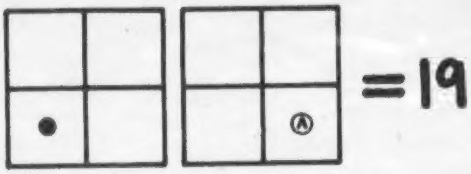
19 29 49

Fourth clue

Display two Minicomputer boards. Hold up one positive checker and one negative checker.

T: Ms. Lockwood's fourth clue is: "The secret number can be put on this Minicomputer with exactly one regular and one negative checker". Which of these numbers can we put on this Minicomputer with one regular and one negative checker?

Allow the students to tell you the secret number with a whisper. Ask for a volunteer to put the secret number on the Minicomputer with one regular and one negative checker.



T: 19 could be the secret number. Can we put 29 or 49 on the Minicomputer with one regular and one negative checker?

Allow students who think they can put either of these numbers on the Minicomputer to try. Perhaps a student will comment that "29 = 30 - 1 = 40 - 11" and "49 = 50 - 1" so neither of these numbers can be put on the Minicomputer with exactly one regular and one negative checker. Conclude that Ms. Lockwood's office is on the 19th floor.

W15 Summer School in the Old Days

CAPSULE LESSON SUMMARY

Read the Storybook SUMMER SCHOOL IN THE OLD DAYS and discuss the illustrations in it.

MATERIALS

Teacher:	One copy of the Storybook SUMMER SCHOOL IN THE OLD DAYS; Minicomputer set
Student:	One copy of the Storybook SUMMER SCHOOL IN THE OLD DAYS for each pair of students

DESCRIPTION OF LESSON

Read the Storybook SUMMER SCHOOL IN THE OLD DAYS with your class as you would read any other story. You may wish to read it aloud yourself or ask students to read it aloud. Most of the content of this storybook is presented in the illustrations, so it is important that the students have time to look at the pictures and to comment on them. Frequently ask students to describe what they see on a particular page of the storybook. The following suggestions as well as the text of the storybook will help you to guide the discussion of the illustrations.

Pages 3-7

Ask the students how many sticks each of the numbers pictured is carrying and how the sticks are grouped. Emphasize that some sticks are loose and some sticks are bundled in tens and in hundreds. For example, 354 is carrying three bundles with 100 sticks, five bundles with 10 sticks, and four loose sticks.

Pages 8-11

Ask the students to suggest number stories for each of the presentations by 12 and 45. Record the number stories on the board.

W 15

Pages 12 and 13

Determine the number of sticks the children stole from 74; then ask how many sticks 74 has left. Encourage the students to calculate this number rather than counting the sticks in the picture.

Pages 14-17

Ask the students to comment on the way 74 has arranged the sticks and to notice how the new sticks are stored. Discuss how 74 might arrange the new sticks which 0 has brought. Emphasize that 74 is 7 tens and 4 ones.

Pages 18-21

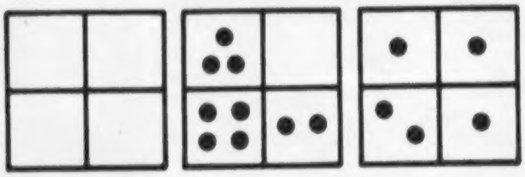
Ask the students what we call the flag of the World of Numbers. ("Minicomputer") Notice any other Minicomputers in the pictures and ask what numbers are displayed when there are checkers on them.

Pages 22 and 23

Encourage the students to explain why the numbers no longer carry any sticks and why 0 does not carry a bag of checkers.

Pages 24-27

Ask the students to suggest a number story for each of the presentations by 9. After discussing the first two pictures for 357, put this configuration on the Minicomputer using only red and yellow checkers.



NOTE: It would be preferable to use all red checkers, but your Minicomputer set does not contain enough red magnetic checkers; therefore, use some yellow checkers or borrow some red checkers from another teacher. Avoid using blue checkers in this configuration.

Invite someone to explain why the number is 357.

Pages 28-32

Encourage the students to think about what 0 did with the red and the blue checkers on page 32. Tell the students that they will learn more about 0's idea next week.

W 16

W16 Summer School: 0's Discovery

CAPSULE LESSON SUMMARY
Extend the ideas of a previous storybook in the Story-Workbook SUMMER SCHOOL: 0'S DISCOVERY

MATERIALS
Teacher: One copy of SUMMER SCHOOL: 0'S DISCOVERY;
Student: SUMMER SCHOOL: 0'S DISCOVERY; colored pencils

DESCRIPTION OF LESSON

The Story-Workbook SUMMER SCHOOL: 0'S DISCOVERY is a continuation of the Storybook SUMMER SCHOOL IN THE OLD DAYS. At the end of SUMMER SCHOOL IN THE OLD DAYS, a boy put a bag of red checkers and blue checkers in 0's lunch box; these checkers gave 0 an idea.

After a brief discussion of SUMMER SCHOOL IN THE OLD DAYS, distribute copies of the story-workbook. Allow the students a few minutes to read it silently and encourage them to try to figure out 0's idea. Ask the students not to write in the story-workbook at this time.

Pages 2-5

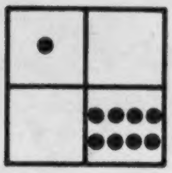
Read, or ask students to read, pages 2 and 3 aloud. Then discuss the configurations for 0 on pages 4 and 5; a sample dialogue follows.

T: Let's look carefully at these posters. What do you notice about the first poster 0 is showing us?

S: There is a red checker and a blue checker on each square.

331

T: Isn't it interesting that 0 did that! Let's look at the next poster. What number is on the Minicomputer with red checkers?



S: 8.

T: . . . with blue checkers?

S: 8.

In the same manner consider each of the other two posters.

T: What is 0's idea? How is 0 using the checkers?

S: 0 is using one color for regular checkers and the other color for negative checkers.

T: That's right! In this story-workbook, red checkers are used to show positive numbers and blue checkers are used to show negative numbers.

Invite students to show other configurations for 0 using red (positive) checkers and blue (negative) checkers on the demonstration Minicomputer.

Pages 6 and 7

After reading these pages aloud, discuss why 200 and 16 are angry with 0.

W 16

Pages 8-11

After reading page 8, ask students to explain why the poster shows a configuration for 1. Allow some time for the students to show (on page 9) several ways to put 1 on the Minicomputer. Let students take turns putting their "pictures" of 1 on the demonstration Minicomputer.

NOTE: Some students may prefer to use negative checkers " Ⓐ " instead of blue checkers to indicate negative numbers on the Minicomputer. Allow this.

When a few students have completed page 9, continue reading on page 10. Discuss the posters prepared by 2 and 3; then ask the students to design another "poster" for each of those numbers using red checkers and blue checkers. As students find solutions, ask them to show their "pictures" on the demonstration Minicomputer.

Pages 12 and 13

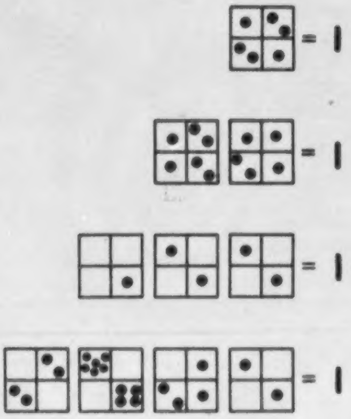
Read, or ask a student to read these pages aloud. Allow about five minutes for the students to work on page 13. Emphasize that the students are to put each of the numbers on the Minicomputer using exactly one red checker and one blue checker. Invite students to show their solutions on the demonstration Minicomputer.

Pages 14 and 15

Finish reading the story-workbook. Ask the class why the numbers formed a circle around the children and cheered. If necessary, remind them that the boy near 23 ("your father" in this story) put the bag of red checkers and blue checkers in 0's lunch box.

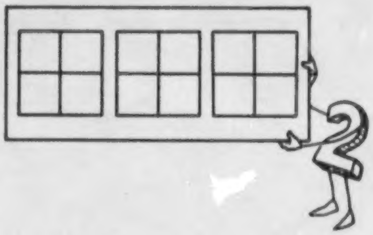
Summer School - 0's Discovery

Show 1 in other ways with red (positive) checkers and blue (negative) checkers.

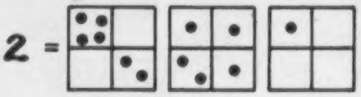


Many other solutions are possible.

"Here I am," said 2, by holding up this poster.

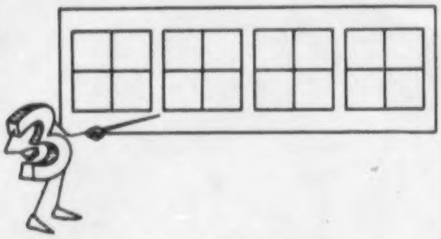


Show 2 in another way with red (positive) checkers and blue (negative) checkers.

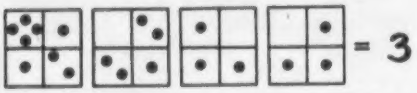


Many other solutions are possible.

"It's fun showing myself with more than three checkers," said 3, pointing to this picture.

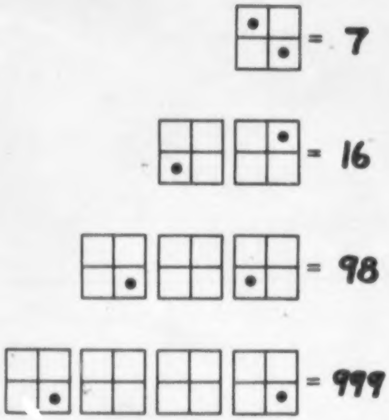


Show 3 in another way with red (positive) checkers and blue (negative) checkers.



Many other solutions are possible

Show each of these numbers with one red (positive) checker and one blue (negative) checker.



W 17

W17 Detective Story #6

CAPSULE LESSON SUMMARY

Solve a detective story in order to find the floor on which you must change elevators in the Empire State Building. The clues involve an arrow picture, a string picture, and the Minicomputer.

MATERIALS

Teacher: Colored chalk; Minicomputer set
Student: None

DESCRIPTION OF LESSON

NOTE: The first exercise of this lesson gives a preliminary exploration of ideas which will be used in the detective story, Exercise 2.

Exercise 1

Begin this lesson with about five minutes of mental arithmetic involving multiples of 4. Try to maintain an atmosphere of exploration rather than turn this activity into a drill. If the students lose interest, go on to Exercise 2.

T: What are some numbers which are multiples of 4?

Encourage the students to name many multiples of 4, including negative numbers and relatively large numbers such as 200 and 400. Occasionally ask what number multiplied by 4 would equal the number which has been suggested. This would be especially appropriate in the following situations:

i) when the suggested number is a multiple of 4 between 4 and 20;

ii) when the number suggested is: 40; 400; 4,000; 4,000,000; . . . ;

Whenever possible, if the suggested number is not a multiple of 4, explain how

338

you know this number is not a multiple of 4. A sample dialogue is given on the next two pages.

S: 16.

T: 4 times what number is 16? (4)
4 × 4 = 16, so 16 is a multiple of 4.

S: 40.

T: 4 times what number is 40? (10)
4 × 10 = 40, so 40 is a multiple of 4.

S: 44.

T: How do you know that 44 is a multiple of 4?

S: I just added 4 to 40.

S: 100.

T: 4 times what number is 100? (25)
4 × 25 = 100, so 100 is a multiple of 4.

S: 200.

T: Yes, 200 is a multiple of 4 because 200 = 100 + 100.

S: 300.

T: 300 is a multiple of 4 because 300 = 100 + 100 + 100.

S: 50.

W 17

T: Is 50 a multiple of 4? We know that 40 is a multiple of 4. If we start at 40 and keep adding 4, will we meet 50? (No) What number is 40 + 4? (44) 44 + 4 = . . . ? (48) 48 + 4 = . . . ? (52). We skipped 50, so 50 is not a multiple of 4.

S: 816.

T: 800 is a multiple of 4 and 16 is a multiple of 4, so you're correct. 816 is a multiple of 4.

S: 32.

T: 32? Is 32 a multiple of 4? We know that 20 is a multiple of 4. What should we add to 20 to get 32? (12) 20 + 12 = 32. Is 12 a multiple of 4? (Yes) 20 and 12 are both multiples of 4 and 20 + 12 = 32, so 32 is a multiple of 4.

T: What are some large numbers which are multiples of 4?

S: 1,000,000.

S: 4,000,000.

T: 4 times what number equals 4,000,000? (1,000,000)

S: 1,000,004.

T: What are some small numbers which are multiples of four?

S: 4.

T: What times 4 is equal to 4?

S: 1.

T: 1 x 4 = 4; 4 is a multiple of four.

S: $\hat{4}$.

S: $\widehat{44}$.

Exercise 2: Detective Story

Ask the students what they remember about the Empire State Building. If necessary, remind the students that the Empire State Building has 102 floors above the ground level.

T: There is an observation area on floor 102. Many tourists visit this observation area because from there they have a marvelous view of New York City. If you start at ground level and ride the elevator to floor 102, how many floors will you go up? (102) Try to imagine going up 102 floors on an elevator.

Allow the class to discuss this briefly.

T: But the Empire State Building does not have an elevator which goes all the way from floor 0 to floor 102. People who wish to go all the way to the 102nd floor must change elevators.

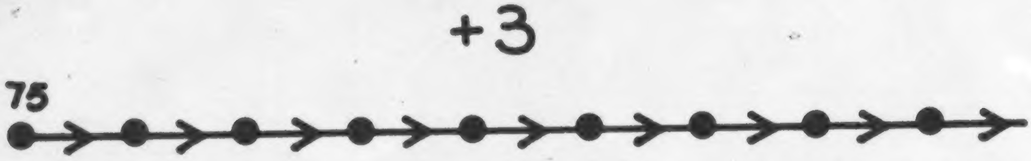
Allow a brief discussion. Be sure the class understands that to go to the 102nd floor you need to get off one elevator and to get on another one.

T: Do you know the number of the floor in the Empire State Building where you must change elevators? You can find out which floor it is by first solving this detective story for a secret number. When you know the secret number, add 2 and you will know the number of the floor where you need to change elevators.

W 17

First clue

Draw the following arrow picture on the board. The last arrow on the right should appear to go off the board.



T: The secret number is in this arrow picture.

Ask students to label the dots until the arrow picture is complete. Point to the arrow on the extreme right.

T: Why do you think this arrow has no ending point?

Be sure the students understand that this arrow road keeps on going.

T: Let's make a list of all the numbers we meet on this +3 arrow road.

Progressively make a list on the board. When you reach the last visible dot along this road, ask the class to imagine that the arrow road continues. Students may want to stop the list when you reach 102 because the Empire State Building only has 102 floors. Accept this suggestion.

If it is not suggested, stop the list when you reach 105. Ask the students if 105 could be the secret number. Someone should suggest that 105 cannot be the secret number because the Empire State Building has only 102 floors. Otherwise, suggest it yourself. Erase "105".

T: Could the secret number be 102? Remember that we must add 2 to the secret number to know the number of the floor where people change elevators.

S: No, because 102 + 2 = 104 and there is no floor 104.

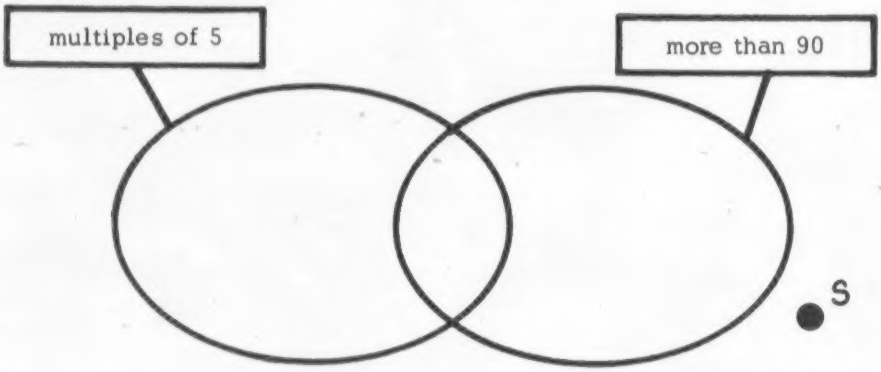
Erase "102". You should have this list on the board.

75, 78, 81, 84, 87, 90, 93, 96, 99

NOTE: Some classes may suggest erasing "99" because there would be no need for an elevator which starts on floor 101. Accept this suggestion, but do not suggest it yourself.

Second clue

Draw the following string picture on the board.



T: What is the blue string for? (Multiples of 5)
What are some multiples of 5?

Accept three or four correct answers.

T: What is the red string for? (More than 90)
What are some numbers which are more than 90?

Accept three or four correct answers.

W 17

T: This dot (point to "s") is for the secret number. What can you tell me about the secret number?

S: It is not more than 90.

S: It is not a multiple of 5.

T: Which of these numbers in our list could be the secret number?

For each number a student suggests, determine where that number would be in the string picture and either cross it off the list if it cannot be the secret number or circle it if it could be the secret number. Continue until all the numbers in your list have been considered.

~~75~~, 78, 81, 84, 87, ~~90~~, ~~93~~, ~~96~~, ~~99~~

T: Now we know that the secret number is 78 or 81 or 84 or 87.

Erase everything in the list which has been crossed off.

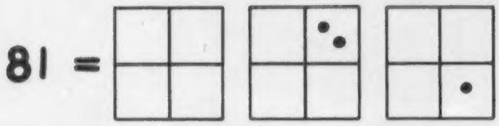
Third clue

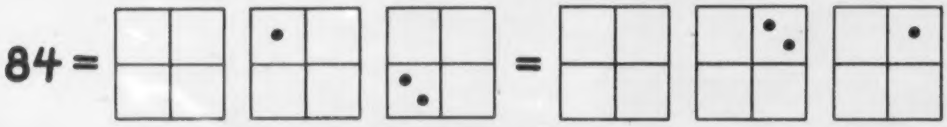
Display three Minicomputer boards.

T: The secret number can be put on the Minicomputer with exactly three regular checkers. Using this information, tell me which of these numbers could be the secret number.

78, 81, 84, 87

When students claim to be able to put a number on the Minicomputer, ask them to do so. Your students should conclude that it is impossible to put 78 or 87 on the Minicomputer with exactly three regular checkers. 81 and 84 can be put on the Minicomputer with three regular checkers.





T: So the secret number is either 81 or 84.

Fourth clue

T: The secret number is a multiple of 4.
Does this new clue help?

Ask the students to write the secret number on a piece of paper when they know it. Check some of their answers.

T: What is the secret number?

S: 84.

T: 80 is a multiple of 4, so 84 is also a multiple of 4.
Since 80 is a multiple of 4, can 81 also be a multiple of 4? (No)
Do you know which floor the special elevator is on?

S: The 86th floor.

W 17

If necessary, remind the students that the secret number plus 2 equals the number of the floor where people change elevators.

T: The elevator to the observation area starts on floor 86 and goes to floor 102. How many floors does it travel?

S: Sixteen.